Effective Density Dependence of Nuclear Interaction
in The Relativistic Hadron Field Theory

ME | Kristian Petrík

SUPERVISOR | Ing. Štefan Gmuca, CSc.

Zimányi 2012

3-7 December 2012

Motivation

Compact Stars

Ground state of dense matter

What is the **composition** of a compact star ?

Any problems ?

Indirect kind of observations – we can't have a look inside the star

Compact stars are an application of **QCD**

strong coupling nature

Impossible to derive a low energy hadronic **Lagrangian** from **QCD**

Density dependence of nuclear interaction – **under construction**

Solution !

Effective hadron field theory

Blueprint

$$\mathcal{L} = \mathcal{L}^{fermi} + \mathcal{L}^{bose} + \mathcal{L}^{int}$$

.DENSITY DEPENDENCE.

NON-LINEAR HFT

$$\mathcal{L}^{int} = \bar{\psi}_B [g_{\sigma B}\hat{\sigma} + g_{\sigma_s B}\hat{\sigma}_s + g_{\delta B}\boldsymbol{\tau}_B \cdot \hat{\boldsymbol{\delta}} - g_{\omega B}\hat{\omega}_\mu \gamma^\mu - g_{\phi B}\hat{\phi}_\mu \gamma^\mu - g_{\rho B}\boldsymbol{\tau}_B \cdot \hat{\boldsymbol{\rho}}_\mu \gamma^\mu]\psi_B$$

$$\mathcal{L}^{selfint} = -\frac{1}{3} b_\sigma m_N (g_{\sigma N}\hat{\sigma})^3 - \frac{1}{4} c_\sigma (g_{\sigma N}\hat{\sigma})^4 + \frac{1}{4} c_\omega (g^2_{\omega N}\hat{\omega}_\mu \hat{\omega}^\mu)^2 + \frac{1}{4} c_\rho (g^2_{\rho N}\hat{\boldsymbol{\rho}}_\mu \cdot \hat{\boldsymbol{\rho}}^\mu)^2 + \frac{1}{2}\Lambda_{vv}(g^2_{\rho N}\hat{\boldsymbol{\rho}}_\mu \cdot \hat{\boldsymbol{\rho}}^\mu)(g^2_{\omega N}\hat{\omega}_\mu \hat{\omega}^\mu) + \frac{1}{2}\Lambda_{vs}(g^2_{\rho N}\hat{\boldsymbol{\rho}}_\mu \cdot \hat{\boldsymbol{\rho}}^\mu)(g^2_{\sigma N}\hat{\sigma}^2),$$

$$g_{\sigma B}\hat{\sigma}$$

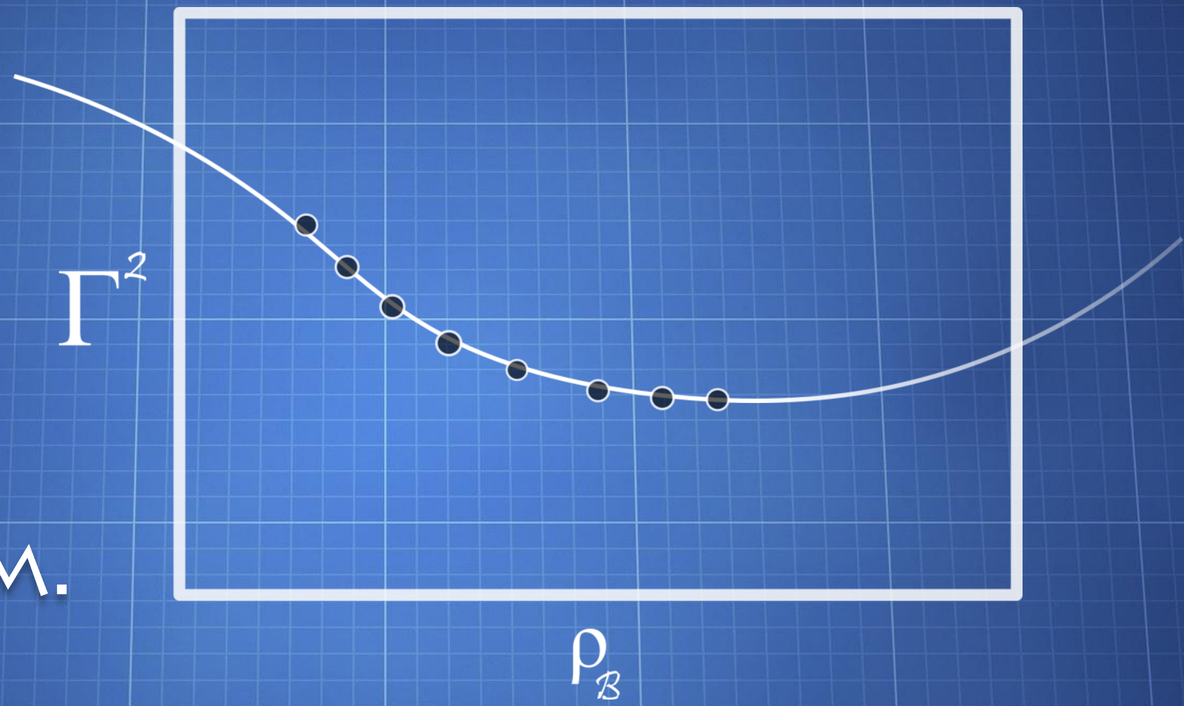
DENSITY DEPENDENT HFT

$$\mathcal{L}^{int} = \bar{\psi}_B [\hat{\Gamma}_{\sigma B}\hat{\sigma} + \hat{\Gamma}_{\sigma_s B}\hat{\sigma}_s + \hat{\Gamma}_{\delta B}\boldsymbol{\tau}_B \cdot \hat{\boldsymbol{\delta}} - \hat{\Gamma}_{\omega B}\hat{\omega}_\mu \gamma^\mu - \hat{\Gamma}_{\phi B}\hat{\phi}_\mu \gamma^\mu - \hat{\Gamma}_{\rho B}\boldsymbol{\tau}_B \cdot \hat{\boldsymbol{\rho}}_\mu \gamma^\mu - e\hat{Q}_B \hat{A}_\mu \gamma^\mu]\psi_B$$

$$\hat{\Gamma}_{\sigma B}\hat{\sigma}$$

.INTERACTION.

