Holographic recipes at finite density and temperature

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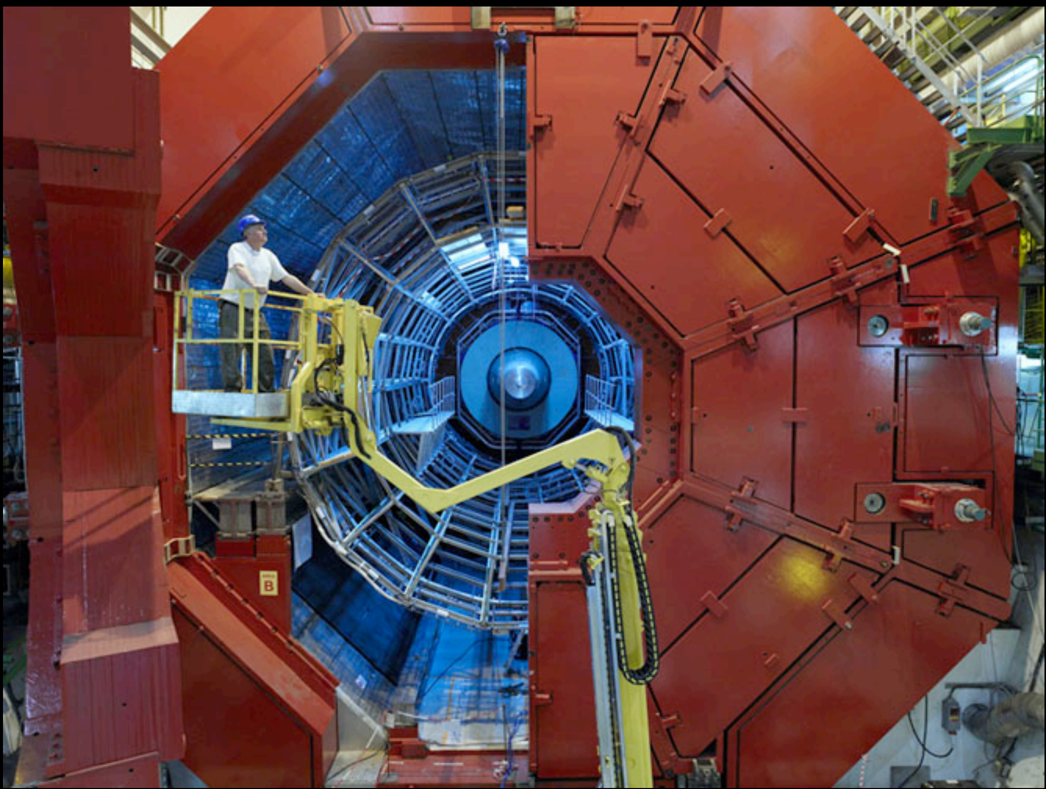
ALICE Detector CERN



Outline

Part I: A (very short) summary of a subset of the gauge/gravity duality results for strongly coupled gauge theories at finite temperature/density

- thermodynamics
- first- and second-order transport coefficients
- RHIC elliptic flow and viscosity/entropy ratio

Part II: Holographic recipes for quantum liquids

- specific heat in the low temperature limit
- zero sound excitation

Over the last several years, holographic (gauge/gravity duality) methods were used to study strongly coupled gauge theories at finite temperature and density

These studies were motivated by the heavy-ion collision programs at RHIC and LHC (ALICE detector) and the necessity to understand hot and dense nuclear matter in the regime of intermediate coupling $\alpha_s(T_{\mathrm{RHIC}}) \sim O(1)$

As a result, we now have a better understanding of thermodynamics and especially kinetics (transport) of strongly coupled gauge theories

Of course, these calculations are done for theoretical models such as N=4 SYM and its cousins (including non-conformal theories etc).

We don't know for QCD

Heavy ion collision experiments at RHIC (2000-current) and LHC (2008-??) create hot and dense nuclear matter known as the “quark-gluon plasma”

(note: qualitative difference between p-p and Au-Au collisions)

Evolution of the plasma “fireball” is described by relativistic fluid dynamics (relativistic Navier-Stokes equations)

Need to know

thermodynamics (equation of state)
kinetics (first- and second-order transport coefficients)
in the regime of intermediate coupling strength:

$$\alpha_s(T_{\rm RHIC}) \sim O(1)$$

initial conditions (initial energy density profile)

thermalization time (start of hydro evolution)

freeze-out conditions (end of hydro evolution)

