Pionic Freezeout Hypersurfaces

in Relativistic Nucleus-Nucleus Collisions

D. Anchishkin[a], V. Vovchenko[b], L.P. Csernai[c]

[a]Bogolyubov Institute for Theoretical Physics
[b]Taras Shevchenko Kiev National University
[c]Institute for Physics and Technology, University of Bergen

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- Calculation algorithm
- Freeze-out hypersurfaces
- Reaction zones
- Conclusions

Introduction

- Relativistic energies, relativistic velocities ($v \approx 0.99c$)
- Lorentz contraction

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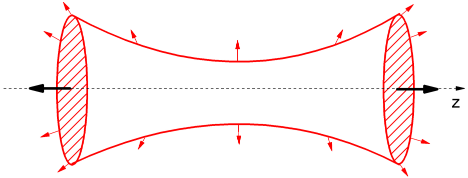
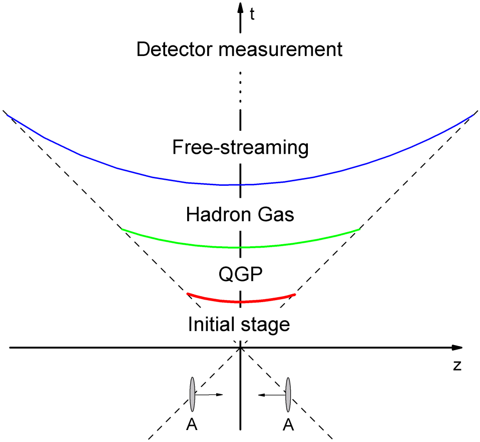


- High excited nonequilibrium system (energy density $\sim$GeV/fm^3)
- High multiplicity of secondary particles
- Short life-time of the system ($\sim$10–20 fm/c)

Stages of Fireball Evolution



Sharp freeze-out hypersurface definition

The sharp freeze-out hypersurface is defined with the help of some parameter $P(t, \boldsymbol{r})$ which takes the critical value P_c on the hypersurface:

$$P(t, \boldsymbol{r}) = P_c$$

1) the density of particles $n(t, \boldsymbol{r})$:

D.Adamova et al. (CERES Collaboration), Phys. Rev. Lett. 90, 022301 (2003).

2) the energy density $\epsilon(t, \boldsymbol{r})$:

V.N. Russkikh and Y.B. Ivanov, Phys. Rev. C 76, 054907 (2007);

J. Sollfrank, P. Huovinen, and P.V. Ruuskanen, Eur. Phys. J. C 6, 525 (1999)

3) the temperature $T(t, \boldsymbol{r})$:

H. von Gersdorff, L. McLerran, M. Kataja, and P.V. Ruuskanen, Phys. Rev. D34, 794 (1986);

P. Huovinen, Eur. Phys. J. A37, 121 (2008)

Particle (pion) four-flow:

$$N^\mu(x) \;=\; \int \frac{d^3p}{p^0}\, p^\mu\, f(x,p) \;=\; (n_{\rm lab}\,, n_{\rm lab}\boldsymbol{v}_E\,)\,.$$

Collective velocity (Eckart definition):

$$u^\mu(x) \;=\; \frac{N^\mu}{(N^\nu N_\nu)^{\frac{1}{2}}} \;=\; (\gamma_E\,, \gamma_E\,\boldsymbol{v}_E\,)\;.$$

Pion energy-momentum tensor:

$$T^{\mu\nu}(x) = \int \frac{d^3p}{p^0}\, p^\mu\, p^\nu\, f(x,p)\,.$$

Invariant particle density

$$n(x) \;=\; N^\mu(x)\, u_\mu(x)\,.$$

Invariant particle energy density

$$\epsilon(x) \;=\; u_\mu(x)\; T^{\mu\nu}(x)\; u_\nu(x)\,.$$

Freezeout hypersurfaces calculation algorithm

Invariant particle density

$$n(x) = N^{\mu}(x)\,u_{\mu}(x)\,.$$

Invariant particle energy density

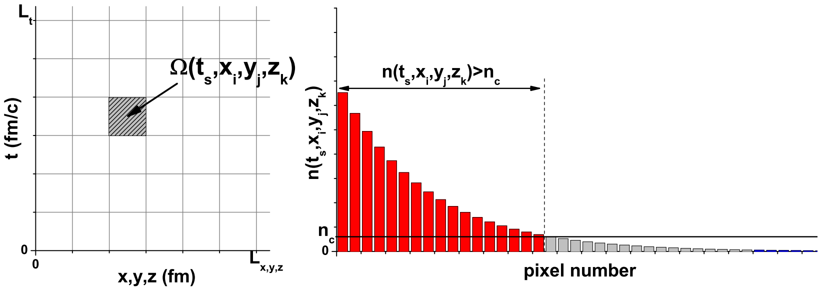
$$\epsilon(x) = u_{\mu}(x)\,T^{\mu\nu}(x)\,u_{\nu}(x)\,.$$