Blueprint



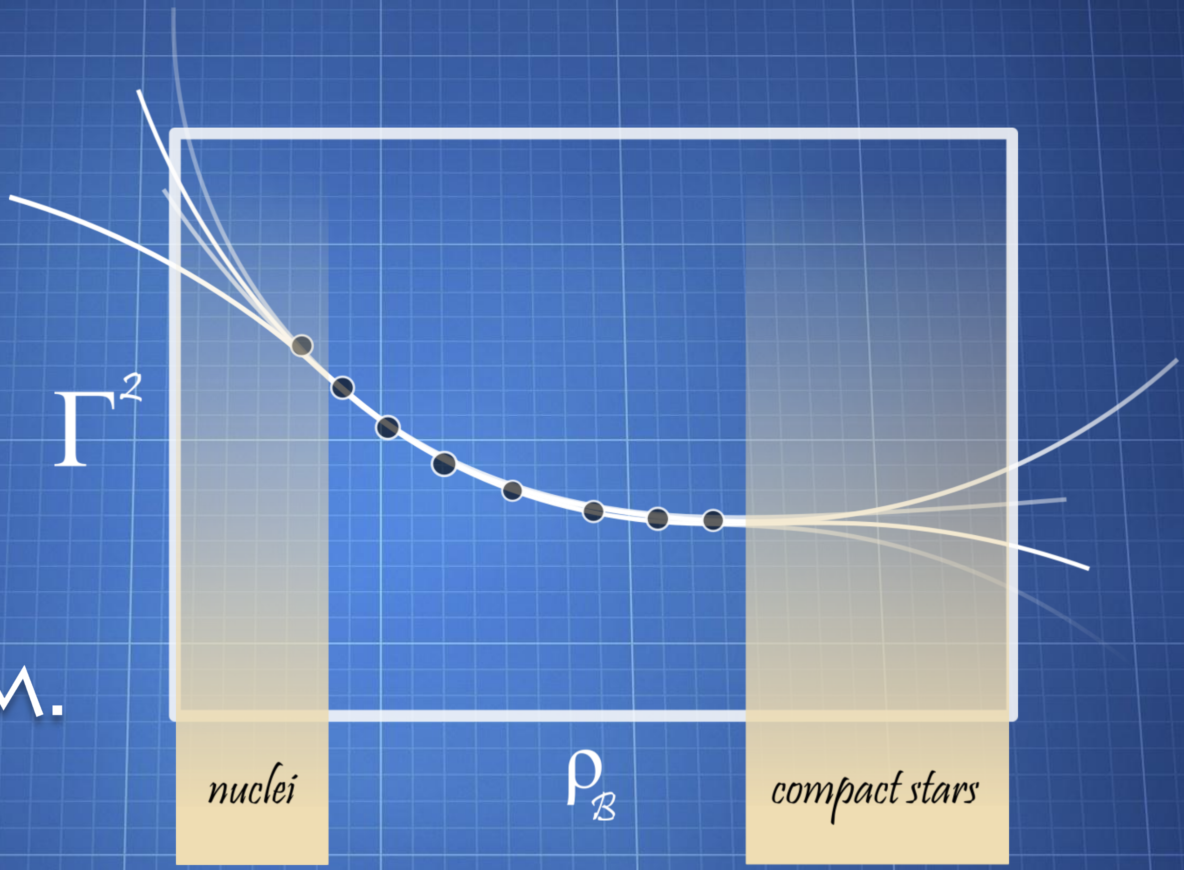
.Problem.

Γ - density dependent vertex

ρ_B - baryon density

● - *DBHF data*

Blueprint



.Problem.

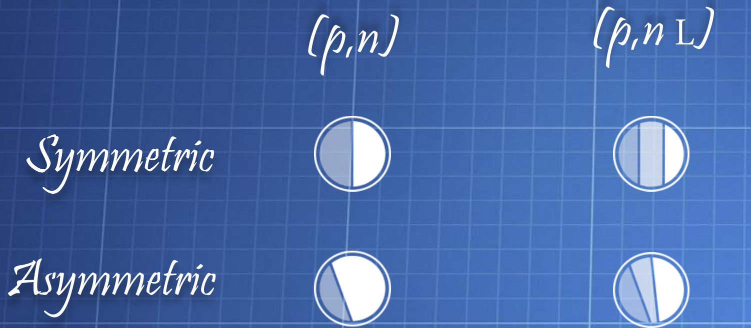
Γ - density dependent vertex

ρ_B - baryon density

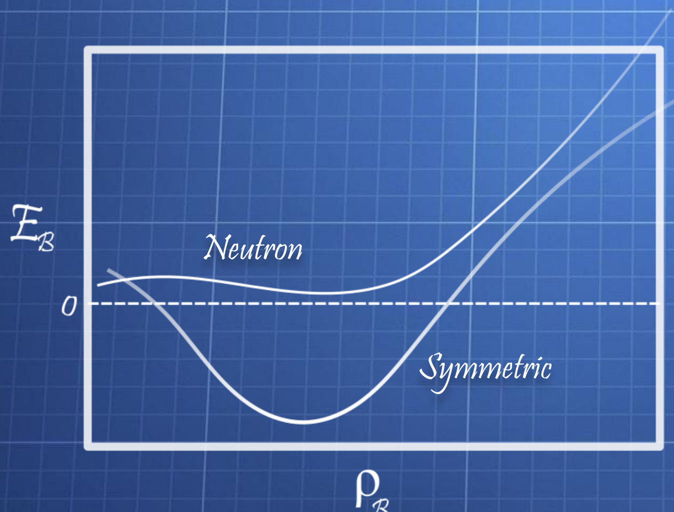
● - DBHF data

Blueprint

Nuclear Lambda

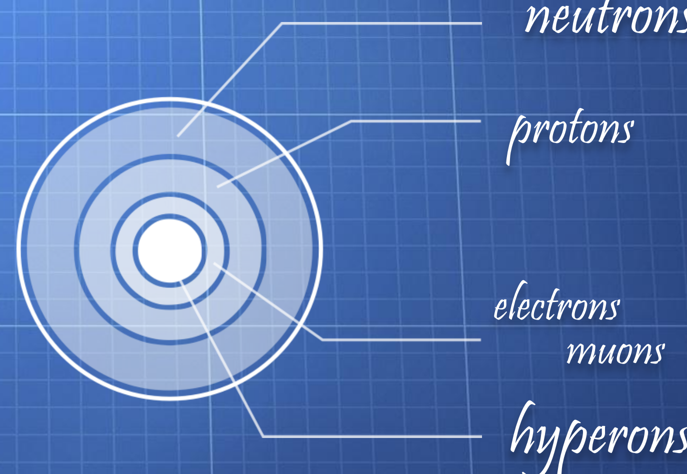


Equation of State



.To do list.

Compact Star Composition



Model

.Density dependence of vertex functions.

Typical choice
(density dependence)

$$\Gamma_i(\rho_B) = a_i \left[\frac{1 + b_i(\rho_B/\rho_{sat} + d_i)^2}{1 + c_i(\rho_B/\rho_{sat} + e_i)^2} \right],$$

5 par

$$x = \frac{\rho_B}{\rho_{sat}}$$

4 par

4–parametric class

$$^{A}\Gamma_i^{\bullet\bullet}(x) = a_i \left[\frac{c_i + (x + d_i)^2}{e_i + (x)^2} \right],$$

$$^{B}\Gamma_i^{\bullet\bullet}(x) = a_i \left[\frac{c_i + (x + d_i)^{1/2}}{e_i + (x)^{1/2}} \right],$$

3 par

3–parametric class

$$^{C}\Gamma_i^{\circ\bullet}(x) = a_i \left[\frac{1 + (x + d_i)^2}{(x + c_i)^2} \right],$$

$$^{D}\Gamma_i^{\circ\bullet}(x) = a_i \left[\frac{1 + (x + d_i)^2}{c_i + (x)^2} \right],$$

$$^{E}\Gamma_i^{\circ\bullet}(x) = a_i \left[\frac{1 + (x + d_i)^2}{c_i + (x + c_i)^2} \right],$$

2–parametric class

$$^{I}\Gamma_i^{\circ\circ}(x) = a_i \left[\frac{\beta_i + x^2}{b_i + x^2} \right],$$

$$^{II}\Gamma_i^{\circ\circ}(x) = a_i \left[\frac{\beta_i + x^{3/2}}{b_i + x^{3/2}} \right],$$

$$^{I}\tilde{\Gamma}_i^{\circ\circ}(x) = a_i \left[\frac{\beta_i + x^2}{b_i + x^2} \right]^{-1},$$

$$^{II}\tilde{\Gamma}_i^{\circ\circ}(x) = a_i \left[\frac{\beta_i + x^{3/2}}{b_i + x^{3/2}} \right]^{-1},$$

New functions
(density dependence)

2 free parameters
1 beta factor

$$^{I}\Gamma_\delta^{\circ\bullet}(x) = a_\delta \left[\frac{\beta_\delta + (x + c_\delta)^2}{b_\delta + x^2} \right],$$

$$^{II}\Gamma_\delta^{\circ\bullet}(x) = a_\delta \left[\frac{\beta_\delta + (x + c_\delta)^{3/2}}{b_\delta + x^{3/2}} \right].$$

Model

.DENSITY DEPENDENT HADRON FIELD THEORY.

