Thermal or not thermal?

(The origin of Tsallis entropy and the T_{qgp})

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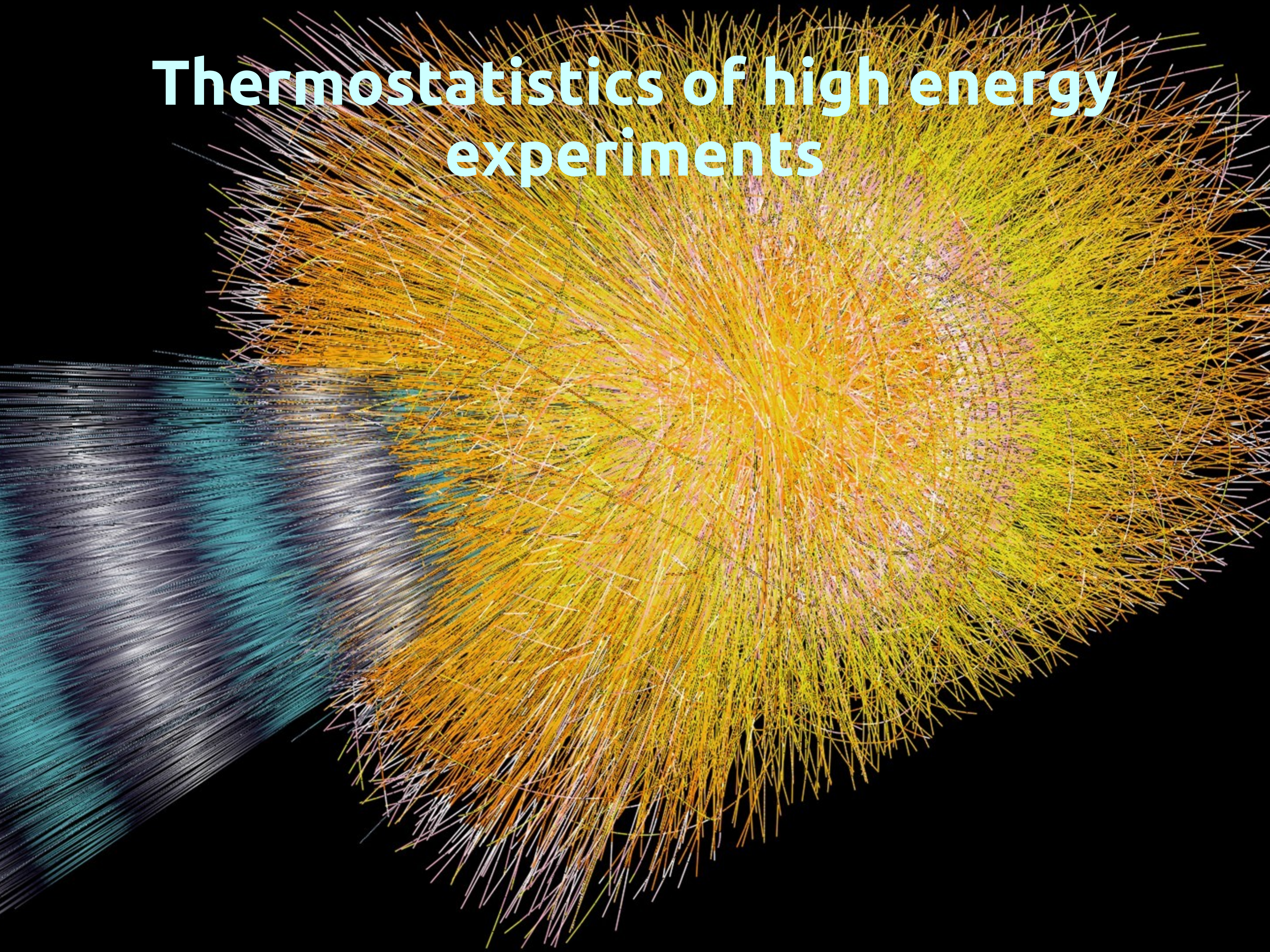
Content

— Experimental and theoretical power law tails

— Formal logarithms designed for Universal Thermostat Independence

— Temperatures galore

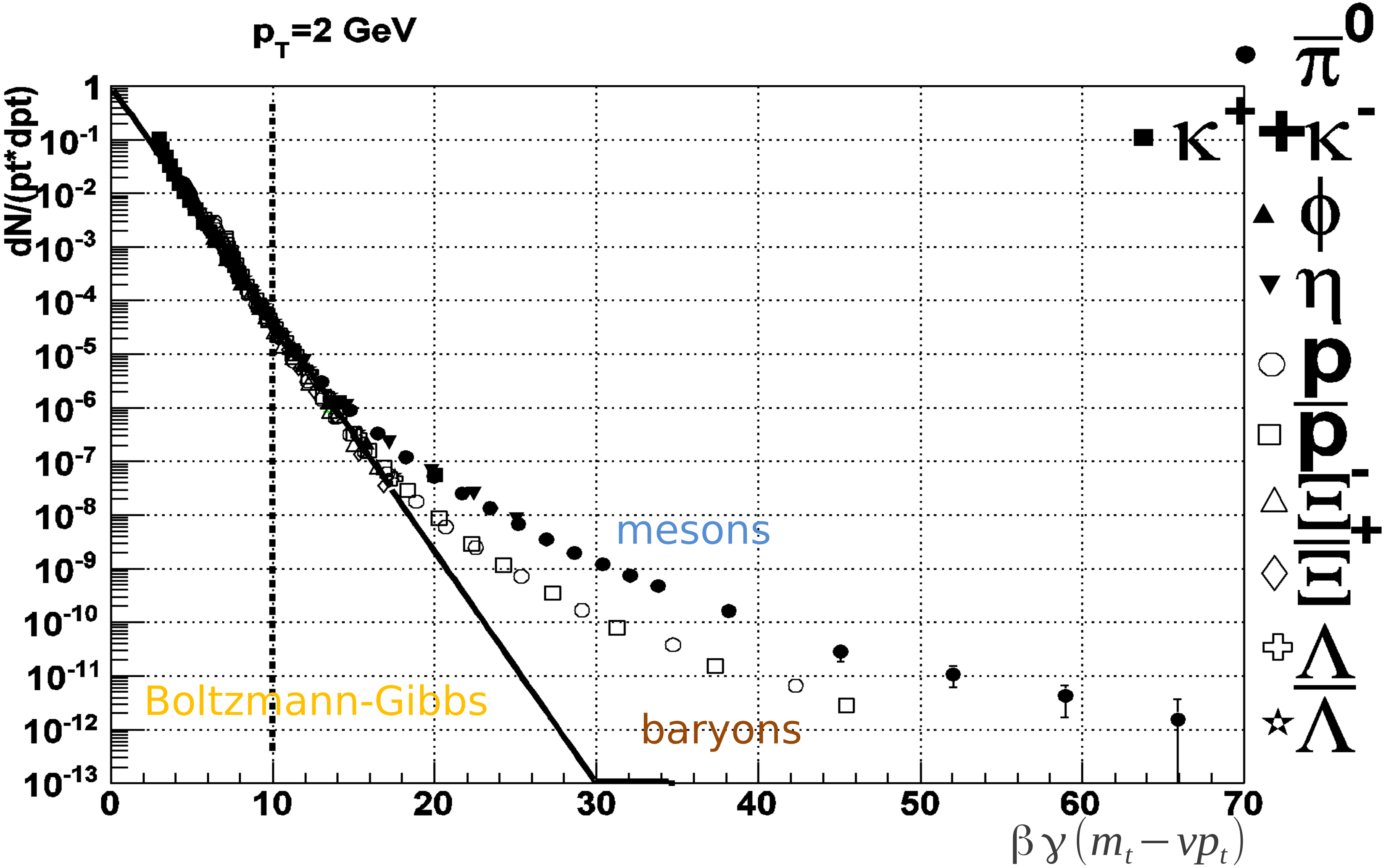
Thermostatistics of high energy experiments