Equation $n(x) = n_c$ or $\epsilon(x) = \epsilon_c$ defines pionic freezeout hypersurface.

Calculations within UrQMD (Ultrarelativistic quantum molecular dynamics) microscopic transport model designed for description of relativistic heavy-ion collisions

- S.A. Bass, M. Belkacem, M. Bleicher et al., Prog. Part. Nucl. Phys. 41, 225 (1998);
- M. Bleicher, E. Zabrodin, C. Spieles et al., J. Phys. G: Nucl. Part. Phys. 25, 1859 (1999).

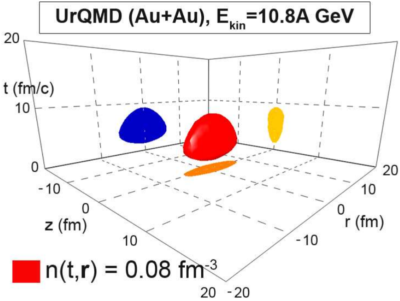
Freezeout hypersurfaces calculation algorithm



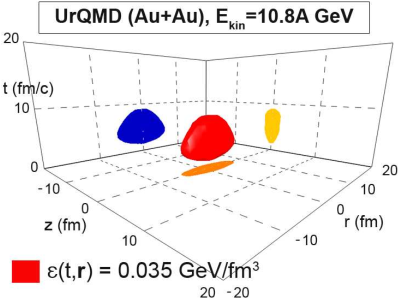
$$N_{\text{pixel}} = 16 \cdot 10^4\,, \quad \Omega_{sijk} = 1\ \text{fm}^4$$

Freezeout Hypersurface in (t, r, z) coordinates for AGS conditions

Invariant π^- density

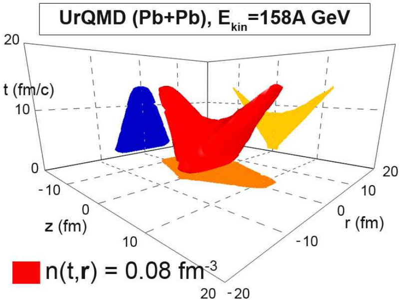


Invariant π^- energy density

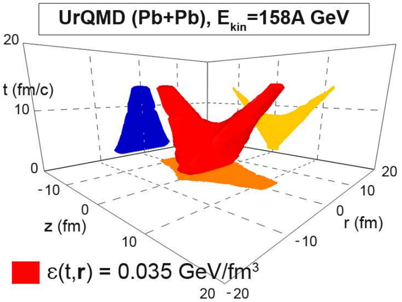


Freezeout Hypersurface in (t, r, z) coordinates for SPS conditions

Invariant π^- density

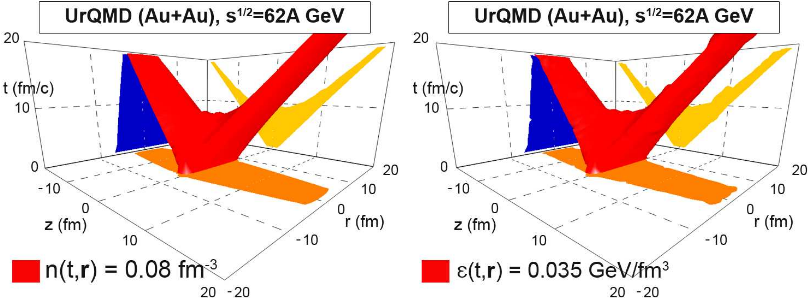


Invariant π^- energy density



Freezeout Hypersurface in (t, r, z) coordinates for RHIC conditions

Invariant π^- density   Invariant π^- energy density



$n(t, \boldsymbol{r}) = n_c$ and $\epsilon(t, \boldsymbol{r}) = \epsilon_c$ give same hypersurfaces.
Specific correspondence between n_c and ϵ_c.