Lagrangian density
(baryons and mesons)

$$\mathcal{L} = \mathcal{L}^{fermi} + \mathcal{L}^{bose} + \mathcal{L}^{int}$$

$$\mathcal{L}^{fermi} = \bar{\psi}_B \left[i\gamma^\mu \partial_\mu - m_B \right] \psi_B + \bar{\psi}_{e^-} \left[i\gamma^\mu \partial_\mu - m_{e^-} \right] \psi_{e^-} + \bar{\psi}_{\mu^-} \left[i\gamma^\mu \partial_\mu - m_{\mu^-} \right] \psi_{\mu^-}$$

$$\begin{aligned}\mathcal{L}^{bose} = & \frac{1}{2}\partial_\mu \hat{\sigma}\partial^\mu \hat{\sigma} - \frac{1}{2} m_\sigma^2 \hat{\sigma}^2 - \frac{1}{4}\hat{\omega}_{\mu\nu}\hat{\omega}^{\mu\nu} + \frac{1}{2} m_\omega^2 \hat{\omega}_\mu \hat{\omega}^\mu \\ & + \frac{1}{2}\partial_\mu \hat{\boldsymbol{\delta}}\partial^\mu \hat{\boldsymbol{\delta}} - \frac{1}{2} m_\delta^2 \hat{\delta}^2 - \frac{1}{4}\hat{\boldsymbol{\rho}}_{\mu\nu}\cdot\hat{\boldsymbol{\rho}}^{\mu\nu} + \frac{1}{2} m_\rho^2 \hat{\boldsymbol{\rho}}_\mu \cdot \hat{\boldsymbol{\rho}}^\mu \\ & + \frac{1}{2}\partial_\mu \hat{\sigma}_s \partial^\mu \hat{\sigma}_s - \frac{1}{2} m_{\sigma_s}^2 \hat{\sigma_s}^2 - \frac{1}{4}\hat{\phi}_{\mu\nu}\hat{\phi}^{\mu\nu} + \frac{1}{2} m_\phi^2 \hat{\phi}_\mu \hat{\phi}^\mu \\ & - \frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{int} = & \bar{\psi}_B [\hat{\Gamma}_{\sigma B}\hat{\sigma} + \hat{\Gamma}_{\sigma_s B}\hat{\sigma}_s + \hat{\Gamma}_{\delta B}\boldsymbol{\tau}_B \cdot \hat{\boldsymbol{\delta}} \\ & - \hat{\Gamma}_{\omega B}\hat{\omega}_\mu \gamma^\mu - \hat{\Gamma}_{\phi B}\hat{\phi}_\mu \gamma^\mu - \hat{\Gamma}_{\rho B}\boldsymbol{\tau}_B \cdot \hat{\boldsymbol{\rho}}_\mu \gamma^\mu - e\hat{Q}_B \hat{A}_\mu \gamma^\mu]\psi_B\end{aligned}$$

Baryon octet
(2 nucleons + 6 hyperons)

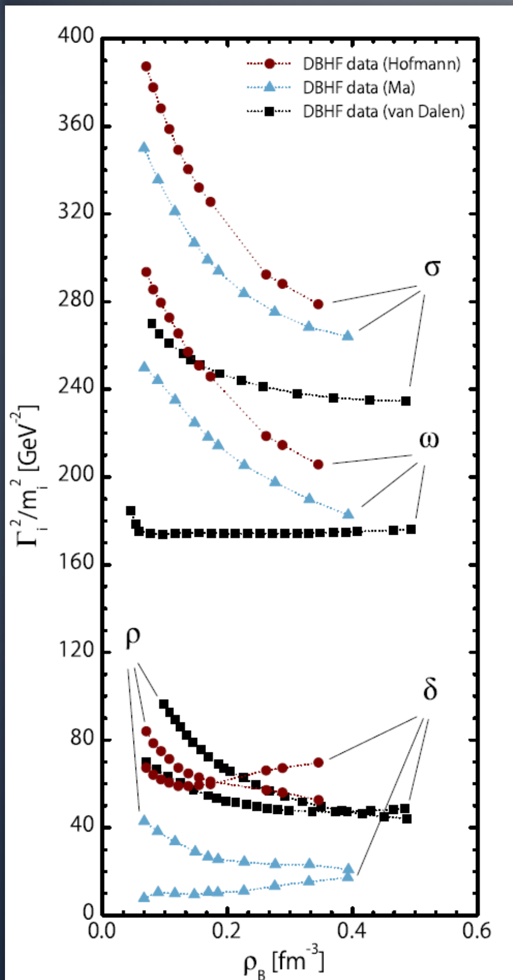
$$\psi_B = \begin{pmatrix} \psi_{\mathbf{N}} \\ \psi_{\boldsymbol{\Lambda}} \\ \psi_{\boldsymbol{\Sigma}} \\ \psi_{\boldsymbol{\Xi}} \end{pmatrix} = (\psi_{\mathbf{N}}, \psi_{\boldsymbol{\Lambda}}, \psi_{\boldsymbol{\Sigma}}, \psi_{\boldsymbol{\Xi}})^T,$$

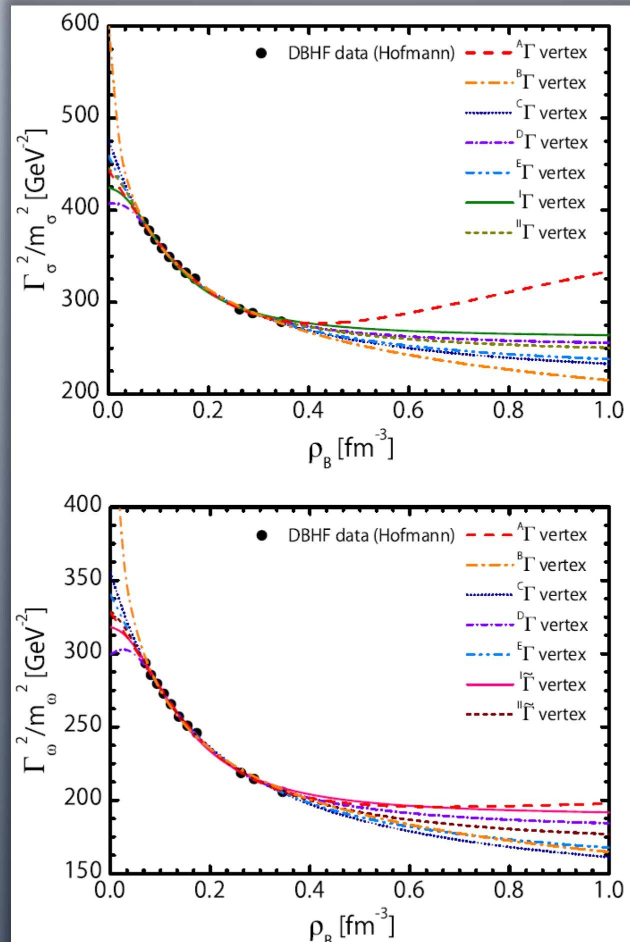
Density dependent couplings
(DD vertex functionals)