Energy density vs temperature for various gauge theories

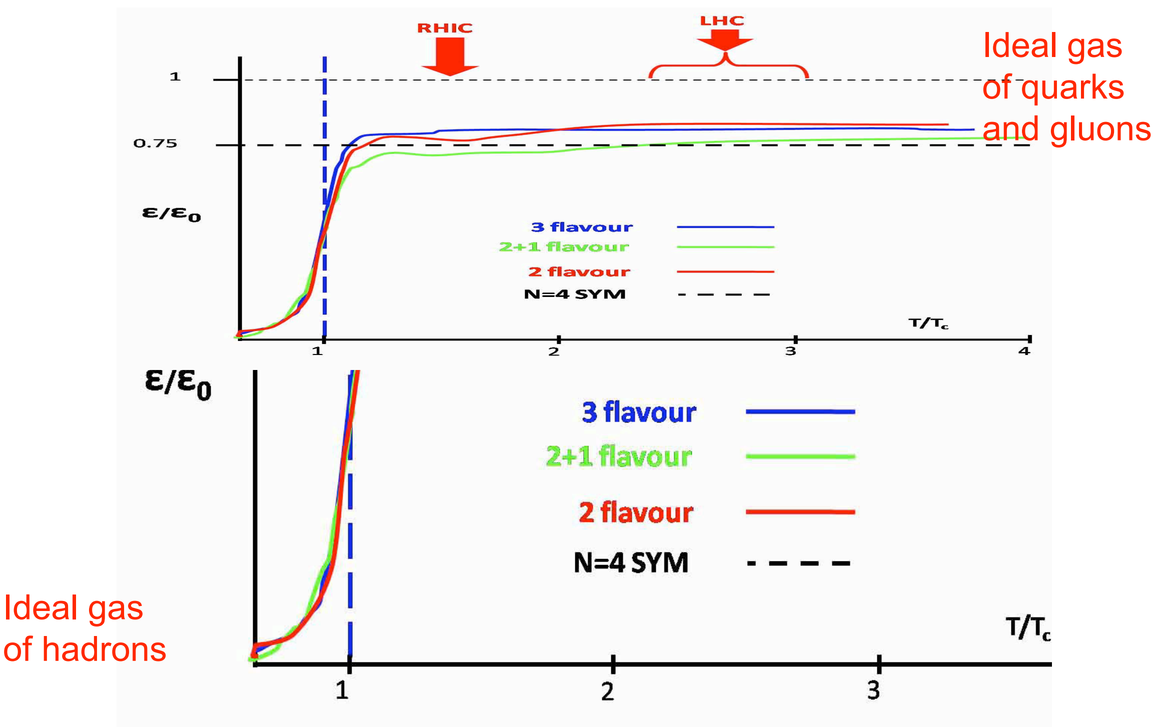


Figure: an artistic impression from Myers and Vazquez, 0804.2423 [hep-th]

First-order transport (kinetic) coefficients

Shear viscosity	η
Bulk viscosity	ζ
Charge diffusion constant	D_Q
Supercharge diffusion constant	D_s
Thermal conductivity	κ_T
Electrical conductivity	σ

* Expect Einstein relations such as $\frac{\sigma}{e^2 \Xi} = D_{U(1)}$ to hold

Computing transport coefficients from dual gravity

Assuming validity of the gauge/gravity duality,
all transport coefficients are completely determined
by the lowest frequencies
in quasinormal spectra of the dual gravitational background

(D.Son, A.S., hep-th/0205051, P.Kovtun, A.S., hep-th/0506184)

This determines kinetics in the regime of a thermal theory
where the dual gravity description is applicable

Transport coefficients and quasiparticle spectra can also be obtained from thermal spectral functions

First-order transport coefficients in N = 4 SYM

in the limit $N_c \to \infty\,, \quad g^2_{YM} N_c \to \infty$

Shear viscosity $\quad \eta = \frac{\pi}{8} N_c^2 T^3 \left[1 + O\left(\frac{1}{(g^2 N_c)^{3/2}}, \frac{1}{N_c^2} \right) \right]$

Bulk viscosity $\quad \zeta = 0$

for non-conformal theories see Buchel et al; G.D.Moore et al Gubser et al.

Charge diffusion constant $\quad D_R = \frac{1}{2\pi T} + \cdots$

Supercharge diffusion constant $\quad D_s = \frac{2\sqrt{2}}{9\pi T}$ NEW! (G.Policastro, 2008)

Thermal conductivity $\quad \frac{\kappa_T \; \mu^2}{\eta \; T} = 8\pi^2 + \cdots$

Electrical conductivity $\quad \sigma = e^2 \frac{N_c^2 \, T}{16\,\pi} + \cdots$

Electrical conductivity in $\mathcal{N} = 4$ SYM

Weak coupling: $\quad \sigma = 1.28349 \, \frac{e^2 \, (N_c^2 - 1)\, T}{\lambda^2 \left[\ln \lambda^{-1/2} + O(1) \right]}$

