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Tsallis–Pareto-like distributions in fragmentation

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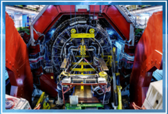
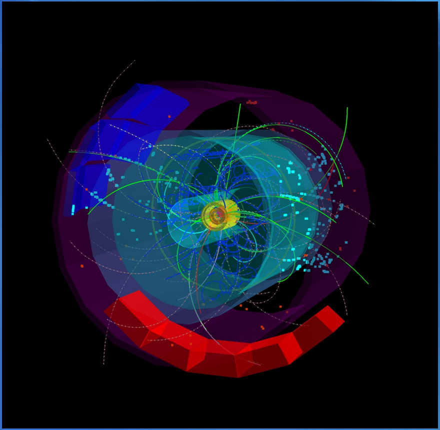
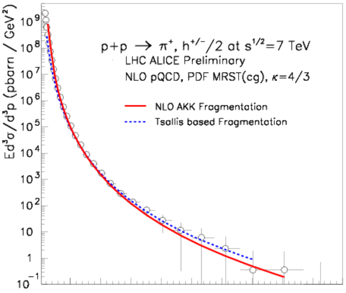


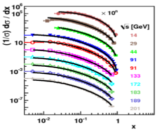
Motivation





e^+e^- annihilations

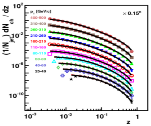
@LEP ($\sqrt{s}$ = 14–200 GeV)



Urmossy et. al.,
Phys. Lett. B, 701, 111-116 (2011),
arXiv:1101.3023

proton-proton collisions

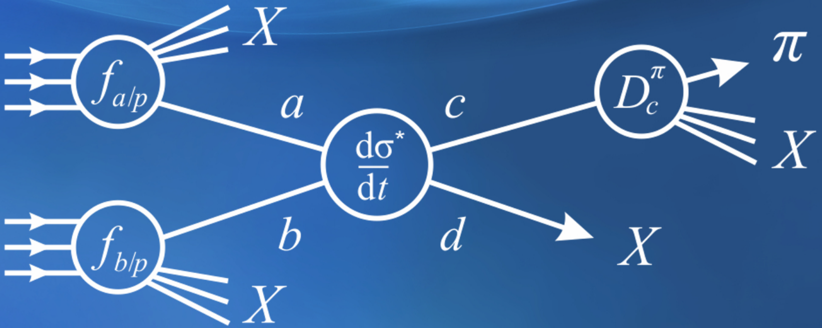
@LHC (p_T = 25–500 GeV/c)



Urmossy et. al.,
arXiv:1204.1508v1

Theoretical background

Proton-proton collision in the **parton model**



$$E_\pi \frac{d\sigma_\pi^{pp}}{d^3p_\pi} \sim f_{a/p}(x_a, Q^2) \otimes f_{b/p}(x_b, Q^2) \otimes \frac{d\sigma^{ab\to cd}}{d\hat{t}} \otimes \frac{D_c^\pi(z_c, Q^2)}{\pi z_c^2}$$

Fragmentation functions in e^+e^- annihilation

Let us define the total fragmentation function:

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{\mathrm{d}\sigma(e^+ + e^- \to h + X)}{\mathrm{d}z}$$

where z is the momentum fraction: $z = E_{hadron}/E_{beam} = 2 \cdot E_h/Q$
It can be shown that:

$$F^h(z,Q^2) = \sum_i C_i(z,\alpha_s) \otimes D_i^h(z,Q^2)$$

where $\otimes$ is the convolution integral:

$$f(z) \otimes g(z) = \int_z^1 \frac{\mathrm{d}y}{y} f(y)\, g\left(\frac{z}{y}\right)$$

The Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equation

The evolution equation for partonic fragmentation functions:

$$\frac{\partial}{\partial \ln Q^2} \begin{pmatrix} D_S^h(z, Q^2) \\ D_g^h(z, Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \begin{pmatrix} P_{qq}(z) & 2N_f P_{qq}(z) \\ P_{qg}(z) & P_{gg}(z) \end{pmatrix} \otimes D^h$$

where $D_S^h(z, Q^2)$ denotes the singlet function:

$$D_S^h(z, Q^2) = \sum_q \left[D_q^h(z, Q^2) + D_{\bar{q}}^h(z, Q^2) \right]$$

Solution:
M. Hirai and S. Kumano, Comput. Phys. C. **183** (2012) 1002-1013

Fragmentation functions

Unfolded: $F^h(x,Q^2) = \sum_q \sigma_{0,f}(Q^2) \cdot D_q^h(x,Q^2) +$

$$+ \int_x^1 \left[\frac{1}{z}\frac{\alpha_s}{2\pi} C_F \cdot (1+z^2) \cdot \sigma_{0,f}(Q^2) \cdot D_q^h(x/z,Q^2)\right] \cdot \left[\frac{\ln(1-z)}{1-z}\right] \mathrm{d}z -$$

$$- \int_x^1 \left[\frac{\alpha_s}{2\pi} C_F \cdot 2 \cdot \sigma_{0,f}(Q^2) \cdot D_q^h(x,Q^2)\right] \cdot \left[\frac{\ln(1-z)}{1-z}\right] \mathrm{d}z -$$

$$- \int_x^1 \left[\frac{1}{z}\frac{\alpha_s}{2\pi} C_F \cdot \frac{3}{2} \cdot \sigma_{0,f}(Q^2) \cdot D_q^h(x/z,Q^2)\right] \cdot \left[\frac{1}{1-z}\right] \mathrm{d}z +$$

$$+ \int_x^1 \left[\frac{\alpha_s}{2\pi} C_F \cdot \frac{3}{2} \cdot \sigma_{0,f}(Q^2) \cdot D_q^h(x,Q^2)\right] \cdot \left[\frac{1}{1-z}\right] \mathrm{d}z +$$

$$+\int_x^1 \frac{1}{z}\frac{\alpha_s}{2\pi} C_F \left[2\frac{1+z^2}{1-z}\ln z\right] \sigma_{0,f}(Q^2)\cdot D_q^h(x/z,Q^2)\,\mathrm{d}z +$$

$$+\int_x^1 \frac{1}{z}\frac{\alpha_s}{2\pi} C_F \left[\frac{3}{2}(1-z)\right] \sigma_{0,f}(Q^2)\cdot D_q^h(x/z,Q^2)\,\mathrm{d}z +$$

$$+\frac{\alpha_s}{2\pi} C_F \cdot \left(\frac{2}{3}\pi^2 - \frac{9}{2}\right) \cdot \sigma_{0,f}(Q^2)\cdot D_q^h(x,Q^2) +$$

$$+\int_x^1 \frac{1}{z}\frac{\alpha_s}{2\pi} C_F \cdot \left[2\frac{1+(1-z)^2}{z}\left[\ln(1-z)\right]\right] \sigma_0(Q^2)\cdot D_g^h(x/z,Q^2)\,\mathrm{d}z +$$

$$+\int_x^1 \frac{1}{z}\frac{\alpha_s}{2\pi} C_F \cdot \left[2\frac{1+(1-z)^2}{z}\left[2\ln z\right]\right] \sigma_0(Q^2)\cdot D_g^h(x/z,Q^2)\,\mathrm{d}z +$$

$$+\sum_q \int_x^1 \frac{1}{z}\frac{\alpha_s}{2\pi} C_F \cdot 1 \cdot \sigma_{0,f}(Q^2)\cdot D_q^h(x/z,Q^2)\,\mathrm{d}z\,.$$

Determining fragmentation functions

Steps for obtaining partonic fragmentation functions:

- Define the $D_i^h(z, Q^2)$ ansatz at some initial Q_0^2 scale
- Evolve it to the desired Q^2 using DGLAP equations
- Compute convolutions with $C_i(z, \alpha_s)$ coefficient functions
- Adjust the parameters of ansatz to fit the data

How to adjust the parameters?

- Define the appropriate merit function (e.g. χ^2)
- Minimize it!