Pionic Freezeout Temperature

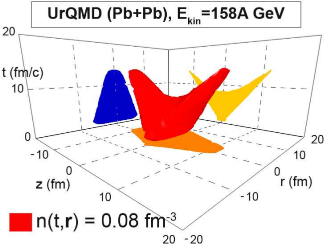
Assuming relativistic ideal dilute gas at freezeout

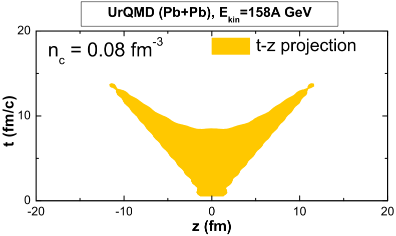
$$\frac{\epsilon_c}{n_c} = 3T_{\rm f} + m_\pi \frac{K_1(m_\pi/T_{\rm f})}{K_2(m_\pi/T_{\rm f})}.$$

Solving the equation yields $T_{\rm f} = 128$ MeV for AGS and SPS energies.

$T_{\rm f}$ increases to 164 MeV at RHIC energy of $\sqrt{s} = 130A$ GeV.

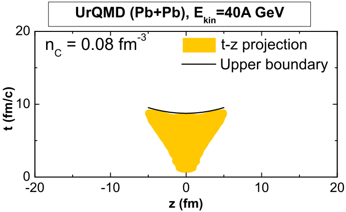
Space-time Evolution Parameters





E_{kin} (A GeV)	$\sqrt{s_{AA}}$ (A GeV)	$A+A$	τ (fm/c)	$R_\perp$ (fm)	$R_\parallel$ (fm)
10.8	4.88	$Au+Au$	9	6	2
20.0	6.41	$Pb+Pb$	9	6	3
40.0	8.86		8.75	6.5	5
80.0	12.39		8.75	6.5	7
158.0	17.32		8.5	6.5	7.5
202.9	19.6	$Au+Au$	8.25	6.5	7.5
2047.0	62.0		8.75	6.5	8.75
9007.0	130.0		10	6.5	10

(z, t) Projection of Freezeout Hypersurface



Upper (space-like) boundary is well approximated by

$$t_{\mathrm{FO}}(z) \;=\; t^{0}_{\mathrm{FO}} \,+\, \sqrt{\tau^{2}_{\mathrm{FO}} \,+\, z^{2}}\,.$$

E_{kin} (A GeV)	$\sqrt{s_{AA}}$ (A GeV)	$A+A$	t^0_{FO} (fm/c)	τ_{FO} (fm/c)
40.0	8.86	$Pb+Pb$	-7	15.75
80.0	12.39		-3	11.75
158.0	17.32		-0.75	9.25
202.9	19.6	$Au+Au$	-0.25	8.5
2047.0	62.0		-0.05	8.8
9007.0	130.0		0	9.25

- D. Anchishkin, V. Vovchenko, L.P. Csernai, arXiv:1211.1927 [nucl-th].

Reaction Zones

Definition of the fireball

- The space-time region where the reactions between the net particles, created particles take place we name as "fireball".
- The investigation of the reaction zones is equivalent to investigation of the fireball.

4-density of the number of reactions $2 \to 2$

$$\Gamma(x) = \int dp_1\, dp_2\, dp_3\, dp_4\; W_{12\to34}\, f_1(x)\, f_2(x)\, [1 \pm f_3(x)]\, [1 \pm f_4(x)]$$

$$x = (t, \boldsymbol{r}), \quad f_i(x) = f(x, \boldsymbol{p}_i), \qquad dp_i = d^3p_i/(2\pi)^3 E_i$$