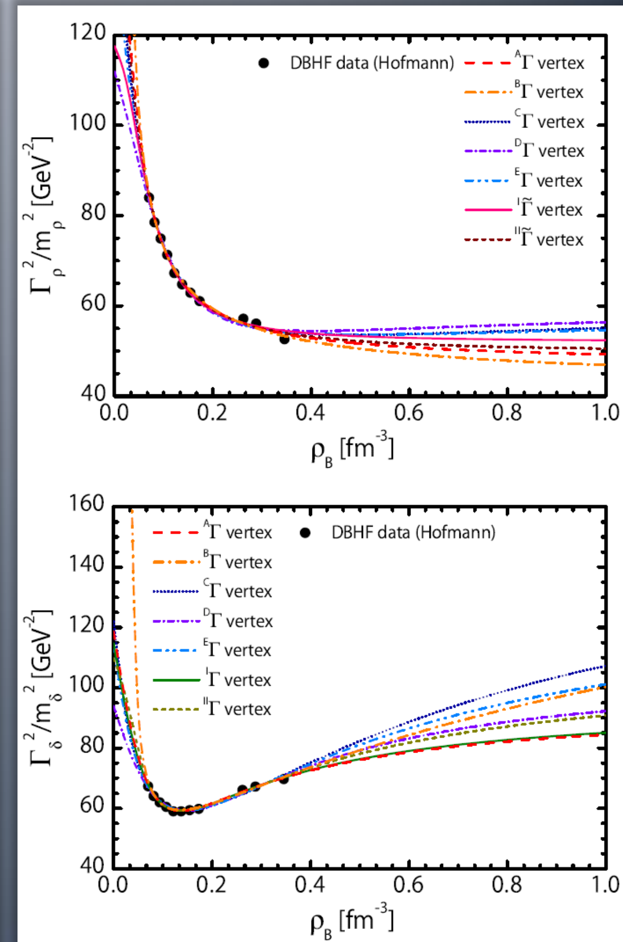
$$\hat{\Gamma}_{\alpha B}(\hat{\rho}_B) \qquad [\alpha = (\sigma, \omega, \rho, \delta, \sigma_s, \phi), B = (N, \Lambda, \Sigma, \Xi)] .$$

Results

.FITS OF VERTEX FUNCTIONALS.







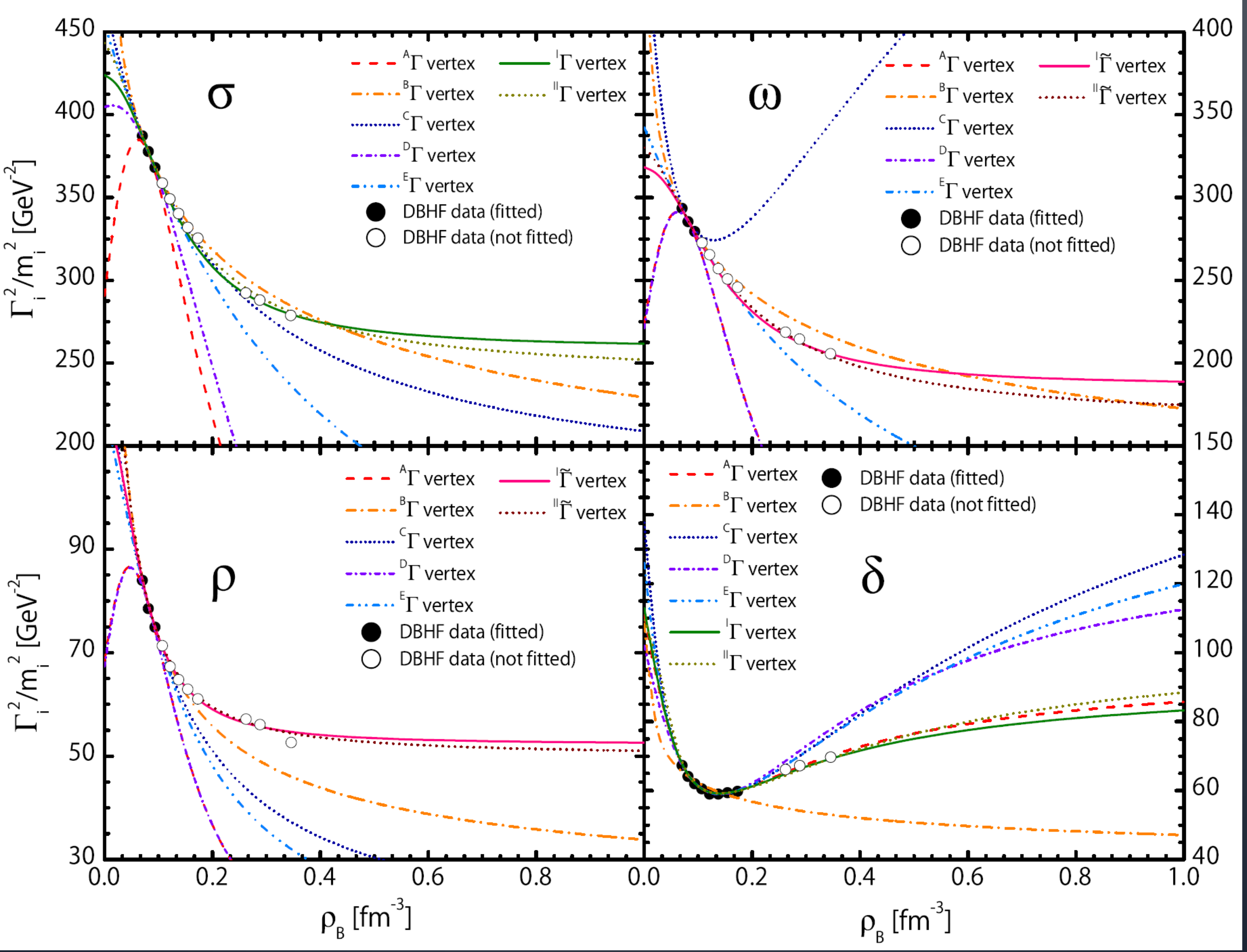
Density dependent hadron field theory for asymmetric nuclear matter and exotic nuclei