Widely used parametrizations

The common ansatz:

$$D_i^h(z, Q^2) = N_i^h \cdot z^{\alpha_i^h} \cdot (1 - z)^{\beta_i^h}$$

Well known examples:

- *AKK:* S. Albino, B. A. Kniehl, and G. Kramer, Nucl. Phys. **B803** (2008) 42-104
- *PKHFF:* S. Kretzer, Phys. Rev. **D62** (2000) 054001
- *HKNS:* M. Hirai, S. Kumano, T.-H. Nagai, and K. Sudoh, Phys. Rev. **D75** (2007) 094009

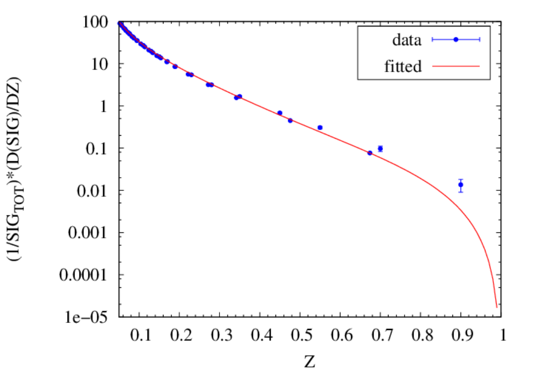
Problem: where is the physics?

Results

Results for common polynomial ansatz

Data: BUSKULIC et al. (ALEPH), Zeit. Phys. **C66**, 355 (1994)

Inclusive $\pi^{\pm}$, $K^{\pm}$ and (p, anti-p) differential cross-sections at the Z resonance



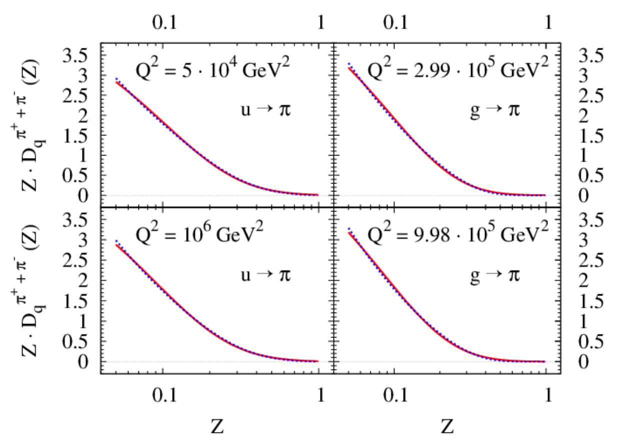
The Tsallis–Pareto-based fragmentation function

We have seen that Tsallis–Pareto distribution fits to partonic fragmentation functions, see:

G. G. Barnaföldi, T. S. Biró, K. Ürmössy, and G. Kalmár, *Tsallis–Pareto-like distributions in hadron-hadron collisions*, Proceedings of the Gribov '80 Memorial Workshop (2010)

The used Tsallis–Pareto-like ansatz: $f(z) = \frac{1}{b} \cdot \frac{1}{z} \cdot \left(1 + \frac{a \cdot z}{T}\right)^{-1/a}$

The Tsallis–Pareto-based fragmentation function



Results for Tsallis–Pareto-like ansatz

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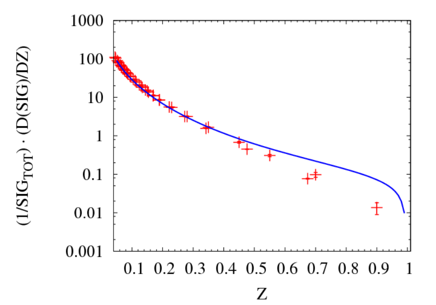
Inclusive $\pi^{\pm}$, $K^{\pm}$ and (p, anti-p) differential cross-sections at the Z resonance



Technical problems, code rewritten, new fits are in progress...

Outlook

Future tasks:

- The code has been rewritten, as there were technical problems with the old one
- We have to run Tsallis–Pareto fits
- We need real *physical interpretation* of the parameters
- We should do Tsallis–Pareto-based *cross section* calculation

Thanks for your attention!