# Charmonium spectral functions in 2+1 flavour lattice QCD

Attila Pásztor [1]
apasztor@bodri.elte.hu
In collaboriation with
S. Borsányi[2] S. Dürr[23] Z. Fodor[123] C. Hoelbling[2] S. D. Katz[1]
S. Krieg[23] D. Nógrádi[1] B. C. Tóth[2] N. Trombitás[1]
K. K. Szabó[23]

[1]Eötvös University

[2]University of Wuppertal

[3]Jülich Supercomputing Center

Zimányi Winter School on Heavy Ion Physics, 2012, Budapest

# Outline of the talk

## Outline of the talk

- Definition of the SPF, what is it good for?
- The Maximum Entropy Method
- Demonstration of the method with mock data analysis
- Recent results on J/Ψ and $\eta_c$

# Preliminaries

## Hadronic current operators

$$J_H(\vec{x},t) = \bar{q}(\vec{x},t)\Gamma_i q(\vec{x},t)$$
$$\Gamma_H = 1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5 \quad \text{for} \quad \mathrm{H} = \mathrm{S}, \mathrm{P}, \mathrm{V}, \mathrm{A}$$

## Lattice → Euclidean correlator

$$G(\tau,\vec{p}) = D_H^{>}(-i\tau,\vec{x}) = \int d^3x e^{ipx} \left\langle T_\tau J_H(-i\tau,\vec{x}) J_H(0,\vec{0}) \right\rangle$$

# Preliminaries

## Spectral function

$$A_H(\omega) = \frac{1}{\pi}\mathrm{Im}\mathrm{D}^{\mathrm{R}}_{\mathrm{H}}(\mathrm{p}_0, \tilde{\mathrm{p}}) = ... =$$

$$\frac{(2\pi)^2}{Z}\sum_{m,n}\left(e^{-E_n/T} \pm e^{-E_m/T}\right)|\langle n|J_H(0)|m\rangle|^2\,\delta^{(4)}(p_\mu - k^n_\mu + k^m_\mu)$$

- $G \to A$ Analytic continuation problem
- Stable particle at T=0 $\to$ $\delta$-peak
- Quasiparticle in matter $\to$ smeared peak
- Accessible by measuring dilepton rates in HI collisions ($T \neq 0$)
- Accessible by measuring R-ratio in lepton collisions ($T = 0$)
- Kubo-formulas $\to$ transport coeffiecients

# Determining the spectral function

## Relation to the Euclidean time correlator

$$G(\tau, \vec{p}) = \int_0^\infty d\omega A(\omega, \vec{p}) K(\omega, \tau)$$

$$K(\omega, \tau) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

## Regularizing

This inversion is ill-defined. Minimum of $\chi^2$ degenerate. To get a unique answer one has to regularize. A commonly used regularization scheme is the Maximum Entropy Method.

# The Maximum Entropy Method

## The method in a nutshell

$$Q = \alpha S - \frac{1}{2}\chi^2$$

$$S = \int d\omega \left( A(\omega) - m(\omega) - A(\omega)\log\left(\frac{A(\omega)}{m(\omega)}\right)\right)$$

$$\chi^2 = \sum_{i,j} (G_i^{\mathrm{fit}} - G_i^{\mathrm{data}}) C_{ij}^{-1} (G_j^{\mathrm{fit}} - G_j^{\mathrm{data}})$$

$$G_i = \int A(\omega) K(\omega, \tau_i) \mathrm{d}\omega$$

$m(\omega)$ is a function, summarizing our prior knowledge of the solution. Then we average over $\alpha$. The conditional probability $P[\alpha|\mathrm{data}, \mathrm{m}]$ is given by Bayes' theorem.

# Analysis with mock data

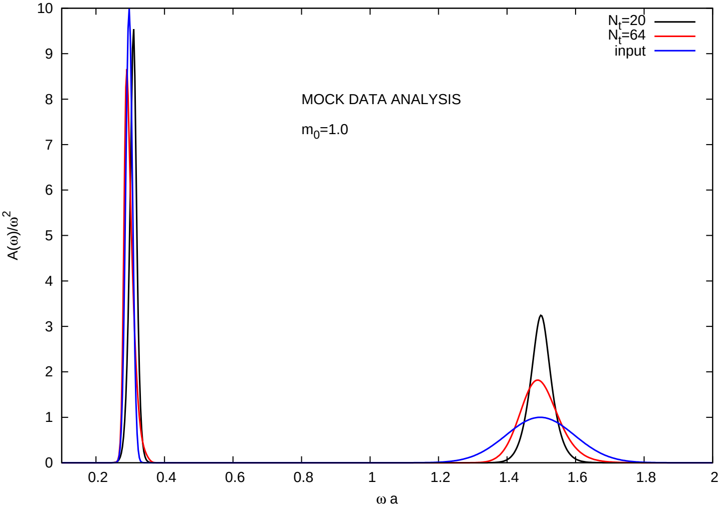
## Strategy

- Write down an input spectral function $A_{in}$
- Generate correlators by integrating