Beyond exponential momentum distributions

Boltzmann-Gibbs Fits to AuAu → h X at s = (200 GeV)2

T = 100 ± 20 MeV, v = 0.5 ± 0.1

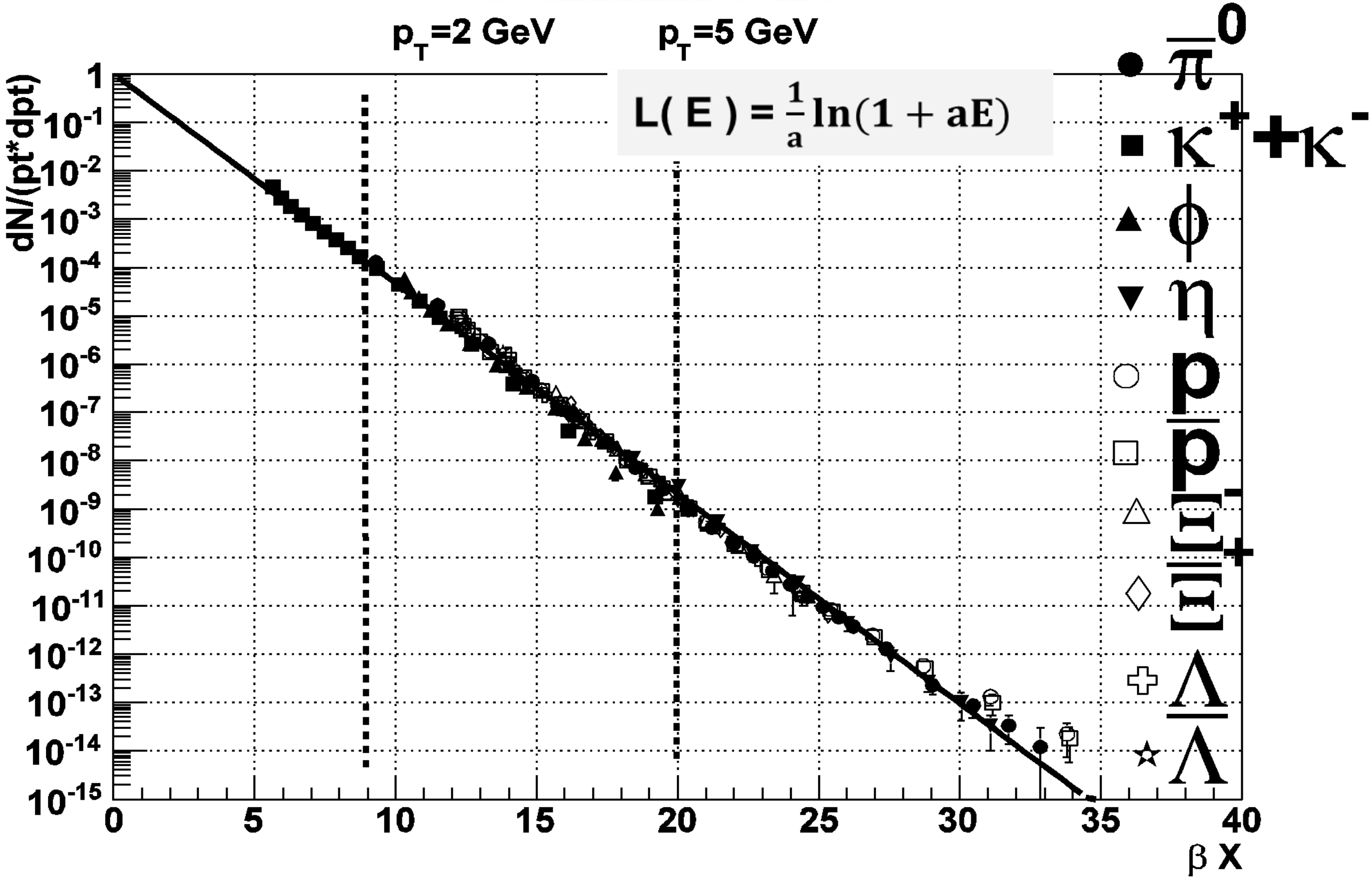


Biró T.S., Ürmössy K. and Schram Zs., J. Phys. G 37, 094027, (2010).