Reaction Zone Calculation Algorithm

Number of reactions in the given space-time region Ω

$$N_{\text{coll}}(\Omega) = \int_\Omega d^4x\, \Gamma(x).$$

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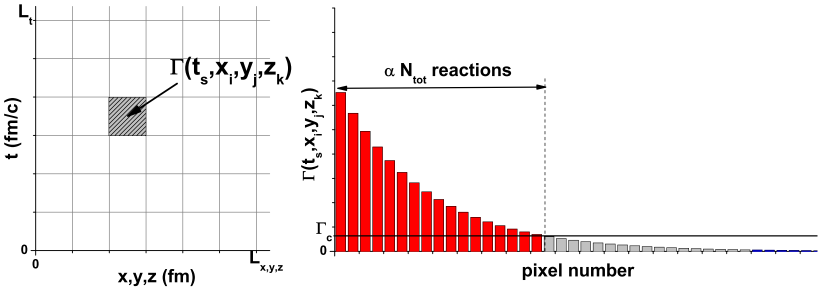
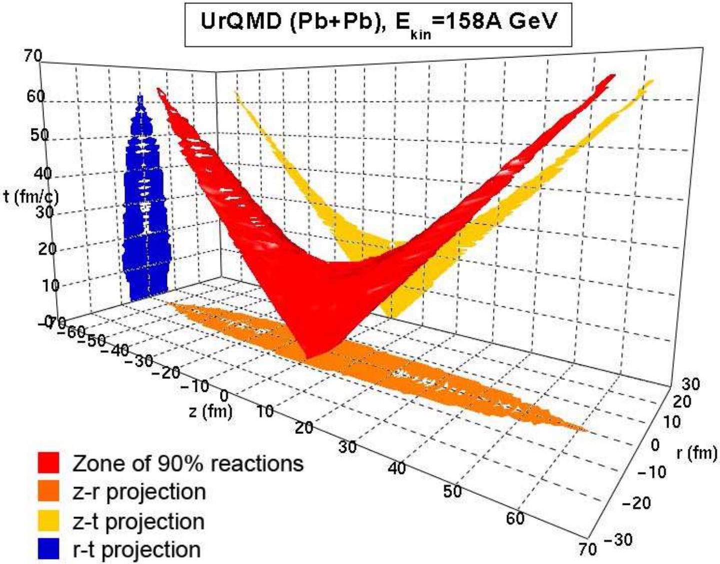
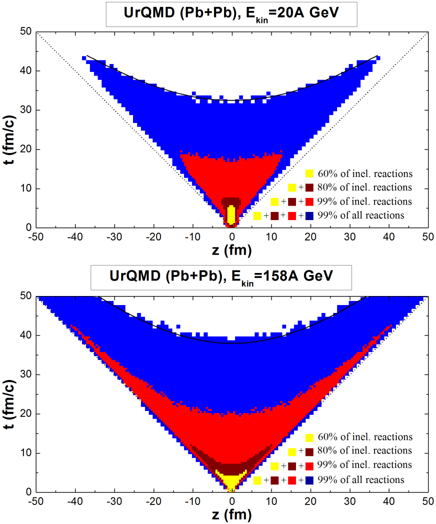


Reaction Zone in (t, r, z) coordinates, $r = \pm\sqrt{x^2 + y^2}$

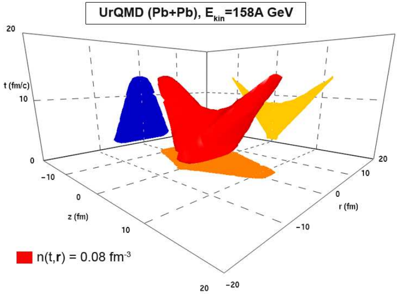


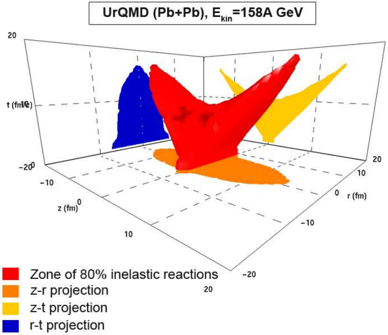
$t - z$ projection of Reaction Zones



Reaction Zone division time is approximately invariant of collision energy

Freezeout Hypersurfaces vs Reaction Zones





Pionic freezeout hypersurfaces can be put into correspondence to inelastic reaction zones

Conclusions

1. We show that pionic freezeout description with the use of pion density and pion energy density are equivalent and there is correspondence between n_c and ϵ_c values.

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2. We show that fireball lifetime and it's maximum spatial size are approximately invariant of collision energy. The approximation of $[t - z]$ projection of freezeout hypersurface in the form of modified hypersurface of constant proper time is introduced.

Conclusions

1. We show that pionic freezeout description with the use of pion density and pion energy density are equivalent and there is correspondence between n_c and ϵ_c values.
2. We show that fireball lifetime and it's maximum spatial size are approximately invariant of collision energy. The approximation of $[t - z]$ projection of freezeout hypersurface in the form of modified hypersurface of constant proper time is introduced.
3. Pionic freezeout hypersurface are compared to inelastic reaction zones and it is shown that they can be put into correspondence.

Thanks for your attention!