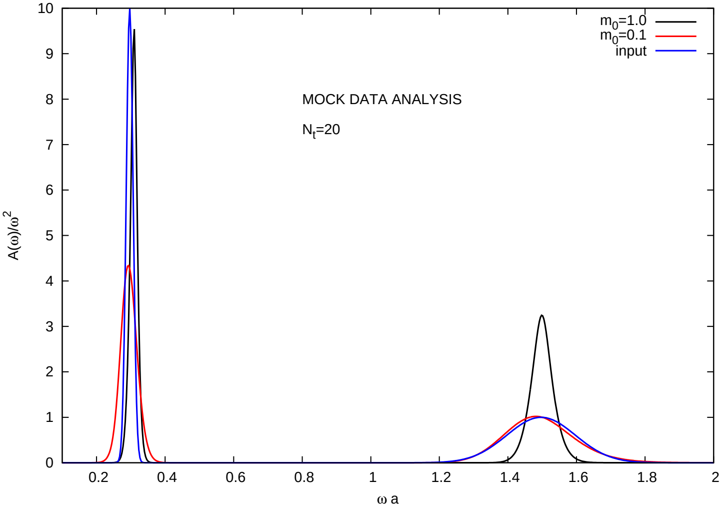
$$G(\tau) = \int K(\tau, \omega) A_{in}(\omega) d\omega$$

- Generate mock data by adding gaussian noise with variance $\sigma(\tau) = \eta \cdot \tau \cdot G_{mock}(\tau)$.
- Reconstruct spectral function with MEM. For now, we will use a prior function: $m(\omega) = m_0 \omega^2$

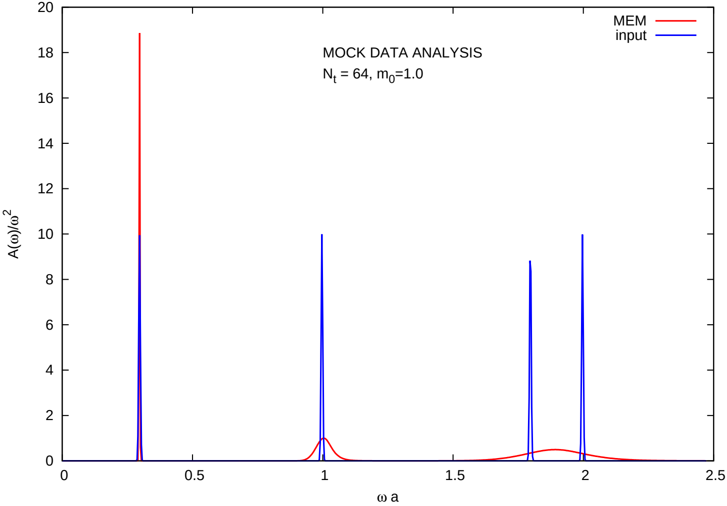
# How many data points are enough?



# Sensitivity on the prior function



# Individual resolution of many peaks



# Conclusion of the mock data analysis

## Lessons learned

- MEM gives the correct qualitative features of the spectral function, but it is not a precise quantitative method.
- The peak positions agree well with the input, the shapes do not
- As long as the data points are not too noisy, O(10) point are enough for reconstruction.
- Features that remain unchanged by varying the prior are restricted by the data.
- Peaks close in position can be merged into one broader peak.

# Analysis with lattice QCD data

## Quenched approximation

In our simulation, **u**,**d**, and **s** quarks, are **dynamical**, but c quarks are not. They are in the quenched approximation. There are no internal charm quark loops, they are only at tree level.

## Mass tuning

We had $m_s$ and $m_{ud}$ already tuned (Science 322 (2008) 1224-1227, `arXiv:0906.3599`) We adopted a charm mass, chosen so that the ratios of the masses of $J/\Psi$, $\eta_c$, $D_s$ and $\varphi$ mesons are (close to) physical.

## Motivation for 2+1 flavour calculation

The quenched approximation in the light quark sector is a good approximation only at zero temperature. Not only the transition temperature, but also the qualitative behaviour is quite different (quenched QCD: 1st order transition, dynamical quarks: cross-over).