$\lambda \ll 1$

Strong coupling: $\quad \sigma = \frac{e^2 \, N_c^2 \, T}{16\,\pi} + O\left(\frac{1}{\lambda^{3/2}} \right)$

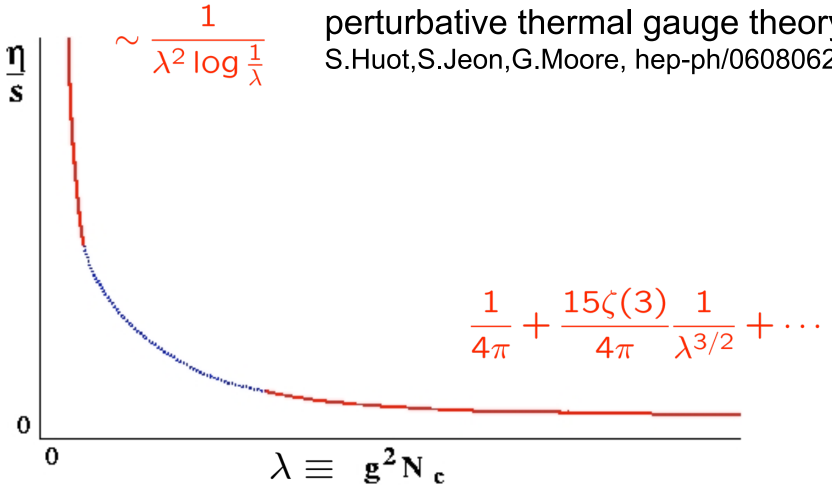
$\lambda \gg 1$

* Charge susceptibility can be computed independently: $\quad \Xi = \frac{N_c^2 T^2}{8}$

D.T.Son, A.S., hep-th/0601157

Einstein relation holds: $\quad \frac{\sigma}{e^2 \Xi} = D_{U(1)} = \frac{1}{2\pi T}$

Shear viscosity in $\mathcal{N} = 4$ SYM



perturbative thermal gauge theory
S.Huot,S.Jeon,G.Moore, hep-ph/0608062

Correction to $1/4\pi$: Buchel, Liu, A.S., hep-th/0406264

Buchel, 0805.2683 [hep-th]; Myers, Paulos, Sinha, 0806.2156 [hep-th]

Second-order transport coefficients in N = 4 SYM

in the limit $N_c \to \infty, \quad g^2_{YM} N_c \to \infty$

Relaxation time

Here we also used results from: S.Bhattacharyya,V.Hubeny,S.Minwalla,M.Rangamani, 0712.2456 [hep-th]

Generalized to CFT in D dim: Haack & Yarom, 0806.4602 [hep-th];

Finite ‘t Hooft coupling correction computed: Buchel and Paulos, 0806.0788 [hep-th]

Question: does this affect RHIC numerics?

Hydrodynamics: fundamental d.o.f. = densities of conserved charges

Need to add constitutive relations!

Example: charge diffusion

Conservation law $$\partial_t j^0 + \partial_i j^i = 0$$

Constitutive relation

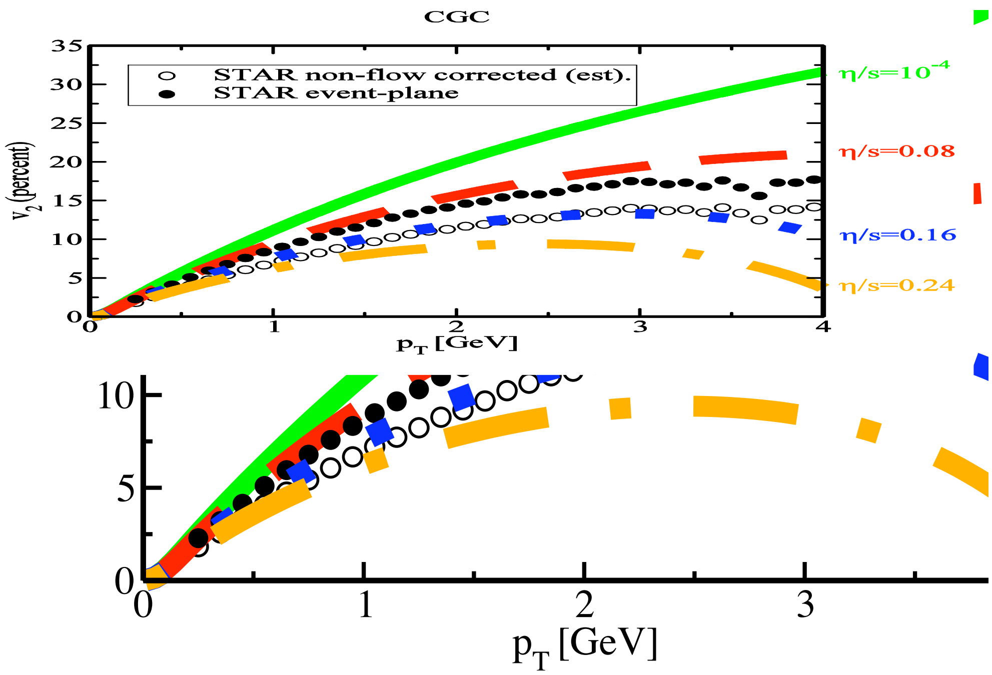
[Fick's law (1855)] $$j_i = -D\,\partial_i j^0 + O[(\nabla j^0)^2, \nabla^2 j^0]$$

