# Scattering Amplitudes via AdS/CFT

Luis Fernando Alday

Utrecht University

Strings 2008 @ CERN

21 August 2008

In collaboration with J. Maldacena





## Motivations

We will be interested in gluon scattering amplitudes of planar $\mathcal{N}=4$ super Yang-Mills.

Motivation: It can give non trivial information about more realistic theories but is more tractable.

- Perturbative computations are easier. Higher loop computations are possible $\rightarrow$ proposal for all loops (MHV) $n$-point amplitudes.
- The strong coupling regime can be studied, by means of the gauge/string duality, through a weakly coupled string sigma model.

## Aim of these project

Prescription for computing scattering amplitudes of planar $\mathcal{N}=4$ super Yang-Mills at strong coupling by using the $AdS/CFT$ correspondence.







# Gauge theory amplitudes, Dixon's talk

$A_n^{L,Full} \sim \sum_\rho Tr(T^{a_{\rho(1)}}...T^{a_{\rho(n)}})A_n^{(L)}(\rho(1),...,\rho(2))$

- Leading $N_c$ color ordered $n-$points amplitude at $L$ loops: $A_n^{(L)}$
- The amplitudes are IR divergent.
- Dimensional regularization $D=4-2\epsilon \rightarrow A_n^{(L)}(\epsilon)=1/\epsilon^{2L}+...$
- Focus on MHV amplitudes and scale out the tree amplitude $M_n^{(L)}(\epsilon)=A_n^{(L)}/A_n^{(0)}$.

## BDS proposal for all loops MHV amplitudes (Bern, Dixon, Smirnov)

$$\log \mathcal{M}_n = \sum_{i=1}^{n}\left(-\frac{1}{8\epsilon^2}f^{(-2)}\left(\frac{\lambda\mu^{2\epsilon}}{s_{i,i+1}^{\epsilon}}\right)-\frac{1}{4\epsilon}g^{(-1)}\left(\frac{\lambda\mu^{2\epsilon}}{s_{i,i+1}^{\epsilon}}\right)\right)+f(\lambda)Fin_n^{(1)}(k).$$

$f(\lambda),\ g(\lambda) \rightarrow$ cusp/collinear anomalous dimension.

# $AdS/CFT$ duality

**$AdS/CFT$ duality** (Maldacena)

| | | |
|---|---|---|
| Four dimensional maximally SUSY Yang-Mills | $\Leftrightarrow$ | Type IIB string theory on $AdS_5 \times S^5$. |

We will study scattering amplitudes at strong coupling by using the $AdS/CFT$ duality.

- Set up the computation: Use a $D - brane$ as IR cut-off.
- Actual computations: Dimensional regularization.

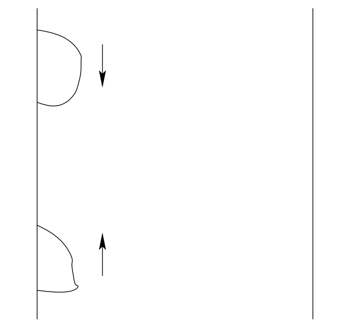




# String theory set up

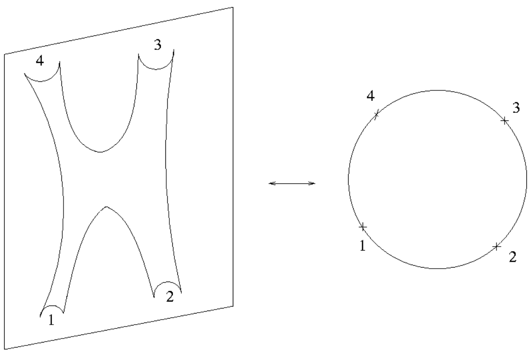
$$ds^2 = R^2 \frac{dx_{3+1}^2 + dz^2}{z^2}$$

- Place a D-brane extended along $x_{3+1}$ and located at $z_{IR} \gg R$.




- The asymptotic states are open strings ending on the D-brane.
- Consider the scattering of these open strings.
- $k_{pr} = k\frac{z_{IR}}{R}$ is very large: fixed angle and very high momentum.
- Intuition from flat space ( Gross and Mende): Amplitude is dominated by a saddle point.

World-sheet with the topology of a disk with vertex operator insertions (corresponding to external states)




- Near each vertex operator, the momentum of the external state fixes the form of the solution.
- The boundary of the world-sheet sits at $z = z_{IR}$.