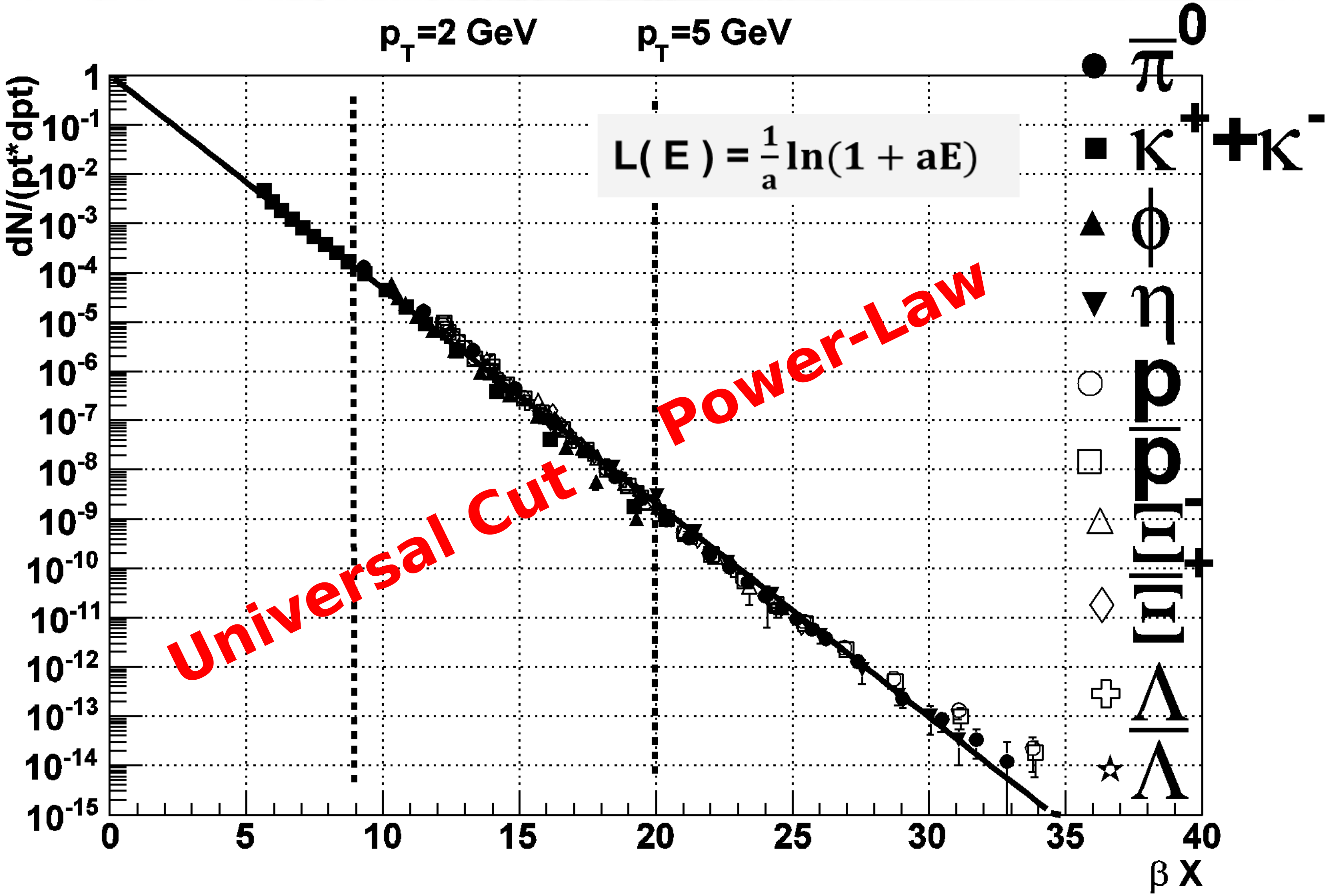
Rescaling: formal logarithms

Tsallis Fits to AuAu → h X at s = (200 GeV)2

T= 51 ±10 MeV, q= 1.062 ±7.65×10^{-3}, v = 0.5 ±0.1

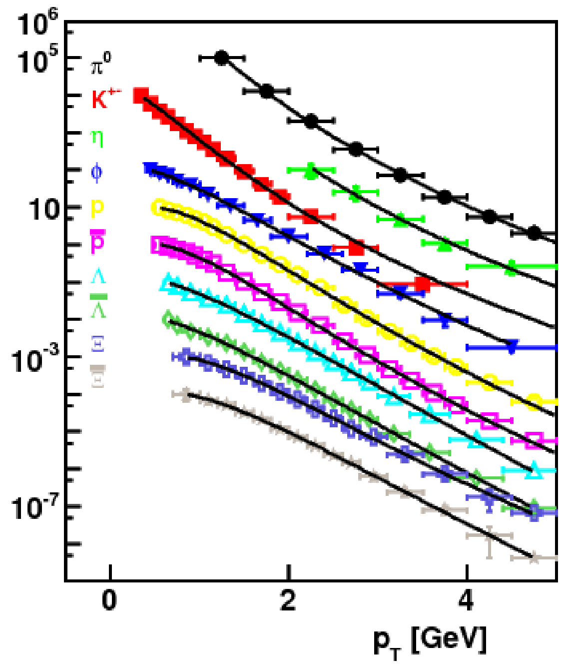


Scales: composition and formal logarithm



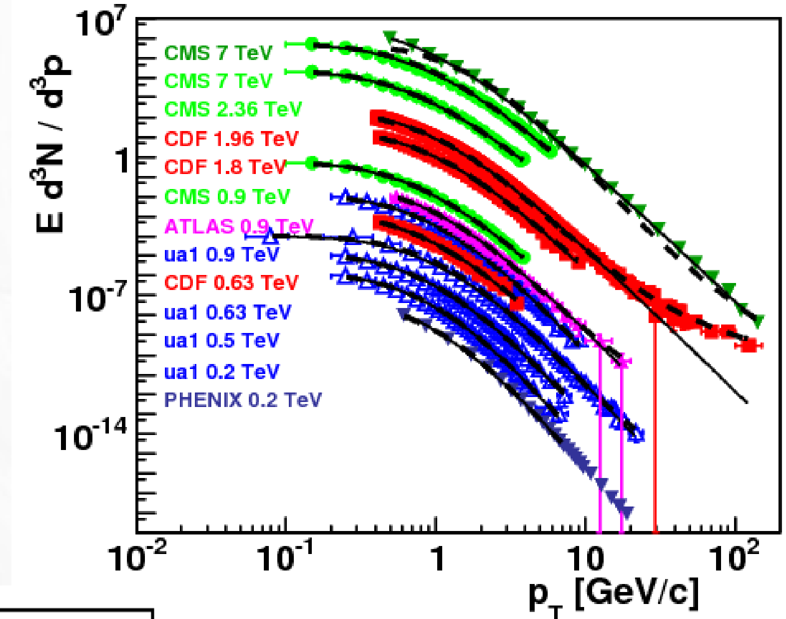
Similar hadron spectra in HEP:

Au-Au, 200 GeV

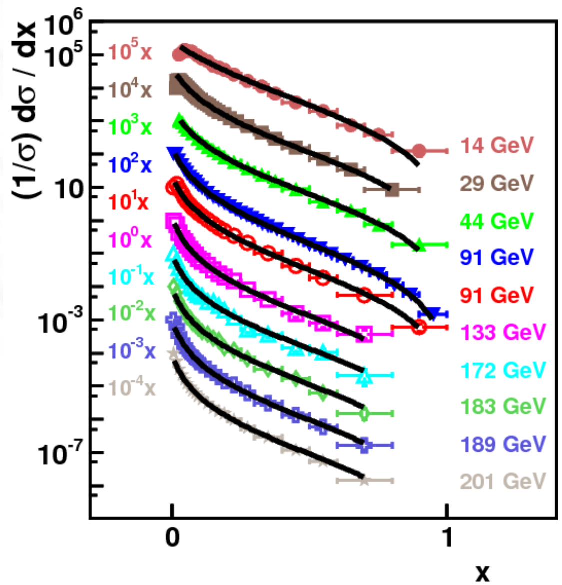


PLB, 689: 14-17, (2010)

p-p, 0.2-7 TeV



JoP, CS, 270 (2011) 012008



e^-e^+, 14-200 GeV

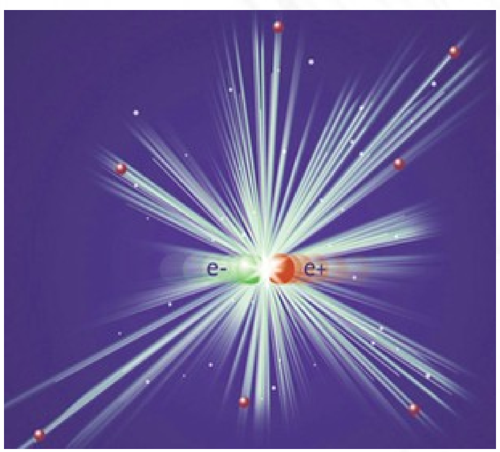
PLB, 701: 111-116 (2011)