Diffusion equation $$\partial_t j^0 = D \nabla^2 j^0$$

Dispersion relation $$\omega = -i\,D\,q^2 + \cdots$$

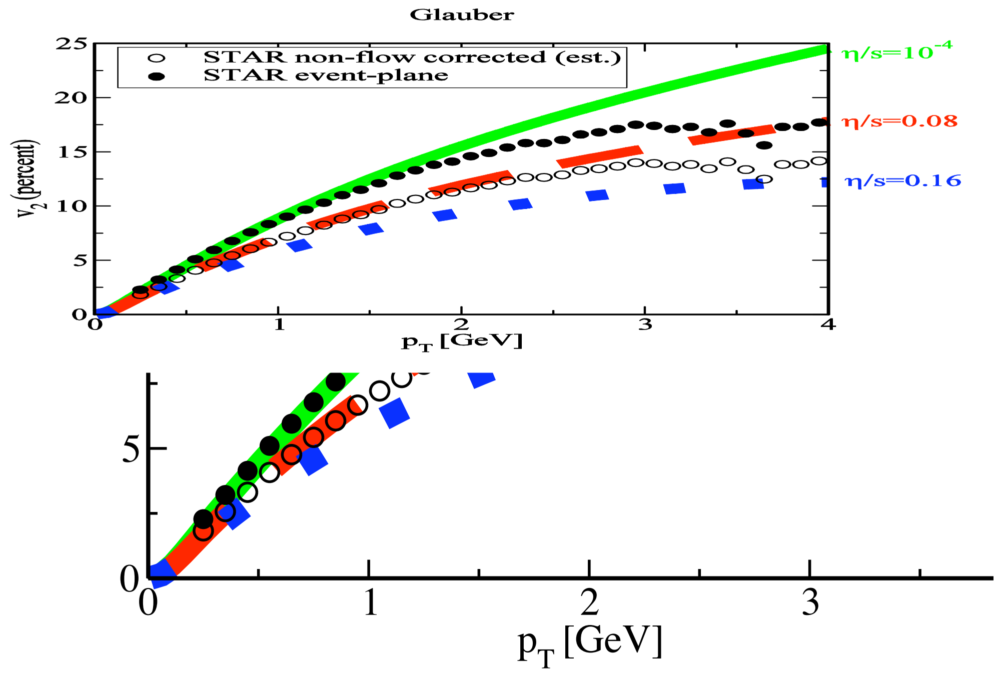
Expansion parameters: $\omega \ll T\,, \quad q \ll T$

Elliptic flow with color glass condensate initial conditions



Luzum and Romatschke, 0804.4015 [nuc-th]

Elliptic flow with Glauber initial conditions



Luzum and Romatschke, 0804.4015 [nuc-th]

Viscosity/entropy ratio in QCD: current status

Theories with gravity duals in the regime where the dual gravity description is valid

(universal limit)

Kovtun, Son & A.S; Buchel; Buchel & Liu, A.S

QCD: RHIC elliptic flow analysis suggests

QCD: (Indirect) LQCD simulations

H.Meyer, 0805.4567 [hep-th]

Trapped strongly correlated cold alkali atoms

T.Schafer, 0808.0734 [nucl-th]

Liquid Helium-3

This part of the talk -

"Holographic recipes at finite density and low temperature"

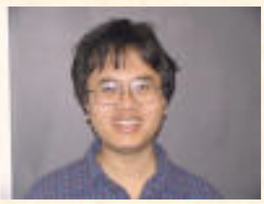
- is based on the paper

"Zero Sound from Holography"

arXiv: 0806.3796 [hep-th]

by

Andreas Karch



Dam Thanh Son

A.S.