F. Hofmann, C. M. Keil, H. Lenske

Phys. Rev C 64 (2001) 034314

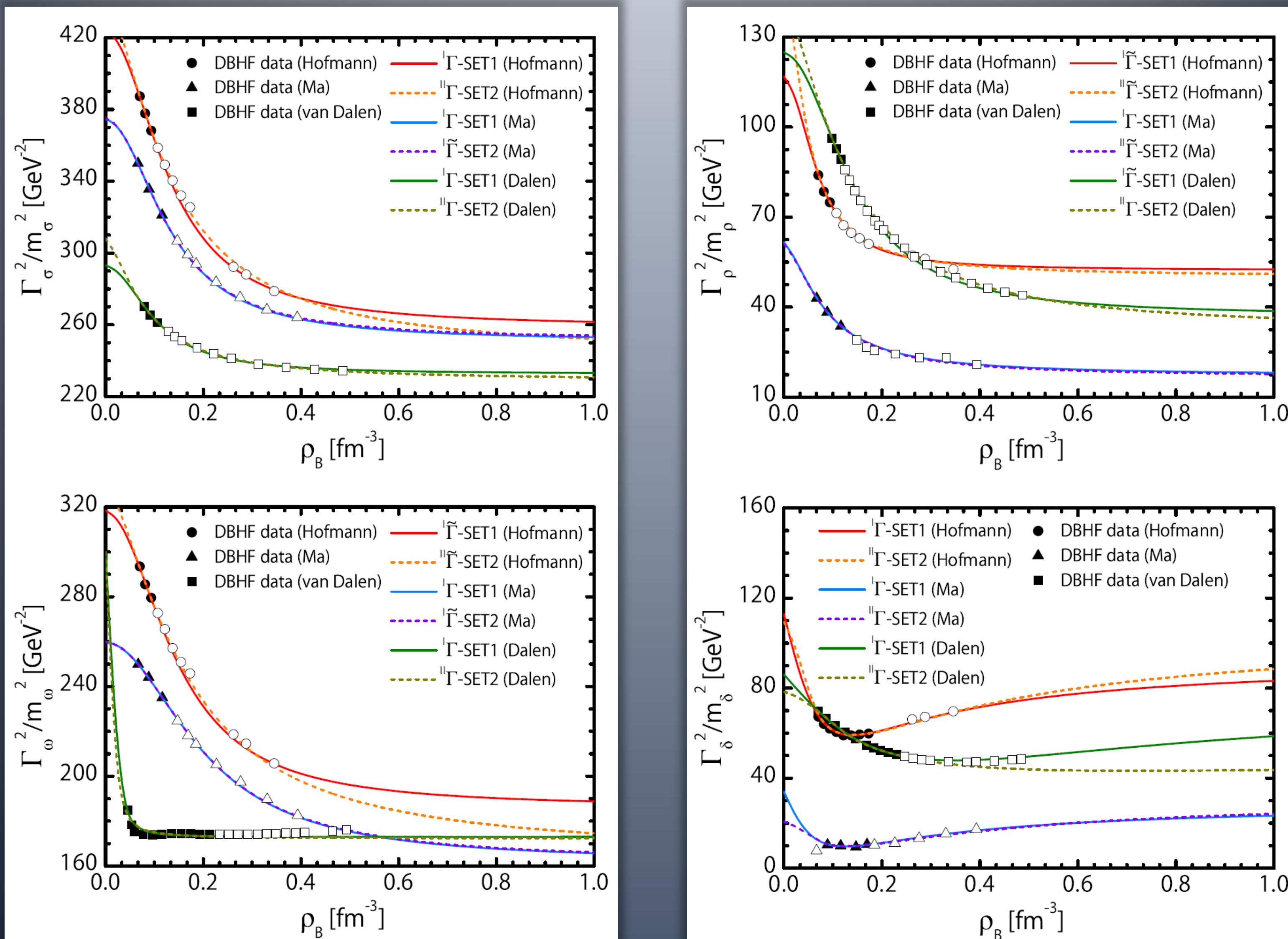
Results

.Partial fits.



Results

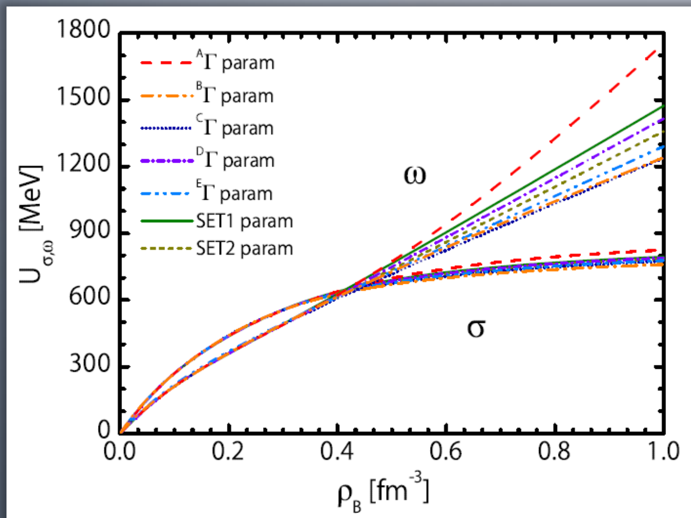
.PARTIAL FITS.



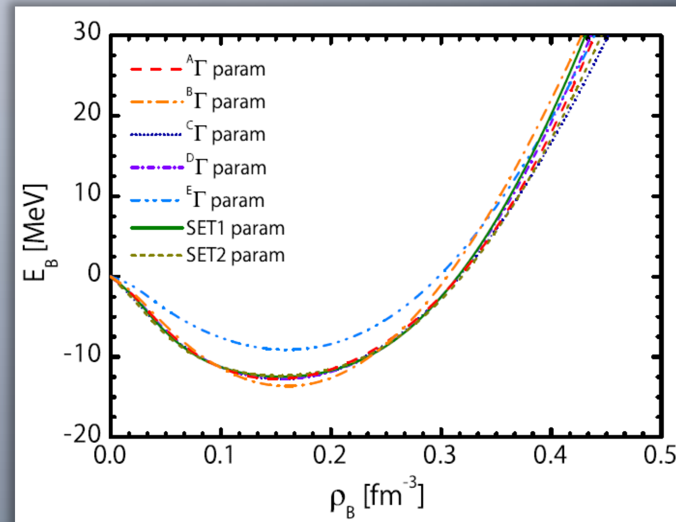
Results

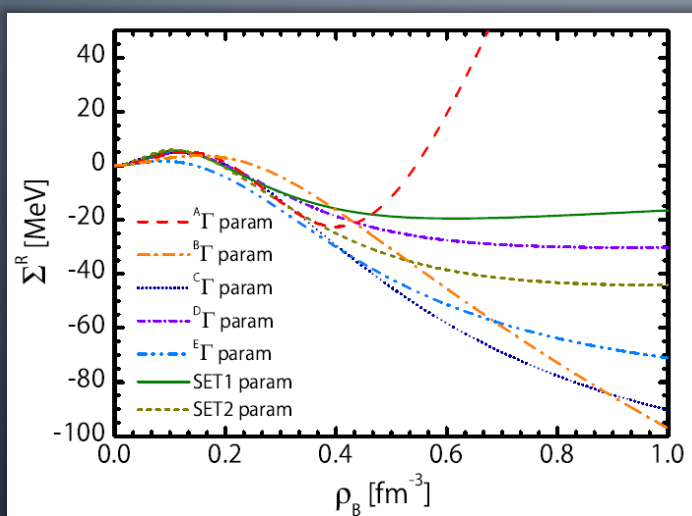
.NUCLEAR MATTER PROPERTIES.