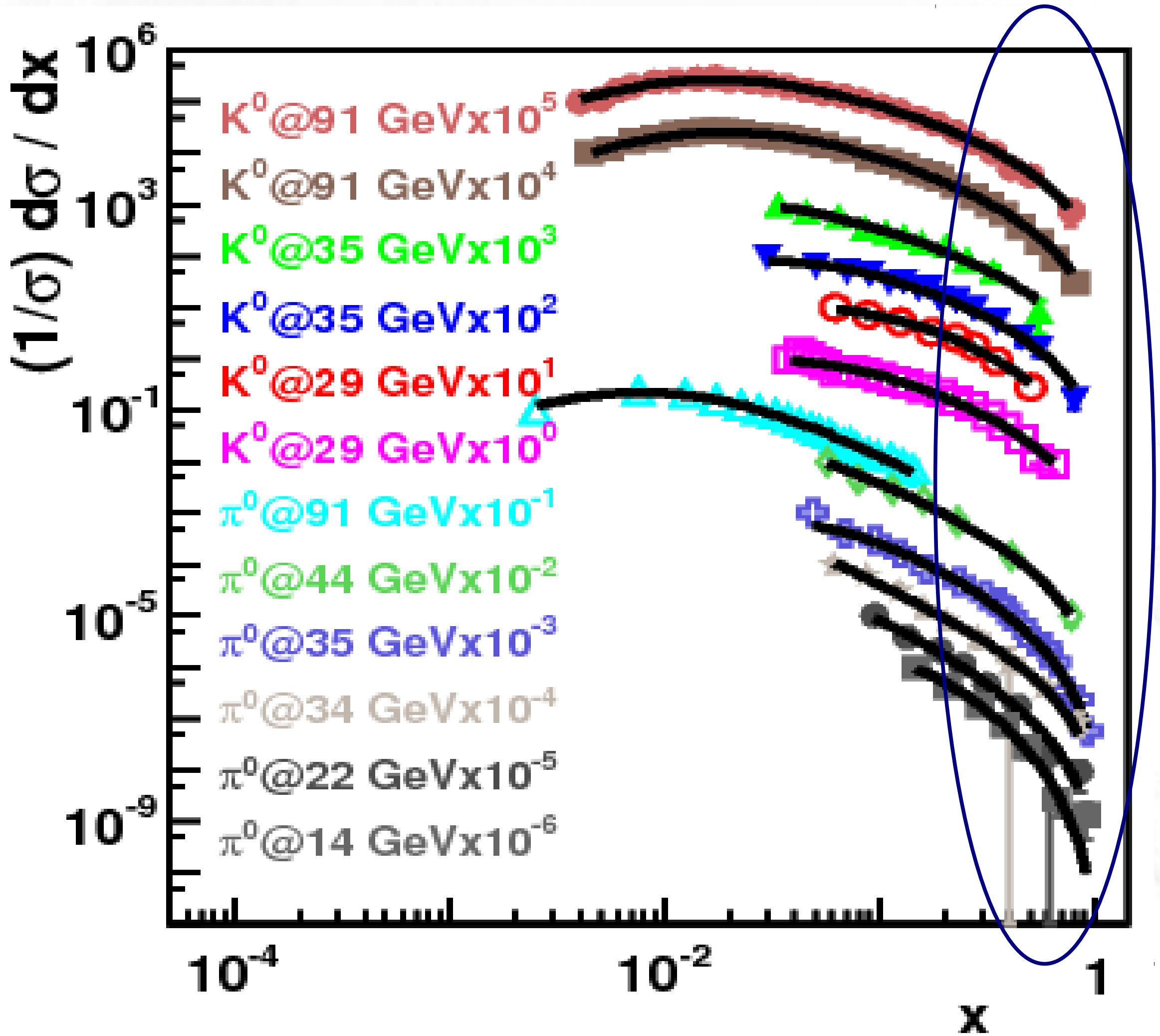
Beyond Tsallis distribution

LEP (DESY) (e^-e^+), pion and kaon energy distributions

x – energy ratio
σ – cross section

small size
and
low energy





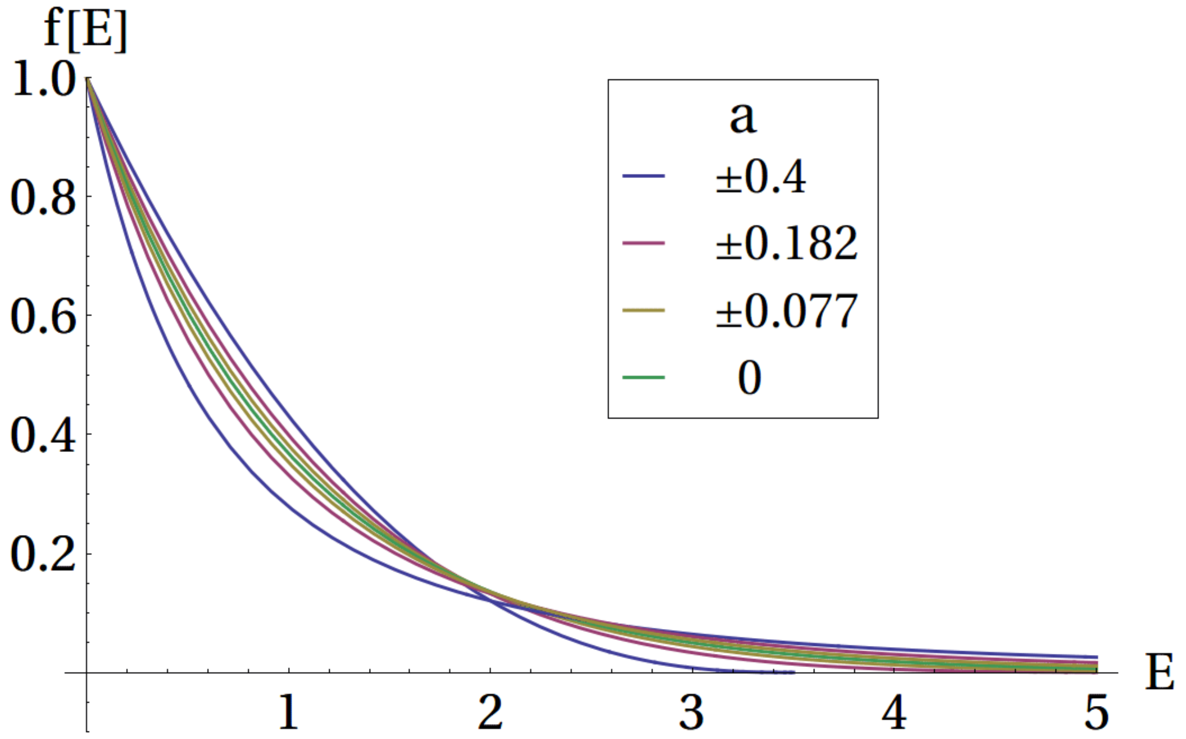
Biró et. al., Acta Phys. Pol., 43, 811-820, (2012)

The distribution

Exponential-power (Tsallis-Pareto):

$$f(E) = \frac{1}{Z}(1 + a\beta E)^{-\frac{1}{a}} \xrightarrow{a \to 0} \frac{1}{Z} e^{-\beta E}$$

$$\xrightarrow{E \to \infty} \frac{1}{Z}(a\beta E)^{-1/a}$$



Microcanonical ideal gas

Uniform distribution on a 3N dimensional sphere:

$$f(E)=\frac{\delta(E-H(p))}{\omega(E)}$$

$$H(p)=\frac{1}{2\mathrm{m}}\sum_{i=1}^{N}\bar{p}_i^2$$

$$p=(\bar{p}_{1,}\dots,\bar{p}_N)$$

$$f_1(\bar{p})=\frac{1}{(2\mathrm{m})^{\frac{3\mathrm{N}}{2}}B'_{3\mathrm{N}}(\sqrt{E})}\int d^3\bar{p}_2\dots d^3\bar{p}_N\,\delta(E-H(p))$$

$$=(2\mathrm{m})^{\frac{-3\mathrm{N}}{2}}\frac{B'_{3(N-1)}\left(\sqrt{E-\frac{\bar{p}^2}{2\mathrm{m}}}\right)}{B'_{3\mathrm{N}}(\sqrt{E})}$$

$$B_{3\mathrm{N}}(r)=\frac{\pi^{\frac{3\mathrm{N}}{2}}}{\Gamma(\frac{3\mathrm{N}}{2}+1)}r^{3\mathrm{N}}$$

volume of the 3N dimensional sphere

$$f_1(p)=(2m\pi)^{-3/2}\frac{N-1}{N}\frac{\Gamma(\frac{3N}{2}+1)}{\Gamma(\frac{3N}{2}-\frac{1}{2})}\frac{\sqrt{(E-\frac{p^2}{2m})}^{3N-4}}{(\sqrt{E})^{3N-1}}$$

$$=A\left(1-\frac{2}{3N-4}\frac{p_1^2}{\frac{2}{3N-4}2mE}\right)^{\frac{3N-4}{2}}$$

$$f_1(\bar{p})=A\left(1-a\beta\frac{\bar{p}_1^2}{2m}\right)^{\frac{1}{a}}$$

$$a=\frac{2}{3N-4}\qquad T=\frac{1}{\beta}=\frac{2E}{3N-4}$$

$$\left(\beta_{mc}=\frac{2E}{3N}\right)$$

$$C=\frac{3N-4}{2}=const.$$

– several other microcanonical distributions

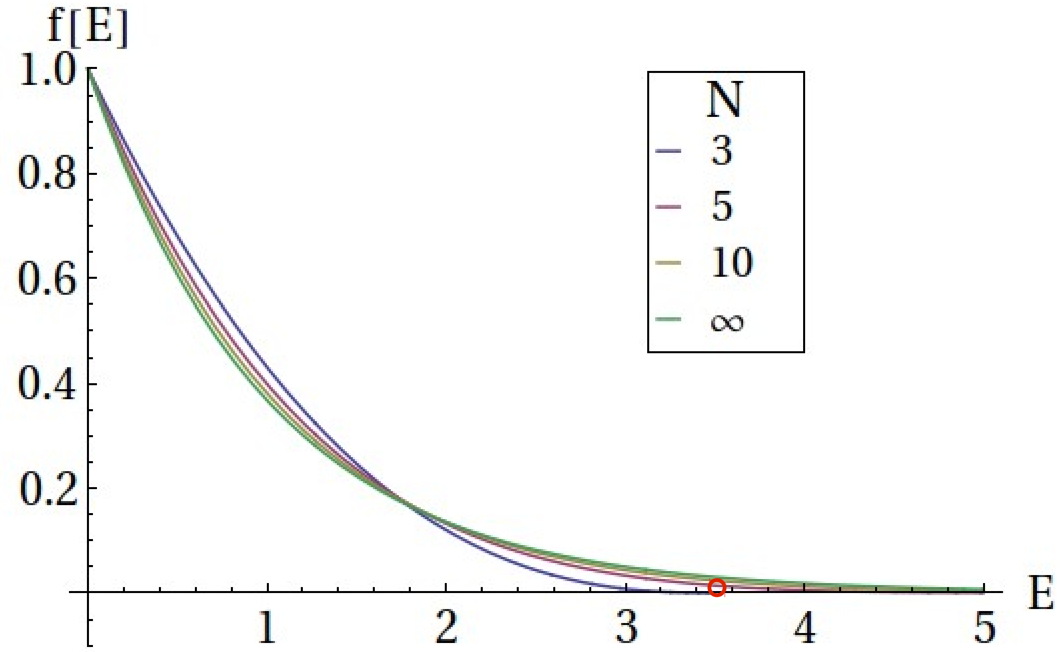
$$f(E)=\frac{1}{Z}(1+a\beta E)^{-\frac{1}{a}}$$