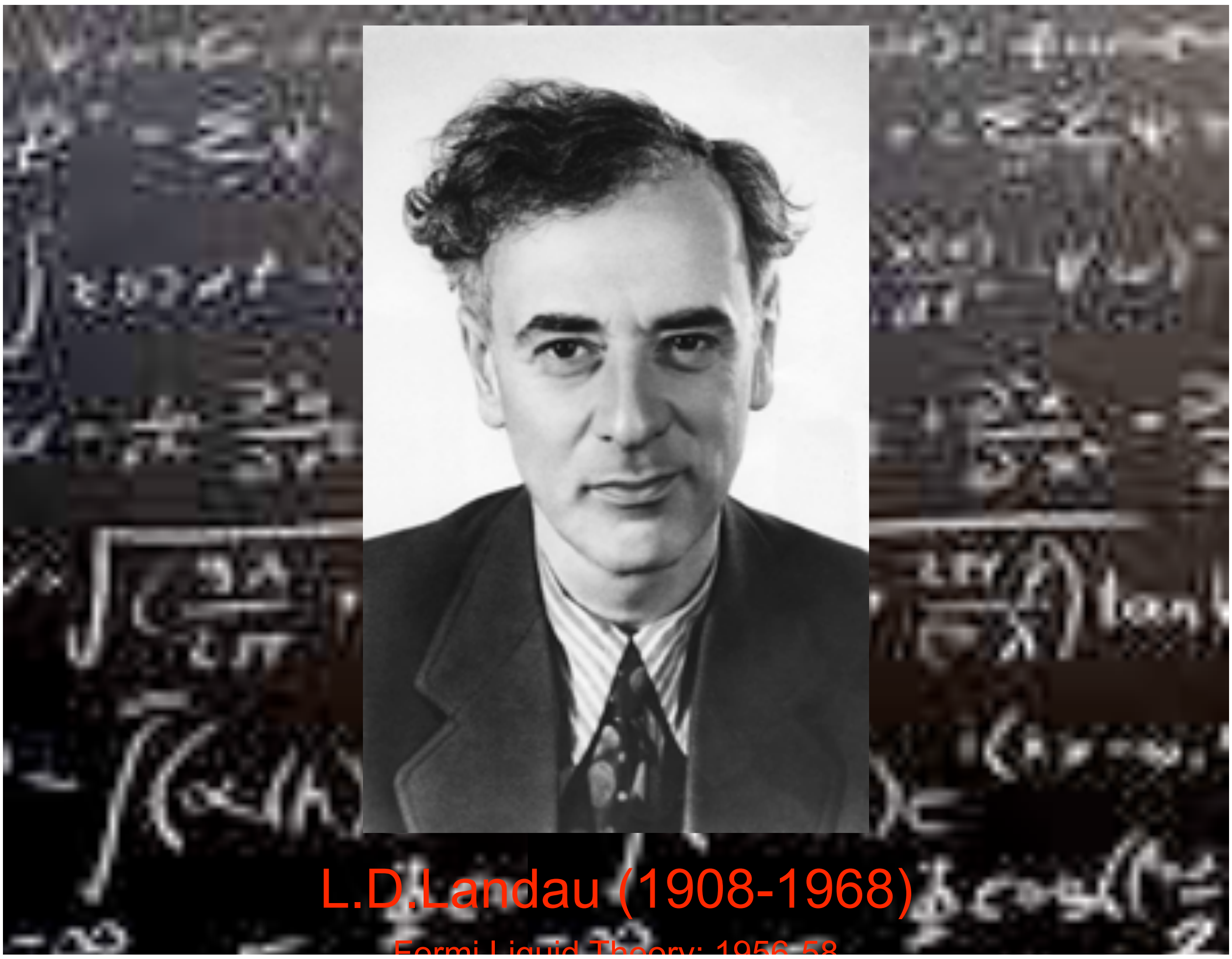
Probing quantum liquids with holography

Quantum liquid in p+1 dim	Low-energy elementary excitations	Specific heat at low T
Quantum Bose liquid	phonons	
Quantum Fermi liquid (Landau FLT)	fermionic quasiparticles + bosonic branch (zero sound)	

Departures from normal Fermi liquid occur in

- 3+1 and 2+1 –dimensional systems with strongly correlated electrons
- In 1+1 –dimensional systems for any strength of interaction (Luttinger liquid)

One can apply holography to study strongly coupled Fermi systems at low T



The simplest candidate with a known holographic description is

at finite temperature T and nonzero chemical potential associated with the "baryon number" density of the charge

There are two dimensionless parameters:

is the baryon number density is the hypermultiplet mass

The holographic dual description in the limit
is given by the D3/D7 system, with D3 branes replaced by the AdS-Schwarzschild geometry and D7 branes embedded in it as probes.

Karch & Katz, hep-th/0205236

AdS-Schwarzschild black hole (brane) background

D7 probe branes

The worldvolume U(1) field couples to the flavor current at the boundary

Nontrivial background value of corresponds to nontrivial expectation value of

We would like to compute

- the specific heat at low temperature

- the charge density correlator

★ The specific heat (in p+1 dimensions):

(note the difference with Fermi and Bose systems)

★ The (retarded) charge density correlator
has a pole corresponding to a propagating mode (zero sound)
- even at zero temperature

(note that this is NOT a superfluid phonon whose attenuation scales as)

New type of quantum liquid?