POTENTIALS

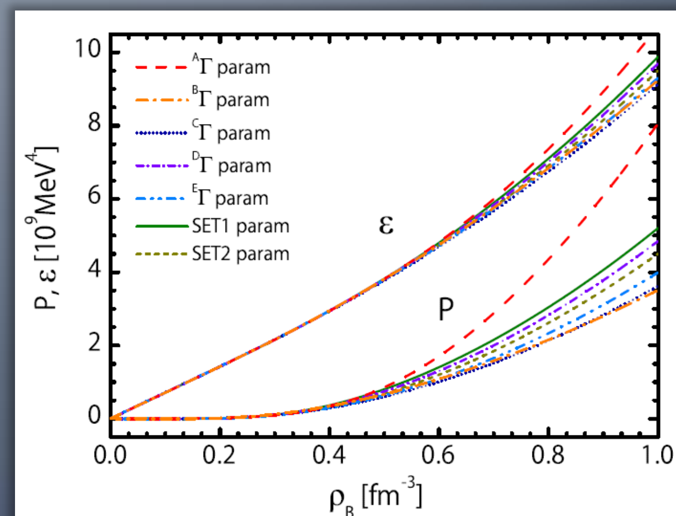


BINDING ENERGY





REARRANGEMENT

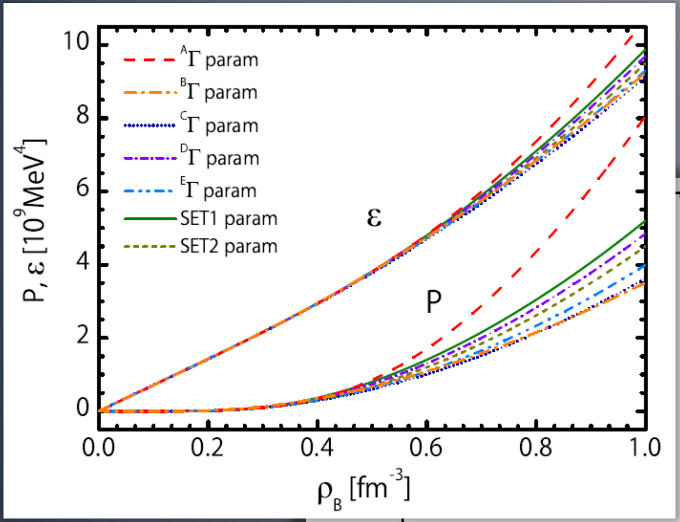


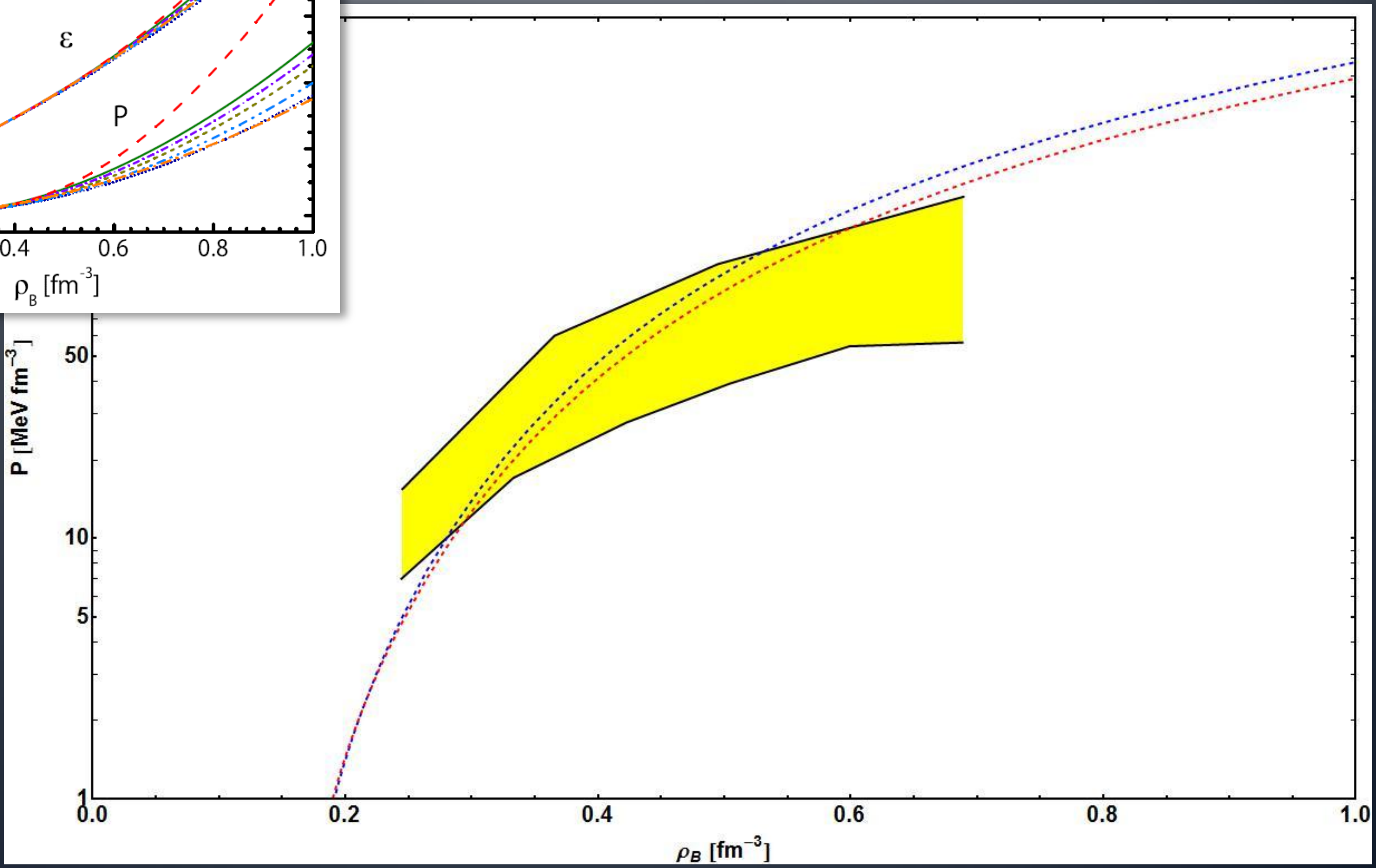
EQTN. OF STATE

Results

.constraints from HIC.

Eqtn. of state



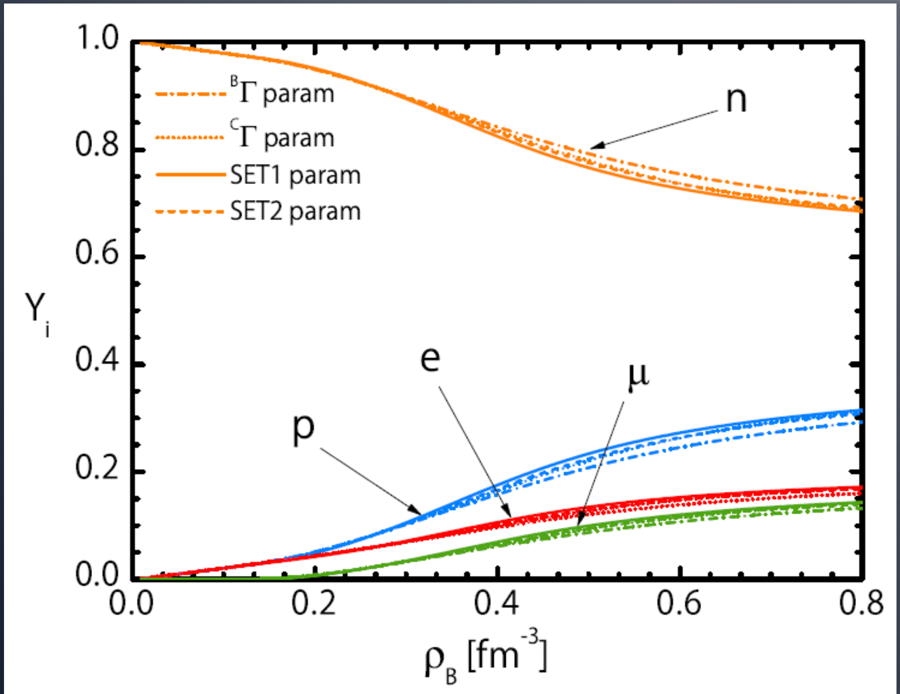


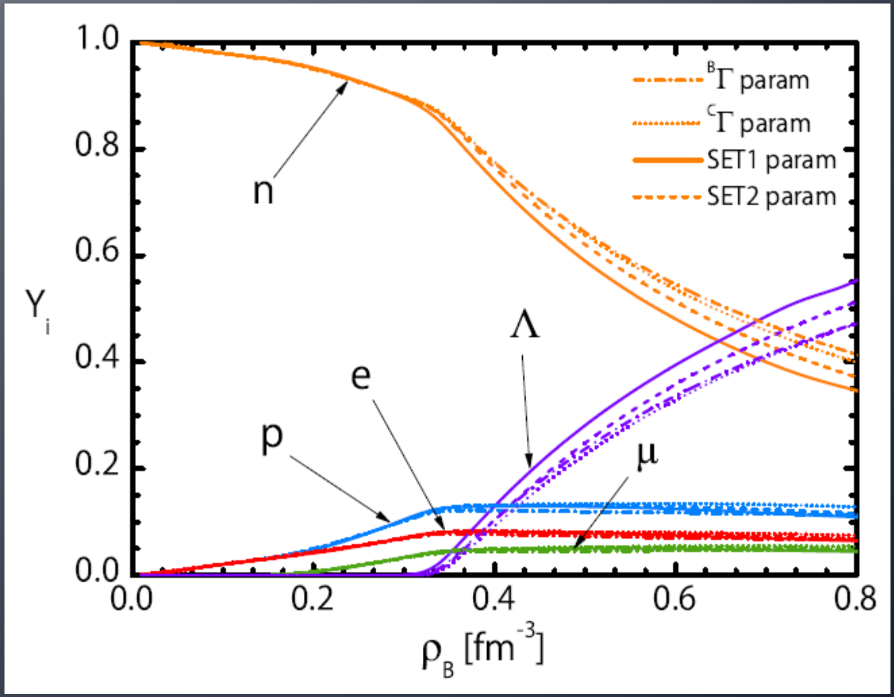
Results

.BETA EQUILIBRIUM.