$$a=\frac{-2}{3N-4}$$

$$E_{max}=\left(1-\frac{1}{a}\right)\beta^{\frac{1}{a-1}}$$

finite support
negative a

$$C=\frac{3N-4}{2}=const.$$



$$f(E)=\frac{1}{Z}(1+a\beta E)^{-\frac{1}{a}}$$

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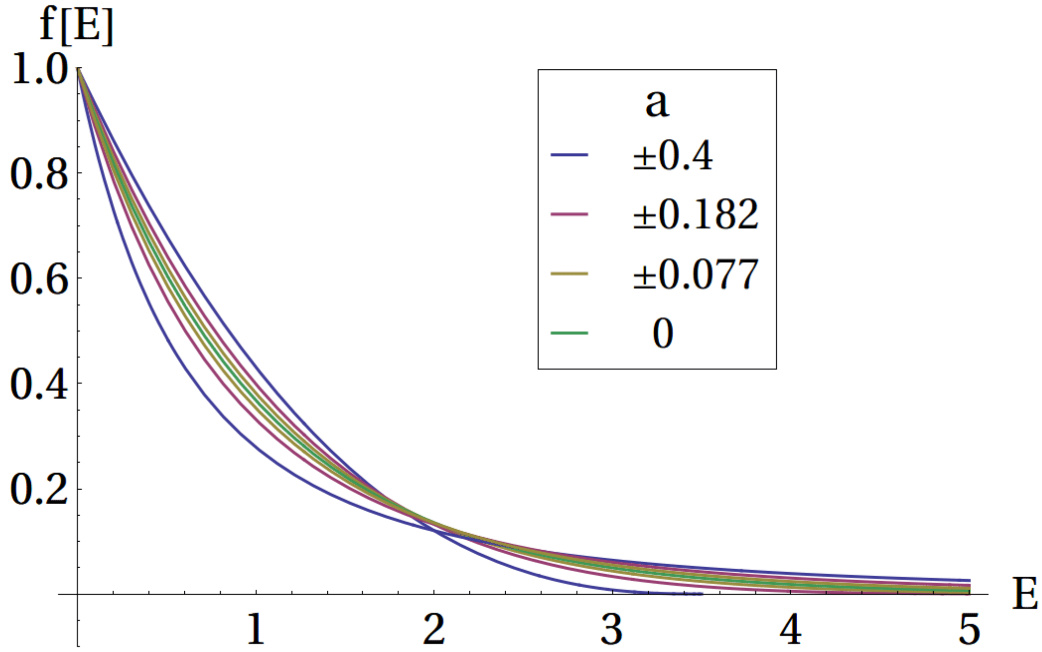


Theory: canonical

Maximum entropy principle with modified entropy:

$$S_T(f) = \frac{1}{1-q} \int (f^q - f)$$

Havrda-Charvat, et al (?)

$$\frac{1}{1-q} \int (f^q - f) - \beta \int E f - \alpha \int f = extr.$$

$$f_{stac}(E) = \frac{1}{Z} \left(1 + \frac{a}{1-a} \beta^{\frac{1}{a-1}} E \right)^{-\frac{1}{a}} = \frac{1}{Z} \left(1 + \frac{1-q}{q} \beta^{\frac{1}{q}} E \right)^{\frac{1}{q-1}}$$

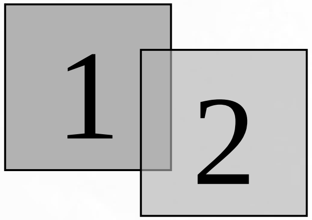
$$q = 1 - a$$

Physical origin?

micro – finite size effect?
independent – correlated

Theory: composition rules

Composition = there are subsystems



$$X_{12}(X_1, X_2)$$

Tsallis-like pseudoadditivity

$$S_{12} = S_1 + S_2 + aS_1S_2$$

series expansion *(a=1-q)*

Formal logarithm:

$$1+aS_{12} = 1+aS_1+aS_2+a^2S_1S_2 = (1+aS_1)(1+aS_2)$$

$$\ln(1+aS_{12}) = \ln(1+aS_1) + \ln(1+aS_2)$$

$$\Rightarrow L(S_T) = \frac{1}{a}\ln(1+aS_T) = S_R$$

additive

Rényi entropy (1963)

Why to use the Tsallis / Rényi entropy formulas?

- It generalizes the Boltzmann-Gibbs-Shannon formula
- It treats correlations – statistical entanglement – between subsystem and reservoir
- It claims to be universal (applicable for whatever material quality of the reservoir)
- It has a cut power-law energy distribution in the canonical treatment

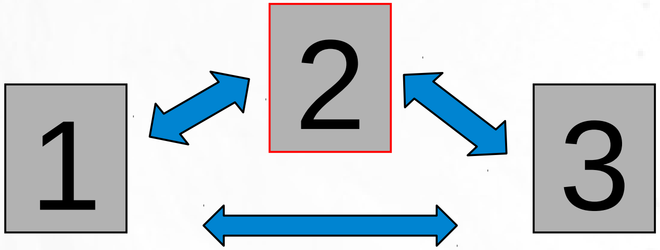
Why not to use the Tsallis / Rényi entropy formulas?

- They lack 200 years of classical thermodynamic foundation (high $T_{Tsallis}$ is hot?)
- Tsallis is not additive, Rényi is not linear
- There is an extra parameter q (mysterious?)
- How do different q systems equilibrate? *(see M. Horváth)*
- Why this and not any other (Landsberg-Vedral, Kaniadakis, etc..)?
- It looks pretty much formal…

Zeroth law and formal logarithms

Zeroth law: transitivity and factorizability:

$$S_{12}(S_1, S_2)$$



Composition and zeroth law → there is formal logarithm

$$L(S_{12}) = L(S_1) + L(S_2)$$

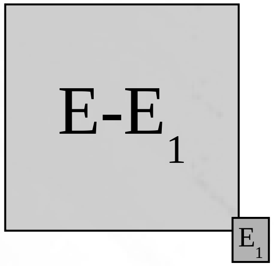
Transitivity: further important conditions
Nonadditive entropy and energy together,
Equilibrium of different non-additive and additive systems, etc…

PRE, 83, 061147 (2011)
J.Phys, Conf. Ser, 394, 012002 (2012)

Canonical and microcanonical?

1) $K\left(S\left(E_1\right)\right)+K\left(S\left(E-E_1\right)\right)=max.\quad E\gg E_1$ microcanonical

2) $K\left(S\left(E_1\right)\right)-\beta E_1=max.$ canonical



Let us find a suitable formal logarithm:

$$\beta_1:=K'(S(E_1))S'(E_1)=K'(S(E-E_1))S'(E-E_1)$$
$$=K'(S(E))S'(E)-\left[\underline{K''(S(E))S'(E)^2+K'(S(E))S''(E)}\right]E_1+O(E_1^2)$$

$$\frac{L''(S)}{L'(S)}=a=-\frac{S''(E)}{S'(E)^2}=T'(E)=\frac{1}{C(E)}$$

The solution is:

$$K(S)=\frac{e^{aS}-1}{a}$$

Biró, Barnaföldi and VP, arXiv: 1208.2533

From additive to nonadditive

The solution is:

$$K(S)=\frac{e^{aS}-1}{a} \qquad (K=L^{-1})$$

This generates:

$$K(-\ln(p_i))=\frac{1}{a}(p_i^{-a}-1)=s_{Tsallis}(p_i)$$

Zeroth law: an additive entropy is necessary.

$$S_{Renyi}=L(S_{Tsallis})=L(\sum_i [p_i L^{-1}(-\ln(p_i))])$$

The canonical maximum entropy principle becomes:

$$\underbrace{\frac{1}{a}\ln\sum_i p_i^{1-a}}_{S_{\text{Rényi}}}-\beta\sum_i p_i E_i-\alpha\sum_i p_i=max.$$

(a=1-q)

Temperatures galore

Temperatures: spectral, classical entropic, Lagrange-multipliers,…

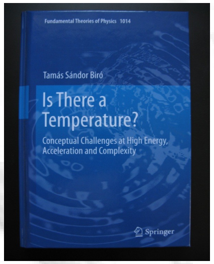
$$\beta = \frac{1}{T} = \frac{\partial L(S)}{\partial E}$$

$$\beta_{clas} = \frac{1}{T_{clas}} = \frac{\partial S}{\partial E}$$

$$\beta_{slope} = \frac{1}{T_{slope}} = \frac{\partial \ln(f(E))}{\partial E}$$

$$\beta \neq \beta_{clas} \neq \beta_{slope}$$

Biró T.S.: Is there a temperature? Springer 2010



$$f^{eq} = \frac{1}{Z}\left(1 + a\hat{\beta}E\right)^{-\frac{1}{a}} = \frac{1}{Z}\left(1 + \frac{Z^{-1/C}}{C-1} e^{S/C} \beta E_i\right)^{-C}$$

Expressed by the physical parameters of the reservoir.