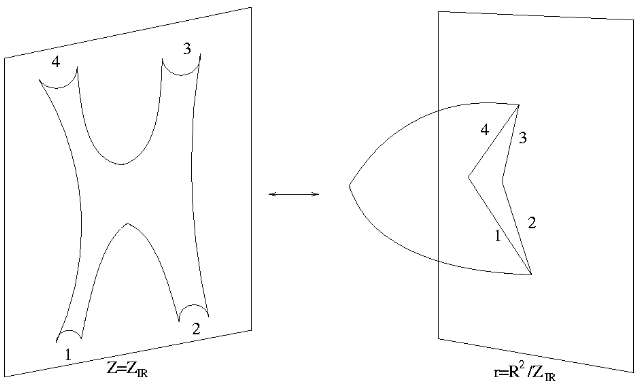
- T-duality in $x^\mu$ directions followed by a change of coordinates $r = R^2/z \longrightarrow$ we end up again with $AdS_5$!

$$ds^2 = R^2 \frac{dx_{3+1}^2 + dz^2}{z^2} \quad \longrightarrow \quad ds^2 = R^2 \frac{dy_{3+1}^2 + dr^2}{r^2}$$



introduction
String theory set up
Explicit example and recent developments
Other processes
Scattering amplitudes vs. twist two operators
Conclusions and outlook

The world-sheet boundary is located at $r = R^2/z_{IR}$ and is a particular line constructed as follows...




- For each particle with momentum $k^\mu$ draw a segment joining two points separated by $\Delta y^\mu = 2\pi k^\mu$
- Concatenate the segments according to the ordering of the insertions on the disk (particular color ordering).

- As $z_{IR} \to \infty$ the boundary of the world-sheet moves to $r = 0$.
- Exactly same computation as when computing the expectation value of a Wilson-Loop given by a sequence of light-like segments!

Luis Fernando Alday Scattering Amplitudes via AdS/CFT

## Prescription

- $\mathcal{A}_n$: Leading exponential behavior of the $n-$point scattering amplitude.
- $A_{min}(k_1^\mu, k_2^\mu, ..., k_n^\mu)$: Area of a minimal surface that ends on a sequence of light-like segments on the boundary.

$$\mathcal{A}_n \sim e^{-\frac{\sqrt{\lambda}}{2\pi} A_{min}}$$

- Prefactors are subleading in $1/\sqrt{\lambda}$, and we don't compute them.
- In particular our computation is blind to helicity (and hence works also for non MHV)

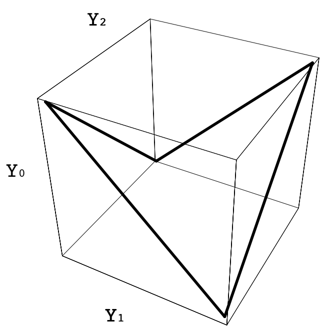




## Four point amplitude at strong coupling

Consider $k_1 + k_3 \longrightarrow k_2 + k_4$

- The simplest case $s = t$.




Need to find the minimal surface ending on such sequence of light-like segments

$$\begin{aligned} r(y_1, y_2) &= \sqrt{(1-y_1^2)(1-y_2^2)} \\ y_0 &= y_1 y_2 \end{aligned}$$

- The "dual" $AdS$ space possesses isometries $SO(2,4)$.
- This dual conformal symmetry takes this solution to the most general one!

Let's compute the area...

- Small problem: The area diverges!
- Dimensional reduction scheme: Start with $\mathcal{N}=1$ in D=10 and go down to $D=4-2\epsilon$.
- For integer $D$ this is exactly the low energy theory living on $Dp-$branes ($p=D-1$)

**Gravity dual**

$$ds^2 = h^{-1/2} dx_D^2 + h^{1/2}\left(dr^2 + r^2 d\Omega_{9-D}^2\right), \qquad h = \frac{c_D \lambda_D}{r^{8-D}}$$

$$\lambda_D = \frac{\lambda \mu^{2\epsilon}}{(4\pi e^{-\gamma})^\epsilon} \qquad c_D = 2^{4\epsilon}\pi^{3\epsilon}\Gamma(2+\epsilon)$$



introduction
String theory set up
Explicit example and recent developments
Other processes
Scattering amplitudes vs. twist two operators
Conclusions and outlook