# Analysis with lattice QCD data

## Lattice details

Gauge action = Symanzik tree-level improved gauge action
Fermion action = 2+1 dynamical Wilson fermions with 6 step stout smearing ($\rho = 0.11$) and tree-level clover improvement

| $a$[fm] | $am_{ud}$ | $am_s$ | $m_\pi$ | $N_s$ | $N_t$ | $T = \frac{1}{N_t a}$ |
|---|---|---|---|---|---|---|
| 0.057(1) | -0.00336 | 0.0050 | 545MeV | 64 | 64 | $\approx$ 0MeV |
| 0.057(1) | -0.00336 | 0.0050 | 545MeV | 64 | 28 | 123MeV |
| 0.057(1) | -0.00336 | 0.0050 | 545MeV | 64 | 20 | 173MeV |
| 0.057(1) | -0.00336 | 0.0050 | 545MeV | 64 | 18 | 192MeV |
| 0.057(1) | -0.00336 | 0.0050 | 545MeV | 64 | 16 | 216MeV |
| 0.057(1) | -0.00336 | 0.0050 | 545MeV | 64 | 14 | 247MeV |
| 0.057(1) | -0.00336 | 0.0050 | 545MeV | 64 | 12 | 288MeV |

# Spectroscopy

## Meson masses

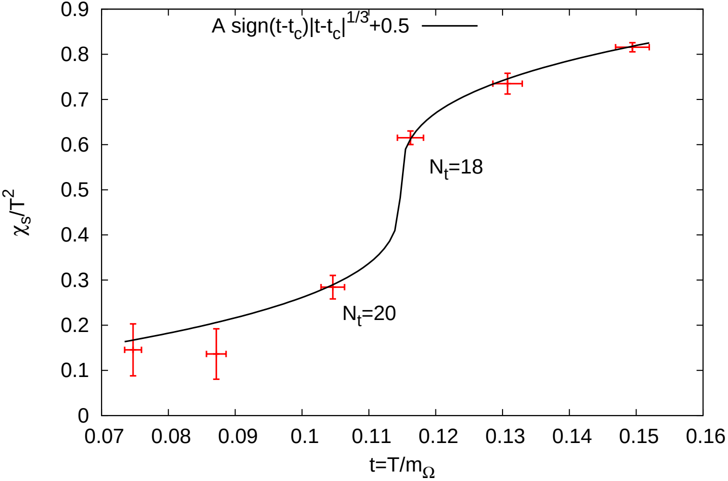
Obtained by fitting $A\cosh(ma(t - N_t/2))$ to the smeared correlators measured on the 0 temp. lattice. Errors from jackknife method.

## Results

| $J^P$ | $m_i$ | name | ma | $ma/m_\Omega a$ | $m_{exp}[MeV]$ | $m_{exp}/m_{D_s^*}$ |
|---|---|---|---|---|---|---|
| $0^-$ | $m_s$,$m_s$ | $\eta$,? | 0.1828(3) | 0.321(1) | ??? | ? |
| $0^-$ | $m_s$,$m_c$ | $D_s$ | 0.54(1) | 0.95(2) | 1968.49 | 0.932 |
| $0^-$ | $m_c$,$m_c$ | $\eta_c$ | 0.8192(7) | 1.437(4) | 2981.0 | 1.411 |
| $1^-$ | $m_s$,$m_s$ | $\phi$,? | 0.270(1) | 0.474(3) | 1019.455, ? | 0.483,? |
| $1^-$ | $m_s$,$m_c$ | $D_s^*$ | 0.570(1) | 1 | 2112.3 | 1 |
| $1^-$ | $m_c$,$m_c$ | $J/\Psi$ | 0.8388(8) | 1.472(2) | 3096.916 | 1.466 |

? - The disconnected diagrams are not taken into account, so these "correlation masses" are not the same as the physical meson masses.

# Strange quark susceptibility → transition temperature



# Outline of MEM procedure

## On the zero temperature lattice

- Drop data points, emulating the number of data points available at finite temperature (remember $T = 1/(N_t a)$)
- Do the same analysis as with the finite temperature correlators. If the ground state peak cannot be reconstructed, the given number of data points is not reliable
- RESULT: $N_t$=12 NOT OK, $N_t$=14,16,18,20 OK Position of ground state peak is always somewhat overerestimated.