Conclusions

The program of computing first- and second-order transport coefficients in N=4 SYM at strong coupling and large number of colors is essentially complete

These calculations are helpful in numerical simulations of non-equilibrium QCD and stimulated developments in other fields such as LQCD, pQCD, condmat

Need better understanding of

- thermalization, isotropisation, and time evolution [Janik et al.]

- corrections from higher-derivative terms [Kats & Petrov; Brigante,Myers,Shenker,Yaida]

[Buchel, Myers, Paulos, Sinha, 0808.1837 [hep-th]]

LHC (ALICE) will provide more data at higher T

At low temperature and finite density, does holography identify a new type of quantum liquid?

(perturbative study of SUSY models at finite temperature and density would be very helpful)

THANK YOU

Is the bound dead?

- Y.Kats and P.Petrov, 0712.0743 [hep-th]
  "Effect of curvature squared corrections in AdS on the viscosity of the dual gauge theory"

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{1}{2N}\right) \qquad \mathcal{N} = 2$$ superconformal Sp(N) gauge theory in d=4

$$S = \int d^D x \sqrt{-g}\left(R - 2\Lambda + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \cdots\right)$$

But see A.Buchel, 0804.3161 [hep-th]

- M.~Brigante, H.~Liu, R.~C.~Myers, S.~Shenker and S.~Yaida,
  ``The Viscosity Bound and Causality Violation,'' 0802.3318 [hep-th],
  ``Viscosity Bound Violation in Higher Derivative Gravity,'' 0712.0805 [hep-th].

- The "species problem"
  T.Cohen, hep-th/0702136, A.Dobado, F.Llanes-Estrada, hep-th/0703132

Second-order hydrodynamics

Hydrodynamics is an effective theory, valid for sufficiently small momenta

$$k\, l_{mfp} \ll 1$$

First-order hydro eqs are parabolic. They imply instant propagation of signals.

This is not a conceptual problem since hydrodynamics becomes “acausal” only outside of its validity range but it is very inconvenient for numerical work on Navier-Stokes equations where it leads to instabilities [Hiscock & Lindblom, 1985]

These problems are resolved by considering next order in derivative expansion, i.e. by adding to the hydro constitutive relations all possible second-order terms compatible with symmetries (e.g. conformal symmetry for conformal plasmas)

Second-order hydrodynamics in $4d\ \mathcal{N}=4$ SYM

Second-order conformal hydrodynamics can be systematically constructed

Using AdS/CFT, all new transport coefficients for N=4 SYM can be computed

$$\eta = \frac{\pi}{8} N_c^2 T^3, \quad \tau_\Pi = \frac{2-\ln 2}{2\pi T}, \quad \kappa = \frac{N_c^2 T^2}{8}$$

$$\lambda_1 = \frac{\eta}{2\pi T}, \quad \lambda_2 = -\frac{\eta \ln 2}{\pi T}, \quad \lambda_3 = 0$$

Here we also used results from: S.Bhattacharyya,V.Hubeny,S.Minwalla,M.Rangamani, 0712.2456 [hep-th]

Generalized to CFT in D dim: Haack & Yarom, 0806.4602 [hep-th];

Finite ‘t Hooft coupling correction computed: Buchel and Paulos, 0806.0788 [hep-th]

Question: does this affect RHIC numerics?

Second-order conformal hydrodynamics

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^{\mu} u^{\nu} \qquad \sigma^{\mu\nu} = 2\nabla^{<\mu}u^{\nu>} \qquad u^{\mu}u_{\mu} = -1$$

$$D \equiv u^{\mu}\nabla_{\mu} \qquad \Omega^{\mu\nu} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\nabla_{\alpha}u_{\beta} - \nabla_{\beta}u_{\alpha}\right)$$

$$A^{<\mu\nu>} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(A_{\alpha\beta} + A_{\beta\alpha}\right) - \frac{1}{d-1}\Delta^{\mu\nu}\Delta^{\alpha\beta}A_{\alpha\beta}$$