**T-dual coordinates**

$$ds^2 = \sqrt{\lambda_D c_D}\left(\frac{dy_D^2 + dr^2}{r^{2+\epsilon}}\right) \to S_\epsilon = \frac{\sqrt{\lambda_D c_D}}{2\pi}\int \frac{\mathcal{L}_{\epsilon=0}}{r^\epsilon}$$

- Presence of $\epsilon$ will make the integrals convergent.
- The eoms will depend on $\epsilon$ but if we plug the original solution into the new action, the answer is accurate enough.
- plugging everything into the action...

$$S \approx -\frac{\sqrt{\lambda_D c_D}}{2\pi a^\epsilon}\,{}_2F_1\left(\frac{1}{2},-\frac{\epsilon}{2},\frac{1-\epsilon}{2};b^2\right) + \mathcal{O}(\epsilon)$$

- Just expand in powers of $\epsilon$...

Luis Fernando Alday Scattering Amplitudes via AdS/CFT

## Final answer

$$\mathcal{A} = e^{iS} = \exp\left[iS_{div} + \frac{\sqrt{\lambda}}{8\pi}\left(\log\frac{s}{t}\right)^2 + \tilde{C}\right]$$

$$S_{div} = 2S_{div,s} + 2S_{div,t}$$

$$S_{div,s} = -\frac{1}{\epsilon^2}\frac{1}{2\pi}\sqrt{\frac{\lambda\mu^{2\epsilon}}{(-s)^\epsilon}} - \frac{1}{\epsilon}\frac{1}{4\pi}(1-\log 2)\sqrt{\frac{\lambda\mu^{2\epsilon}}{(-s)^\epsilon}}$$

- Should be compared to the field theory answer

$$\mathcal{A} \sim (\mathcal{A}_{div,s})^2(\mathcal{A}_{div,t})^2\exp\left\{\frac{f(\lambda)}{8}(\ln s/t)^2 + const\right\}$$

$$\mathcal{A}_{div,s} = \exp\left\{-\frac{1}{8\epsilon^2}f^{(-2)}\left(\frac{\lambda\mu^{2\epsilon}}{s^\epsilon}\right) - \frac{1}{4\epsilon}g^{(-1)}\left(\frac{\lambda\mu^{2\epsilon}}{s^\epsilon}\right)\right\}$$



introduction
String theory set up
Explicit example and recent developments
Other processes
Scattering amplitudes vs. twist two operators
Conclusions and outlook

- $SO(2,4)$ transformations fixed somehow the kinematical dependence of the finite piece.
- This dual conformal symmetry constrains the form of the amplitude

## Dual Ward identity

$$\mathcal{O}_K\mathcal{A} = 0 \quad \rightarrow \quad \mathcal{O}_k Fin = -\mathcal{O}_k Div$$

For $n = 4, 5$ the solution is unique and agrees with BDS! for $n = 6$ there is some freedom.

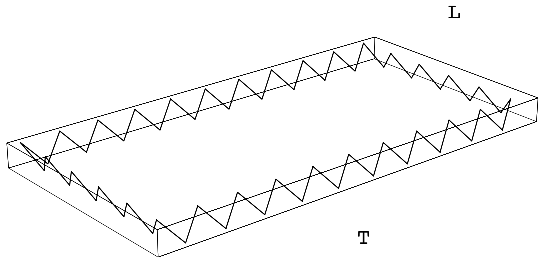
- Also perturbatively and even dual super-conformal symmetry on NMHV (Sokatchev's talk).
- Dual super-conformal symmetry present for all values of the coupling! (Berkovits's talk)

Luis Fernando Alday Scattering Amplitudes via AdS/CFT

What about the BDS ansatz?

- Symmetries "protect" BDS from corrections, we need to consider $n > 5$. What about $n = \infty$?

We choose a zig-zag configuration that approximates the rectangular Wilson loop.




- $\log < W_{rect}^{weak} >= \frac{\lambda}{8\pi}\frac{T}{L}$
- $\log < W_{rect}^{strong} >= \sqrt{\lambda}\frac{4\pi^2}{\Gamma(\frac{1}{4})^4}\frac{T}{L}$

- While BDS $\rightarrow \log < W_{rect}^{strong} >= \frac{\sqrt{\lambda}}{4}\frac{T}{L}$ . Impresive explicit computations showed that indeed BDS fails for 6 gluons at two loops! ( Bern, Dixon, Kosower, Roiban, Spradlin, Vergu, Volovich)



introduction
String theory set up
Explicit example and recent developments
Other processes
Scattering amplitudes vs. twist two operators
Conclusions and outlook

- This computation shows a relation between Wilson loops and scattering amplitudes.