## Finite temperature

- Systematic error analysis: vary $\Delta\omega$, $N_\omega$, the shape of the prior function: $m_0$, $m_0\omega^2$, $1/(m_0 + \omega)$ and $m_0$=0.01, 0.1, 1.0, 10.0.
- Statistical error analysis: given set of parameters, 20 jackknife samples

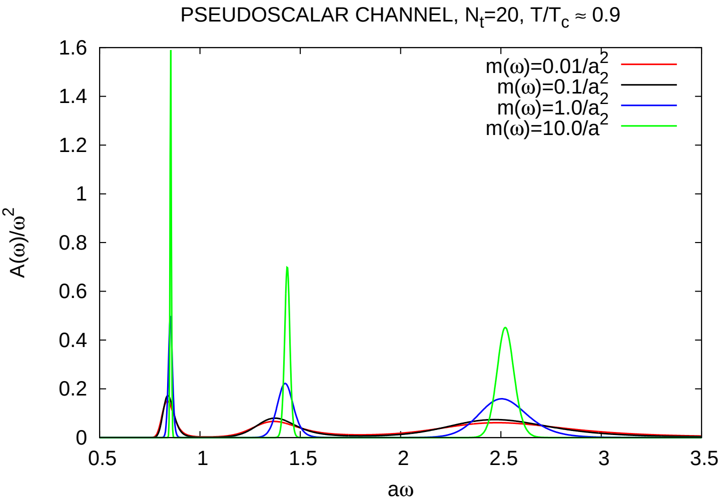
# Preliminary results on J/$\Psi$ and $\eta_c$

## Expectations of heavy ion physicists

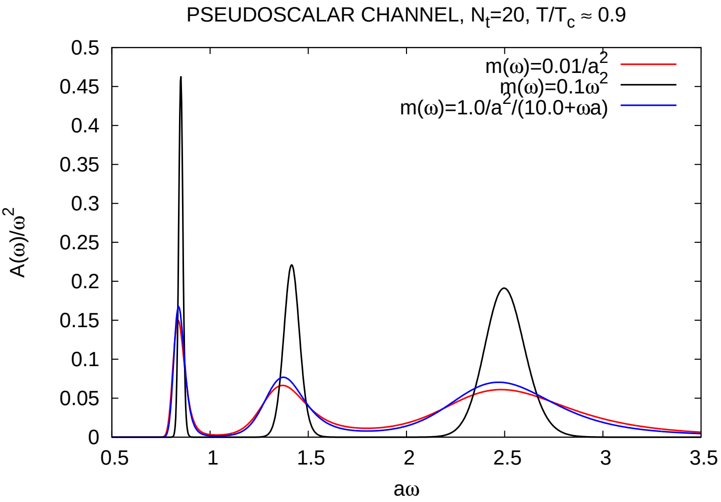
- At sufficiently high temperature, QCD undergoes a transition to a deconfined phase.
- Unlike light mesons, heavy mesons like J/$\Psi$ may survive in the hot medium up to higher temperatures, before dissociating because of colour screening, and collisions within the medium.
- Their supression may be a good experimental signal on the formation of QGP.

Original paper on the idea:
T. Matsui, H. Satz, Phys. Lett. B178, 416 (1986).