BG limit:

$$f^{eq} \xrightarrow[a \to 0]{} \frac{1}{Z_0} e^{-\frac{E}{T_{fit}}}$$

Apparent temperature and reservoir temperature

$$T_{fit} = T \lim_{C \to \infty} e^{-S/C}$$

infinite heat capacity limit

Application:

MIT bag model:

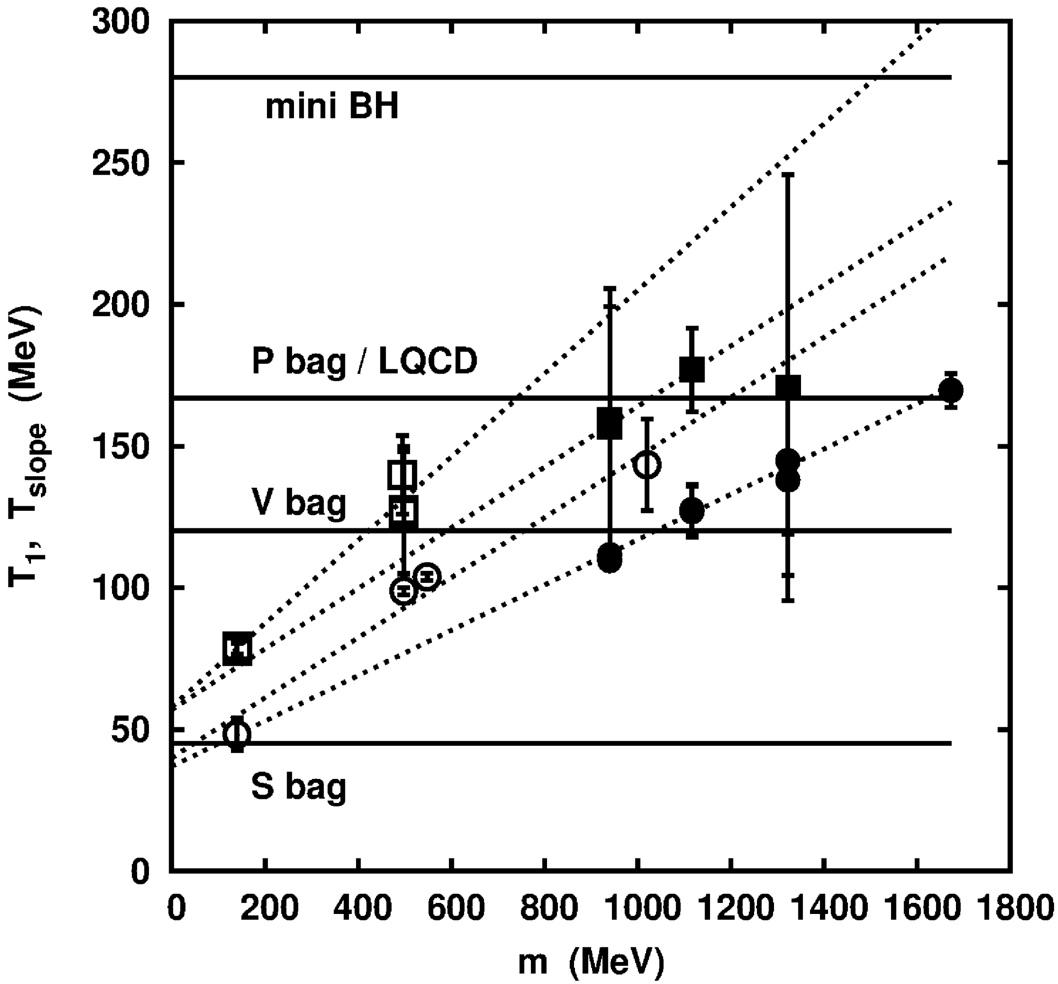
$$E = V\left(\sigma T^4 + B\right), \quad p = \sigma T^4/3 - B, \quad S = 4\sigma V T^3/3$$

$$C = \frac{dE}{dT} = 4\sigma V T^3 + \left(\sigma T^4 + B\right)\frac{dV}{dT}$$

$$C_V = 4\sigma V T^{3,} \quad C_p = \infty \quad C_S = \frac{3}{4} S\left(1 - \frac{T^4}{T_0^4}\right)$$

$$\frac{T_{fit}}{T} = \lim_{C \to \infty} e^{-S/C_V} = e^{-1/3} \approx 0.7$$

statistical / lattice



$$T_{slope} = T_0 + \frac{E}{C_V}$$

circles – RHIC AuAu 200GeV
boxes – LHC pp 900GeV

open – mesons
filled – barions

Summary

- *Zeroth law* and composition rules: formal logarithm
- *Universal Thermostat Independence*: entropy factory
- BG entropy based canonical treatment:
  - exponential (Gaussian) distribution
  - approximation by independent
- Tsallis-Rényi entropy based canonical treatment:
  - Tsallis-Pareto distribution
  - approximation by correlated (microcanonical ideal)
- Temperatures: QGP T = 175 MeV V=const. fit T = 125 MeV

Conclusion

Why Tsallis / Rényi entropy ?

What is q ?

What is T ?

Conclusion

Why Tsallis / Rényi entropy ?

What is q ?

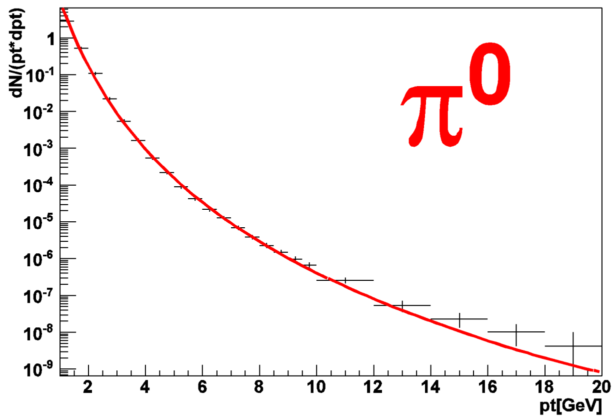
What is T ?

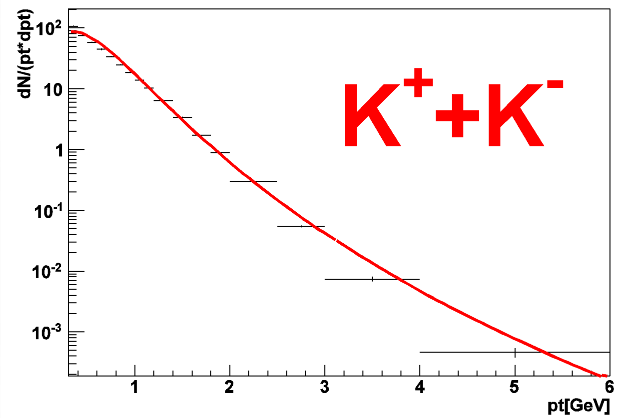
Thank you for the attention!

PRE, 83, 061147 (2011)
J.Phys, Conf. Ser, 394, 012002 (2012)
ArXiv: 1208.2533
ArXiv: 1211.5284

Backup

Tsallis quark matter + transverse flow + quark coalescence fits to hadron spectra





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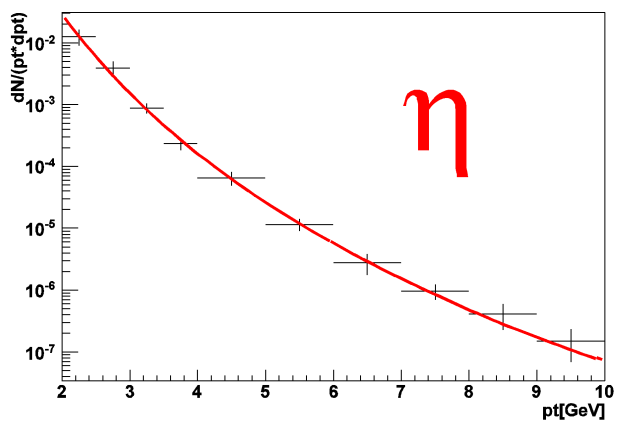
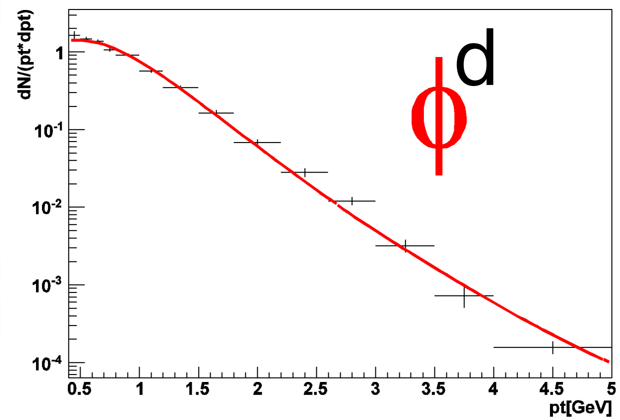
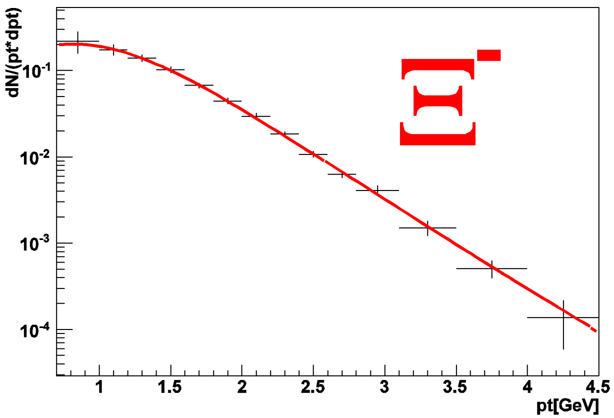
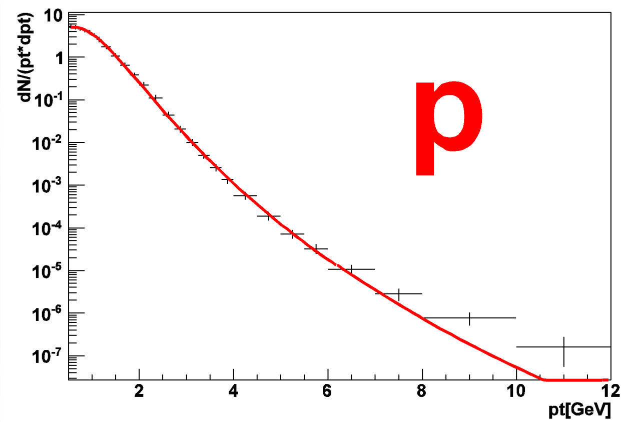