NUCLEAR MATTER
(NUCLEONS)

LAMBDA MATTER
(LAMBDA HYPERON)

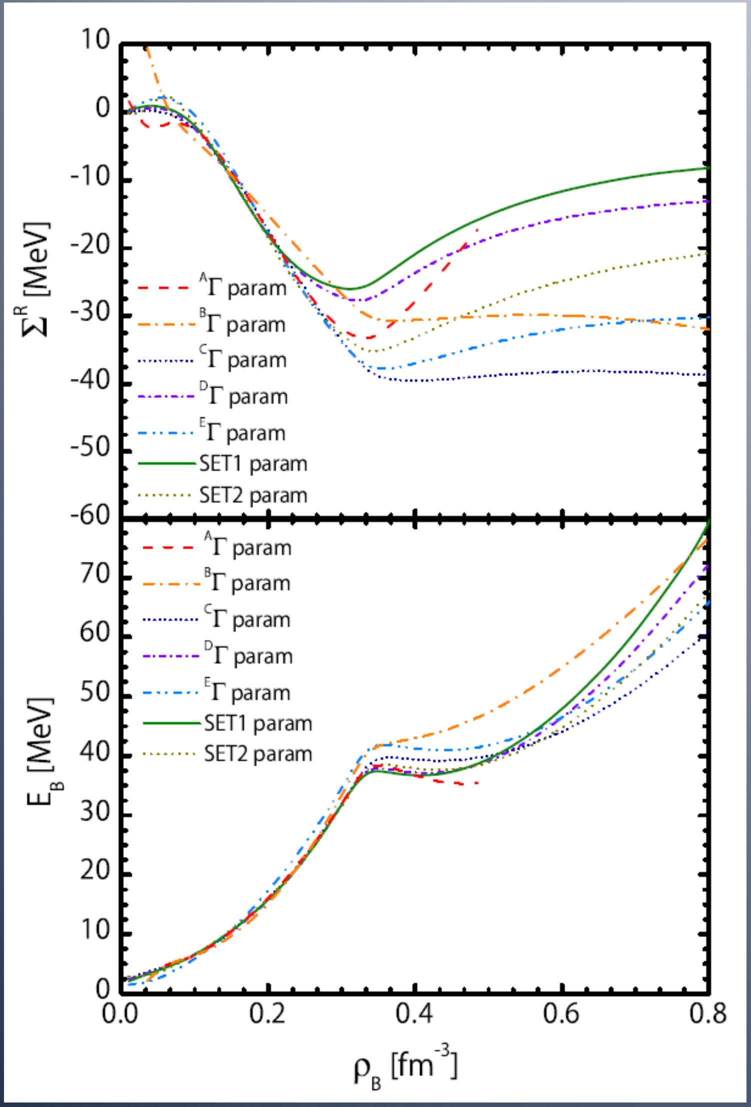




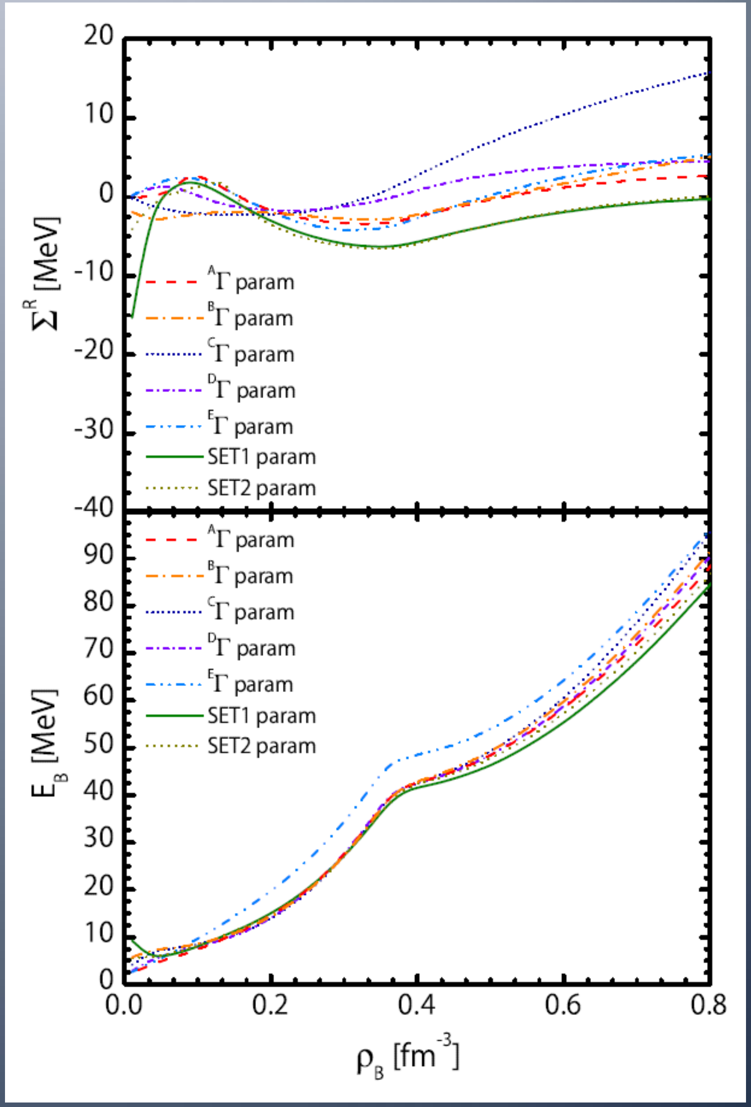
Results

.BETA EQUILIBRIUM.

Hofmann et al.



van Dalen et al.

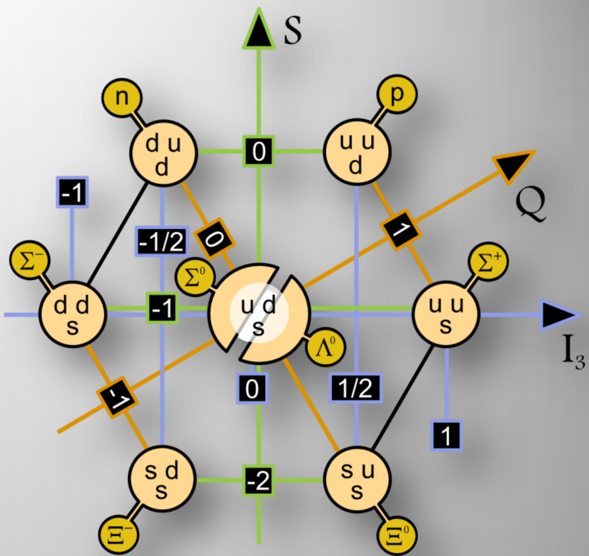


Conclusions

Compact Stars

Full baryonic octet - hyperons

What is the **composition** of a hyperon star and **how does it depend on the density** ?



Density dependence

New reliable **DBHF data**

Exact Hartree-Fock density dependence

Constraints from **astrophysics** or **heavy-ion collisions physics**

Thank you !