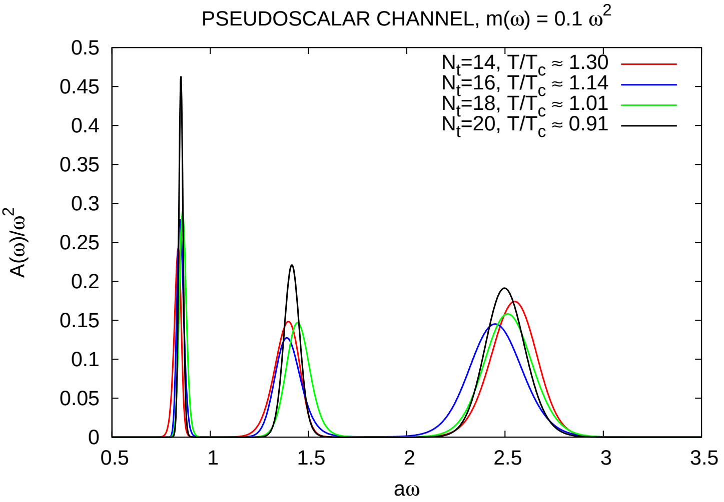
# Prior function sensitivity



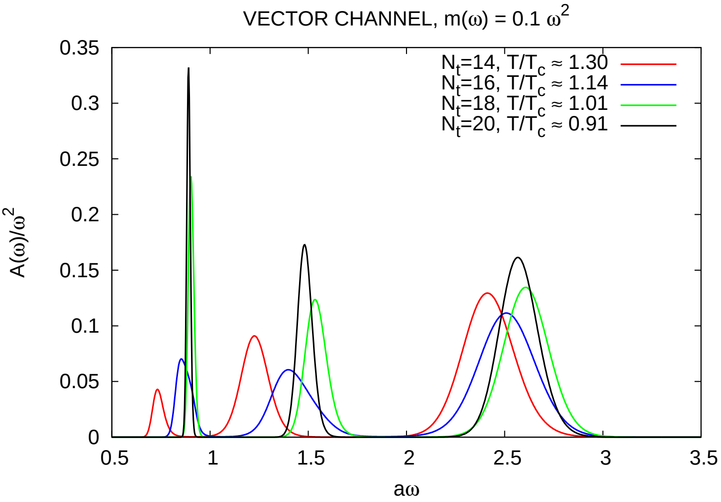
# Prior function sensitivity



# Temperature dependence, $\eta_c$



# Temperature dependence, $J/\Psi$



# Qualitative remarks

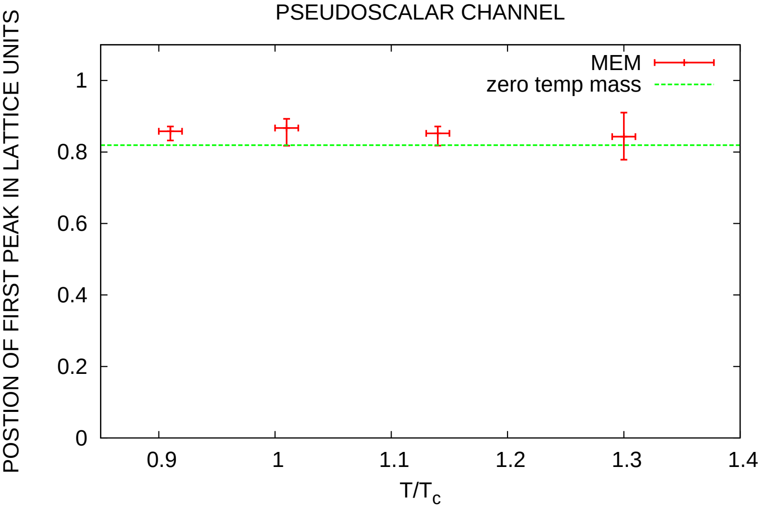
## Pseudoscalar channel

The ground state peak is clearly seen on all available temperatures. The position is at a slightly higher energy than the mass at zero temperature (most likely an artifact).

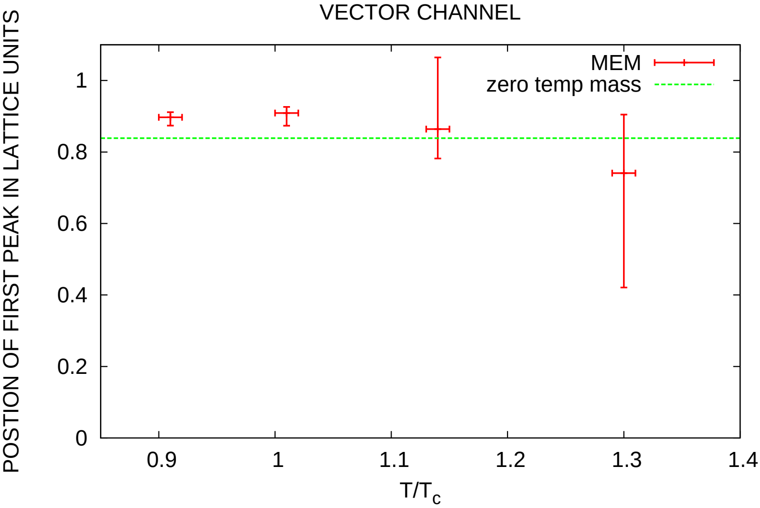
## Vector channel

The ground state peak is clearly seen on the $N_t$=16,18,20 lattices. The position is at a slightly higher energy than the mass at zero temperature. On the $N_t$=14 lattice, for some reconstruction parameters, the first peak is seen at a considerably lower energy than the $J/\Psi$ mass, for other parameters, the first two peaks merge into one peak.

# Temperature dependence, $\eta_c$



# Temperature dependence, $J/\Psi$



# Conclusions and future work

## What's next?

- No clear indication of $\eta_c$ melting up to about $1.3T_c$
- Some indications of $J/\Psi$ melting at about $1.3T_c$
- To access higher temperatures, anisotropic lattices are needed
- Anisotropy tuning with dynamical quarks is hard
- Anisotropy tuning with Wilson flow : hep-lat/1205.0781