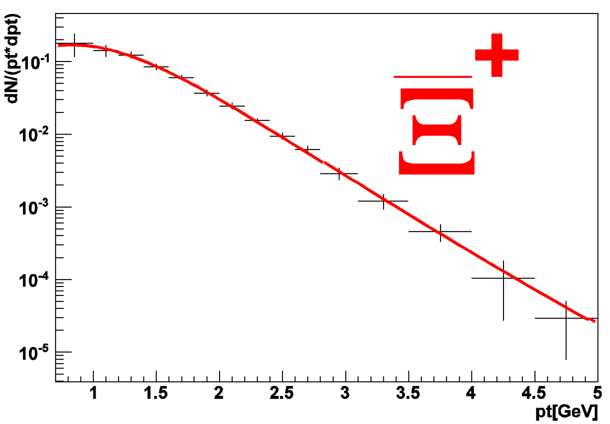


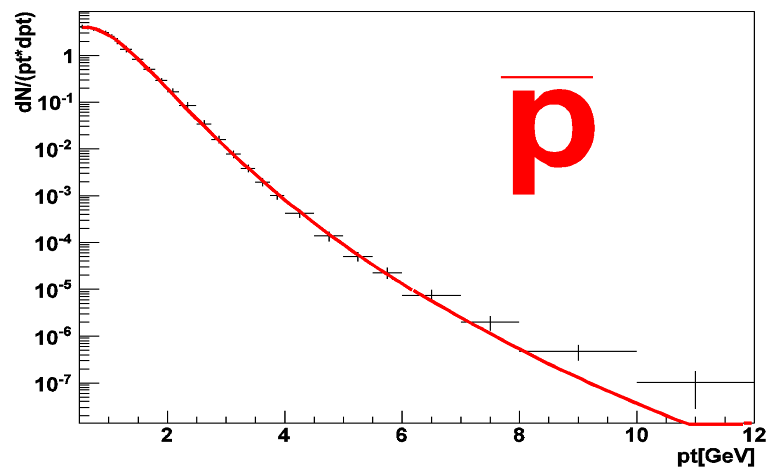
T.S.Biró, K.Ürmössy, JPhysG 36, 064044, 2009 (SQM 2008, Beiji

Tsallis quark matter + transverse flow + quark coalescence fits to hadron spectra



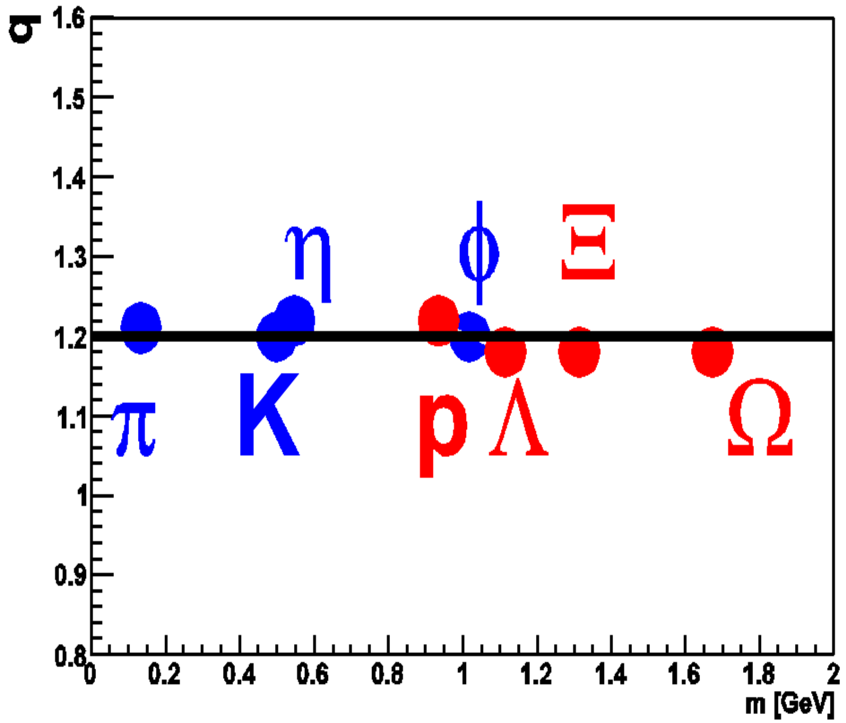


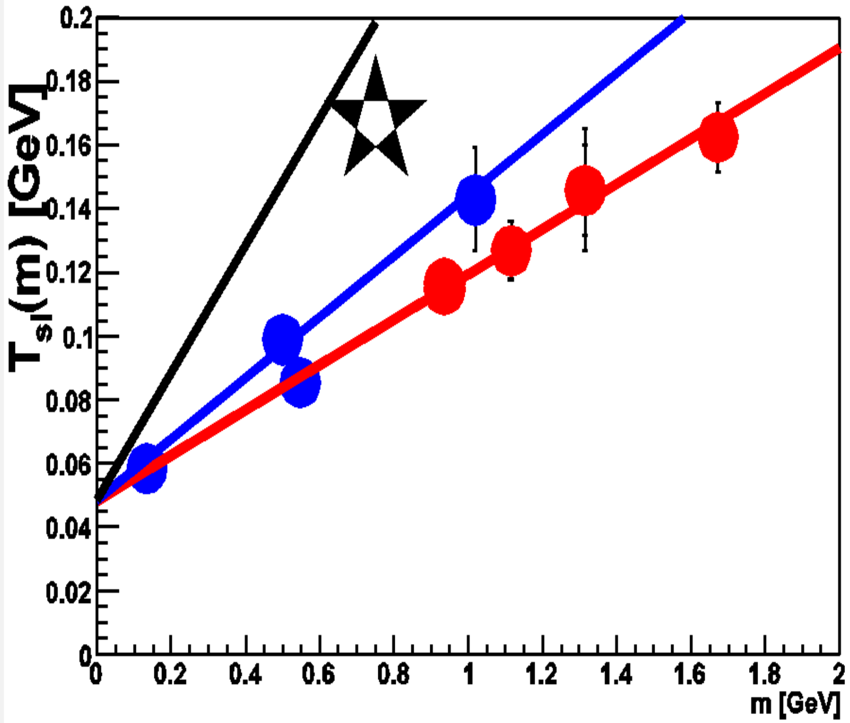




T.S.Biró, K.Ürmössy, JPhysG 36, 064044, 2009 (SQM 2008, Beijing)

Blast wave fits and quark coalescence





Meson q-1 = quark (q-1)/2;
Baryon q-1 = quark (q-1)/3;
Mini BH q-1 = $2/\pi^2$

T.S.Biró, K.Ürmössy, JPhysG 36, 064044, 2009 (SQM 2008, Beiji

Non-additive, but composable

Extensive composition:

$$\lim_{N\to\infty} h\left(h\left(....h\left(h\left(\frac{x}{N}, \frac{x}{N} \right), \frac{x}{N} \right)..., \frac{x}{N} \right), \frac{x}{N} \right) < \infty \qquad \left(\lim_{N\to\infty} \frac{X(N)}{N} < \infty \right)$$