Derivative expansion in hydrodynamics: first order

Hydrodynamic d.o.f. = densities of conserved charges

$\partial_{\mu}T^{\mu\nu} = 0$ (4 equations) $\qquad T^{00}\,,\; T^{0i}$ (4 d.o.f.) or $\varepsilon\,,\; u^{\mu}$

$$T^{\mu\nu} = \varepsilon\, u^{\mu} u^{\nu} + P(\varepsilon)\,\Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta(\varepsilon)\,\sigma^{\mu\nu} - \zeta(\varepsilon)\,\Delta^{\mu\nu}\,(\nabla\cdot u) + \cdots$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^{\mu} u^{\nu} \qquad \sigma^{\mu\nu} = 2\nabla^{<\mu}u^{\nu>}$$

$$A^{<\mu\nu>} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(A_{\alpha\beta} + A_{\beta\alpha}\right) - \frac{1}{d-1}\Delta^{\mu\nu}\Delta^{\alpha\beta}A_{\alpha\beta}$$

$$u^{\mu}u_{\mu} = -1$$

Supersymmetric sound mode ("phonino") in $4d$ $\mathcal{N}=4$ SYM

Conserved charge $\Longrightarrow$ Hydrodynamic mode (infinitely slowly relaxing fluctuation of the charge density) $\Longrightarrow$ Hydro pole in the retarded correlator of the charge density

$\partial_\mu T^{\mu\nu} = 0$ $T^{\mu\nu}_{equib} + \delta T^{\mu\nu}$ $\langle T_{\mu\nu}(-k)\, T_{\rho\sigma}(k)\rangle$

Sound wave pole: $\omega = \pm v_s\, q - i\frac{2}{3sT}\left(\eta + \frac{3}{4}\zeta\right) q^2 + \cdots$ $v_s = \sqrt{\frac{\partial P}{\partial \epsilon}}$

$\partial_\mu\, S^\mu_\alpha = 0$ $S^\mu_\alpha + \delta S^\mu_\alpha$ $\langle \bar{S}^\mu_{\dot\alpha}(-k) S^\nu_\beta(k)\rangle$

Supersound wave pole: $v_{SS} = \frac{P}{\epsilon}$

Lebedev & Smilga, 1988 (see also Kovtun & Yaffe, 2003)

Sound and supersymmetric sound in $4d$ $\mathcal{N}=4$ SYM

In 4d CFT

$\Longrightarrow$

Sound mode:

Supersound mode:

Quasinormal modes in dual gravity

Graviton: $\Longrightarrow$

Gravitino: $\Longrightarrow$

Second-order conformal hydrodynamics

$$T^{\mu\nu}_{\text{conformal}} = \varepsilon\, u^\mu\, u^\nu + \frac{\varepsilon}{d-1}\Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\Pi \left[D\Pi^{<\mu\nu>} + \frac{d}{d-1}\Pi^{\mu\nu}(\nabla\cdot u)\right]$$
$$+\kappa\left[R^{<\mu\nu>} - (d-2)u_\alpha R^{\alpha<\mu\nu>\beta}u_\beta\right]$$
$$+\frac{\lambda_1}{\eta^2}\Pi^{<\mu}_{\lambda}\Pi^{\nu>\lambda} - \frac{\lambda_2}{\eta}\Pi^{<\mu}_{\lambda}\Omega^{\nu>\lambda} + \lambda_3\,\Omega^{<\mu}_{\lambda}\Omega^{\nu>\lambda}$$

$$D \equiv u^\mu \nabla_\mu \qquad \Omega^{\mu\nu} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\nabla_\alpha u_\beta - \nabla_\beta u_\alpha\right)$$

Second-order Israel-Stewart conformal hydrodynamics

$$T^{\mu\nu}_{\text{conformal}} = \varepsilon\, u^\mu\, u^\nu + \frac{\varepsilon}{d-1}\Delta^{\mu\nu} + \Pi^{\mu\nu}$$

Israel-Stewart

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\Pi \left[D\Pi^{<\mu\nu>} + \frac{d}{d-1}\Pi^{\mu\nu}(\nabla\cdot u)\right]$$
$$+\kappa\left[R^{<\mu\nu>} - (d-2)u_\alpha R^{\alpha<\mu\nu>\beta}u_\beta\right]$$
$$+\frac{\lambda_1}{\eta^2}\Pi^{<\mu}_{\lambda}\Pi^{\nu>\lambda} - \frac{\lambda_2}{\eta}\Pi^{<\mu}_{\lambda}\Omega^{\nu>\lambda} + \lambda_3\,\Omega^{<\mu}_{\lambda}\Omega^{\nu>\lambda}$$

$$D \equiv u^\mu \nabla_\mu \qquad \Omega^{\mu\nu} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\nabla_\alpha u_\beta - \nabla_\beta u_\alpha\right)$$

First-order conformal hydrodynamics

Weyl transformations: $g_{\mu\nu} \to e^{-2\omega} g_{\mu\nu}$

$$T^{\mu\nu} \to e^{(d+2)\omega} T^{\mu\nu}$$

$$T^{\mu}_{\mu} = 0$$

In first-order hydro this implies: $\varepsilon = (d-1)P\,,\ \zeta = 0$

$$u^{\mu} \to e^{\omega} u^{\mu} \qquad T \to e^{\omega} T \qquad \sigma^{\mu\nu} \to e^{3\omega} \sigma^{\mu\nu}$$