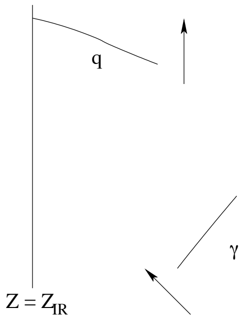
**Scattering amplitudes vs. WL**

$$\mathcal{M}(k_1, ..., k_n) \approx < W(x_1, ..., x_n) >, \qquad k_i = x_{i+1} - x_i$$

- This relation holds also at weak coupling! (for MHV amplitudes)
  - Four legs at one loop ( Drummond, Korchemsky, Sokatchev)
  - $n$ legs at one loop ( Brandhuber, Heslop, Travaglini )
  - Up to six legs at two loops ( Drummond, Henn, Korchemsky, Sokatchev)

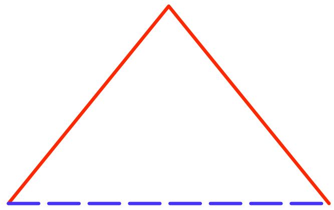
Luis Fernando Alday Scattering Amplitudes via AdS/CFT

## Other processes ( something like *meson/photon* $\longrightarrow q + \bar{q}$)




- Brane $I$ extending along $(x^0, x^1)$ at $Z = Z_{IR}$.
- Brane $II$ extending along $(x^0, x^1, z)$
- meson $\longrightarrow (II, II)$, quarks $\longrightarrow (I, II)$

- $(0, \kappa) \longrightarrow (\kappa/2, \kappa/2) + (-\kappa/2, \kappa/2)$
- After T-duality, a triangle in the $(y^0, y^1)$ plane with boundary conditions for $r$
- $r = 0$ in the red lines (quarks), $r = \infty$ in the blue line (meson).







- Also possible to consider *meson* $\longrightarrow q + \bar{q} + gluons$.
- Other processes like singlets into gluons.
- We could not find the solutions but the singular behavior can be understood

$$g_{quark}(\lambda) = \frac{\sqrt{\lambda}}{4\pi}(1 - 3\log 2)$$

# Scattering amplitudes vs. twist two operators

Consider a massless gauge theory

**High spin, twist two operators**

$$\mathcal{O}_S = Tr(\phi D^S \phi), \qquad S \gg 1 \qquad \Rightarrow \qquad \Delta_{\mathcal{O}_s} = f(\lambda)\log S - B(\lambda) + \ldots$$

**Scattering amplitudes**

$$\log \mathcal{A} = \frac{f(\lambda)}{\epsilon^2} + \frac{g(\lambda)}{\epsilon} + \ldots$$

- Is there any relation between $B(\lambda)$ and $g(\lambda)$?





- They are not equal but $g_R - B_R = C_R X$ , where $X$ is a universal function ( Dixon, Magnea, Sterman).
- All this quantities can be computed at strong coupling in planar $\mathcal{N} = 4$ SYM for gluons (adjoint) and quarks (fundamental).

$$B_{gg} = \frac{\sqrt{\lambda}}{\pi}(\log\left(\frac{\sqrt{\lambda}}{2\pi}\right) + 1 - 2\log 2)$$

$$B_{qq} = \frac{\sqrt{\lambda}}{2\pi}(\log\left(\frac{\sqrt{\lambda}}{2\pi}\right) + 1 - 3\log 2)$$

- Universality seems to hold at strong coupling!

## What needs to be done?

- Try to make explicit computations for $n > 4$, *e.g.* $n = 6$ is a good one.
- Subleading corrections in $1/\sqrt{\lambda}$? Information about helicity of the particles, etc.
- Gross and Mende computed higher genus amplitudes (in flat space) using similar ideas, can we do the same?
- Can we repeat the computation in other backgrounds?
- Deeper relation between Wilson loops and scattering amplitudes?
- Some powerful alternative to BDS?





## Conclusions and Outlook

- A lot of structure ( some discovered and hopefully more waiting to be discovered) behind scattering amplitudes of planar MSYM.
- For $n = 4, 5$, we think we know them to all values of the coupling!
- We haven't assume/use at all the machinery of integrability.

Talk by someone at strings 2009:

- Expression for all planar MSYM amplitudes at all values of the coupling.