N

associative and symmetric

$$h(h(x,y),z) = h(x,h(y,z))$$

$$h(x,y) = h(y,x)$$

Abstract Composition Rules

$$X_1 \oplus X_2 = X_{12}(X_{1,} X_2)$$

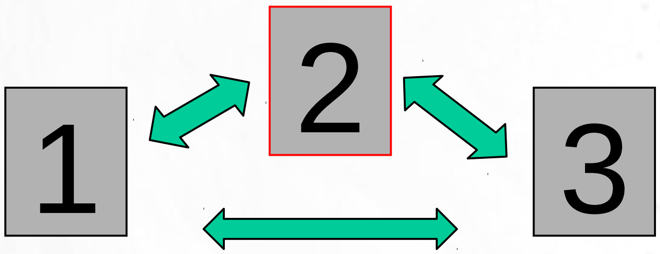
and

zeroth law of thermodynamics

Abe S., Physica A 269 403 (1999)/300 417 (2001)/368 430 (2006)
Johal, R.S., PLA 318 48 (2003)/332 345 (2004)
Scarfone, A.M., PLA 374 2701 (2010)

Additivity + second law ⟹ *zeroth law*

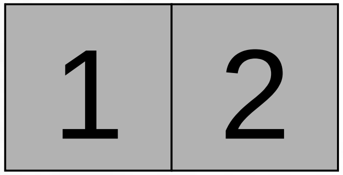
It is the last …   Maxwell (1871), Fowler (1931):



Additive, conserved energy:

$E_{12}(E_1, E_2) = E_1 + E_2$ and $dE_{12}(E_1, E_2) = dE_1 + dE_2 = 0$

Additive, maximal entropy



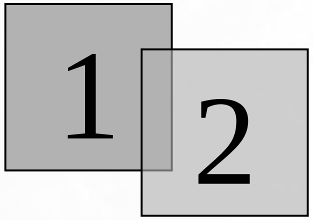
$S_{12}(E_1, E_2) = S_1(E_1) + S_2(E_2)$

$$dS_{12} = \frac{dS_1}{dE_1} dE_1 + \frac{dS_2}{dE_2} dE_2 = \left(\frac{dS_1}{dE_1} - \frac{dS_2}{dE_2} \right) dE_2 = 0 \quad \Rightarrow \quad \frac{1}{T}(E) = \frac{dS}{dE}$$

Every monotonous f(T) is a good empirical temperature…

Composition = there are subsystems

$$X_{12}(X_1, X_2)$$



Composition rules:

$E_{12}(E_1, E_2)$ and $$dE_{12} = \frac{\partial E_{12}}{\partial E_1} dE_1 + \frac{\partial E_{12}}{\partial E_2} dE_2 = 0$$

Additive, maximal entropy:

$S_{12}(E_1, E_2) = S_1(E_1) + S_2(E_2)$ and

$$dS_{12} = \frac{dS_1}{dE_1} dE_1 + \frac{dS_2}{dE_2} dE_2 \Rightarrow \left(\frac{\partial E_{12}}{\partial E_1} S_2' - \frac{\partial E_{12}}{\partial E_2} S_1' \right) dE_1 = 0$$

$$\frac{\partial E_{12}}{\partial E_1}(E_1, E_2) S_2'(E_2) = \frac{\partial E_{12}}{\partial E_2}(E_1, E_2) S_1'(E_1)$$

factorizable for subsystems?

$$\frac{\partial E_{12}}{\partial E_1}(E_1,E_2)\frac{dS_2}{dE_2} = \frac{\partial E_{12}}{\partial E_2}(E_1,E_2)\frac{dS_1}{dE_1}$$

$$A_1(E_1)\,B_2(E_2)\,C(E_1,E_2)\,S'(E_2) = A_2(E_2)\,B_1(E_1)\,C(E_1,E_2)\,S'(E_1)$$

$$\frac{B_2(E_2)}{A_2(E_2)}S'(E_2) = \frac{B_1(E_1)}{A_1(E_1)}S'(E_1)$$

$$E_{12}(E_1,E_2) = \Phi\left(\int\frac{A_1}{B_1}dE_1 + \int\frac{A_2}{B_2}dE_2\right)$$

Formal logarithm:

$$L(E_{12}) = L_1(E_1) + L_2(E_2)$$

Transitivity: further important conditions.
Nonadditive entropy and energy together +…

Biró-Ván: PRE, 83, 061147 (2011)

Example 1: Tsallis-like pseudoadditivity

$$S_{12} = S_1 + S_2 + aS_1S_2$$ ⟸ series expansion *(a=1-q)*

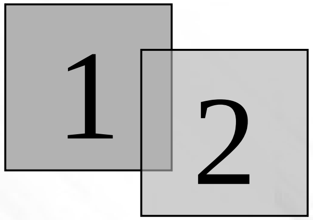
Formal logarithm:

$$1+aS_{12} = 1+aS_1+aS_2+a^2S_1S_2 = (1+aS_1)(1+aS_2) \quad \Rightarrow \quad L(S) = \frac{1}{a}\ln(1+aS)$$

(Rényi)

Example 2: heterogeneous systems

$$S_{12} = \frac{1}{a_{12}}\left((1+a_1S_1)^{a_{12}/a_1}(1+a_2S_2)^{a_{12}/a_2} - 1\right)$$



$a_{12} = a_1 = a_2$ – homogeneous *(a=1-q)* ⟹ $S_{12} = S_{Tsallis}$

– body and interaction

Associativity – generalized additivity

$$h(h(x,y),z) = h(x,h(y,z))$$

$$\Rightarrow \quad \exists L : h_\infty(x,y) = L^{-1}(L(x)+L(y))$$

(*L – formal logarithm*

(unique up to a constant mulitplier)

Construction:

$$L(x) = \int_0^x \left(\frac{\partial h}{\partial y}(z,0) \right)^{-1} dz$$

E.g. Tsallis rule is associative:

$$h(S_1,S_2) = S_1 + S_2 + aS_1S_2$$

$$L(x) = \int_0^x \frac{1}{1+az}\,dz = \frac{1}{a}\ln(1+ax)$$

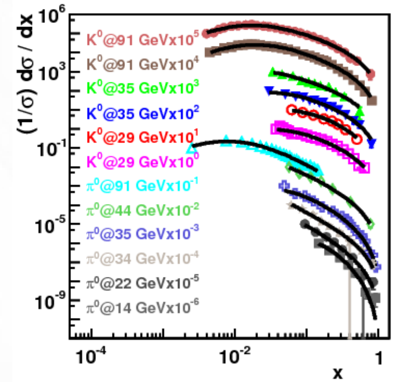
$$\Rightarrow \quad L(S_T) = \frac{1}{a}\ln(1+aS_T) = S_R$$

… and this is Rényi entropy.

Canonical energy distribution:

$$\frac{1}{\hat{a}} \ln \sum_i p_i^{1-\hat{a}} - \beta \sum_i p_i \frac{1}{a} \ln(1 + a E_i) - \alpha \sum_i p_i = max.$$

$$p_i^{eq} = \frac{1}{Z} \left(1 + \hat{a}\hat{\beta} \frac{1}{a} \ln(1 + a E_i)\right)^{\frac{-1}{\hat{a}}}$$



where $\hat{\beta} = \frac{\beta}{1 - \hat{a}(1 + \beta \langle L \rangle)}$

$a \to 0$ $$p_i^{eq} = \frac{1}{Z_S} (1 + \hat{a}\hat{\beta} E_i)^{\frac{-1}{\hat{a}}}$$ nonadditive S, additive E

$\hat{a} \to 0$ $$p_i^{eq} = \frac{1}{Z_E} (1 + a E_i)^{\frac{-\beta}{a}}$$ nonadditive E, additive S

finite maximal/limiting temperature (a>0)!

Tsallis universality:

– additivity, T-pseudoadditivity and hiperT-pseudoadditivity

$$h(x,y) = \boxed{\boxed{\boxed{h_0 + cx + c_1 y} + axy + a_1 x^2 + a_2 y^2} + bxy^2 + b_1 x^2 y + b_2 y^2 x^2} + \ldots$$

$$h(0,0) = 0$$

$$h(x,0) = x$$

1) $\boxed{h_\infty(x,y) = x + y}$ additív

2) $\boxed{h_\infty(x,y) = x + y + axy}$ Tsallis-additív

3) $\boxed{h_\infty(x,y) = \dfrac{x + y + axy}{1 - \frac{b}{2}xy}}$ hiperTsallis-additív