MODEL

.QUANTUM FIELD EQUATIONS.

EQUATION OF MOTION
(SPIN 0 AND SPIN 1 MESONS)

The Klein–Gordon equations for the meson fields **(spin 0)** are

$$\hat{\Gamma}_{\sigma B}\bar{\psi}_B\psi_B = (\Box + m_\sigma^2)\hat{\sigma},$$
$$\hat{\Gamma}_{\sigma_s B}\bar{\psi}_B\psi_B = (\Box + m_{\sigma_s}^2)\hat{\sigma}_s,$$
$$\hat{\Gamma}_{\delta B}\bar{\psi}_B\boldsymbol{\tau}_B\psi_B = (\Box + m_\delta^2)\hat{\boldsymbol{\delta}}.$$

The Proca equations for the meson fields **(spin 1)** are

$$\hat{\Gamma}_{\omega B}\bar{\psi}_B\gamma_\mu\psi_B = (\Box + m_\omega^2)\hat{\omega}_\mu - \partial_\mu\partial^\nu\hat{\omega}_\nu = (\partial^\nu\hat{\omega}_{\nu\mu} + m_\omega^2\hat{\omega}_\mu),$$
$$\hat{\Gamma}_{\phi B}\bar{\psi}_B\gamma_\mu\psi_B = (\Box + m_\phi^2)\hat{\phi}_\mu - \partial_\mu\partial^\nu\hat{\phi}_\nu = (\partial^\nu\hat{\phi}_{\nu\mu} + m_\phi^2\hat{\phi}_\mu),$$
$$\hat{\Gamma}_{\rho B}\bar{\psi}_B\boldsymbol{\tau}_B\gamma_\mu\psi_B = (\Box + m_\rho^2)\hat{\boldsymbol{\rho}}_\mu - \partial_\mu\partial^\nu\hat{\boldsymbol{\rho}}_\nu = (\partial^\nu\hat{\boldsymbol{\rho}}_{\nu\mu} + m_\rho^2\hat{\boldsymbol{\rho}}_\mu).$$

DIRAC EQUATION
(BARYON FIELD)

$$\left[\gamma^\mu(i\partial_\mu - \hat{\Sigma}_\mu^\tau) - (m_B + \hat{\Sigma}_s^\tau)\right]\psi_B = 0$$

SELF - ENERGIES
(SCALAR AND VECTOR)

$$\hat{\Sigma}_s^\tau = -\hat{\Gamma}_{\sigma B}\hat{\sigma} - \hat{\Gamma}_{\sigma_s B}\hat{\sigma}_s - \hat{\Gamma}_{\delta B}\boldsymbol{\tau}_B\cdot\hat{\boldsymbol{\delta}},$$
$$\hat{\Sigma}_\mu^\tau = \hat{\Gamma}_{\omega B}\hat{\omega}_\mu + \hat{\Gamma}_{\phi B}\hat{\phi}_\mu + \hat{\Gamma}_{\rho B}\boldsymbol{\tau}_B\cdot\hat{\boldsymbol{\rho}}_\mu + e\hat{Q}_B\hat{A}_\mu + \hat{\Sigma}_\mu^{\mathcal{R}}.$$

Model

.Relativistic mean-field approximation.

Why do we need it ?
(RMF approximation)

How does it look like ?
(operators to functions)

$$\hat{\Gamma}_{\alpha B}(\hat{\rho}_{\mathcal{B}}) \longmapsto \langle \hat{\Gamma}_{\alpha B}(\hat{\rho}_{\mathcal{B}}) \rangle = \Gamma_{\alpha B}(\rho_{\mathcal{B}}),$$
$$\hat{\sigma} \longmapsto \langle \hat{\sigma} \rangle = \sigma,$$
$$\hat{\omega}_\mu \longmapsto \langle \hat{\omega}_\mu \rangle = \omega_0,$$
$$\hat{\boldsymbol{\rho}}_\mu \longmapsto \langle \hat{\boldsymbol{\rho}}_\mu \rangle = \rho_{03},$$
$$\hat{\boldsymbol{\delta}} \longmapsto \langle \hat{\boldsymbol{\delta}} \rangle = \delta_3,$$
$$\hat{\sigma}_s \longmapsto \langle \hat{\sigma}_s \rangle = \sigma_s,$$
$$\hat{\phi}_\mu \longmapsto \langle \hat{\phi}_\mu \rangle = \phi_0,$$
$$\hat{A}_\mu \longmapsto \langle \hat{A}_\mu \rangle = A_0.$$

Simplification
(solvable equations !)

The Klein-Gordon equations for the meson fields **(spin 0)** are

$$\Gamma_{\sigma B}\rho_s = m_\sigma^2 \sigma,$$
$$\Gamma_{\sigma_s B}\rho_s = m_{\sigma_s}^2 \sigma_s,$$
$$\Gamma_{\delta B}\rho_{is} = m_\delta^2 \delta_3.$$

The Proca equations for the meson fields **(spin 1)** are

$$\Gamma_{\omega B}\rho_{\mathcal{B}} = m_\omega^2 \omega_0,$$
$$\Gamma_{\phi B}\rho_{\mathcal{B}} = m_\phi^2 \phi_0,$$
$$\Gamma_{\rho B}\rho_{iv} = m_\rho^2 \rho_{03}.$$

The Dirac equation for baryon field in the RMF approximation

$$\left[(i\gamma^\mu \partial_\mu - \gamma^0 \Sigma_0^\tau) - (m_B + \Sigma_s^\tau)\right] \psi_B = 0,$$

Model

.EQUATION OF STATE.

Solution of meson equations
(meson potentials)

$$U_\sigma = \Gamma_{\sigma N}\sigma, \qquad U_\omega = \Gamma_{\omega N}\omega_0, \qquad U_\rho = \Gamma_{\rho N}\rho_{03},$$

$$U_\delta = \Gamma_{\delta N}\delta_3, \qquad U_{\sigma_s} = \Gamma_{\sigma_s\Lambda}\sigma_s = \frac{\sqrt{2}}{3}\Gamma_{\sigma N}\sigma_s, \qquad U_\phi = \Gamma_{\phi\Lambda}\phi_0 = -\frac{\sqrt{2}}{3}\Gamma_{\omega N}\phi_0.$$

Equation of State
(energy density and pressure)

$$\epsilon = +\frac{1}{2}U_\sigma\sum_B \rho_{sB}R_{\sigma B} + \frac{1}{2}U_\omega\sum_B \rho_{vB}R_{\omega B} + \frac{1}{2}U_\rho\sum_B \rho_{ivB}R_{\rho B} + \frac{1}{2}U_\delta\sum_B \rho_{ivB}R_{\delta B} + \frac{1}{2}U_{\sigma_s}\sum_{B=\Lambda,\Sigma,\Xi}\rho_{sB} + \frac{1}{2}U_\phi\sum_{B=\Lambda,\Sigma,\Xi}\rho_{vB} + \epsilon^B_{kin} + \epsilon^L_{kin},$$

$$P = -\frac{1}{2}U_\sigma\sum_B \rho_{sB}R_{\sigma B} + \frac{1}{2}U_\omega\sum_B \rho_{vB}R_{\omega B} + \frac{1}{2}U_\rho\sum_B \rho_{ivB}R_{\rho B} - \frac{1}{2}U_\delta\sum_B \rho_{ivB}R_{\delta B} - \frac{1}{2}U_{\sigma_s}\sum_{B=\Lambda,\Sigma,\Xi}\rho_{sB} + \frac{1}{2}U_\phi\sum_{B=\Lambda,\Sigma,\Xi}\rho_{vB} + P^B_{kin} + P^L_{kin} + \rho_{\mathcal{B}}\Sigma^{\mathcal{R}}_0.$$