Thus, in first-order hydro:

$$T^{\mu\nu}_{\text{conformal}} = \varepsilon\, u^{\mu} u^{\nu} + \frac{\varepsilon}{d-1} \Delta^{\mu\nu} - \eta\, \sigma^{\mu\nu}$$

Still, acausality is unpleasant, especially for numerical simulations, where it leads to instabilities

It would be convenient to have a UV completion (at strong and weak coupling)

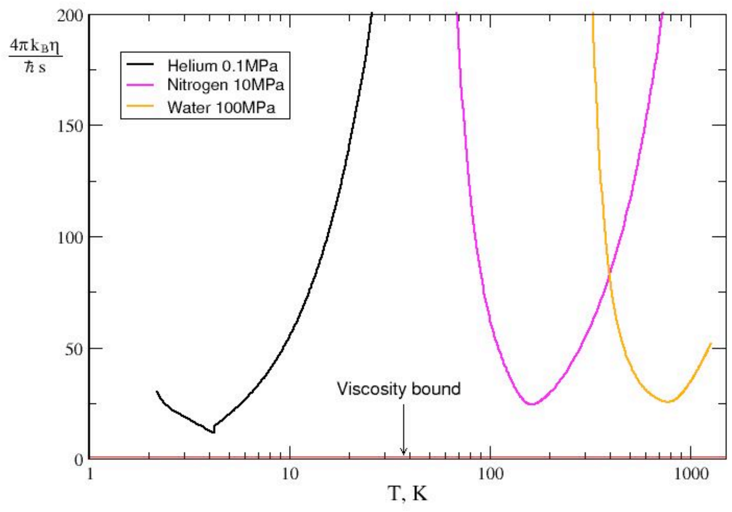
Israel-Stewart theory is such a completion built with e.g. Boltzmann eq

AdS/CFT shows that at strong coupling, I-S theory is incomplete

A viscosity bound conjecture

? ? ? ?

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} K \cdot s$$



Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

Xe 84

Kr 57

CO_2 32

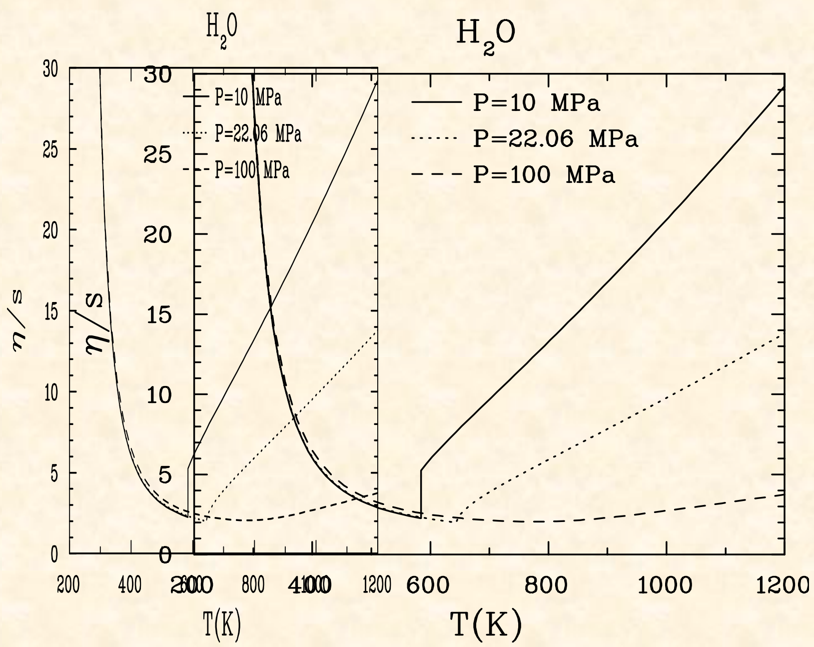
H_2O 25

C_2H_5OH 22

Ne 17

He 8.8

P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231



$(\eta/s)_{\text{min}} \sim 25$ in units of $\frac{\hbar}{4\pi k_B}$

Chernai, Kapusta, McLerran, nucl-th/0604032

A hand-waving argument

$$\eta \sim \rho v l \sim \rho v^2 \tau \sim n m v^2 \tau \sim n \epsilon \tau$$

$$s \sim n$$

Thus $\quad \dfrac{\eta}{s} \sim \epsilon\tau \geq \hbar \quad$?

Gravity duals fix the coefficient: $\quad \dfrac{\eta}{s} \geq \hbar/4\pi$

The "species problem"

Classical dilute gas with a LARGE number of components

has a large Gibbs mixing entropy

$$s = n \ln\left[\frac{1}{n}\left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}\right] + \frac{5}{2}n + n \ln N_f$$

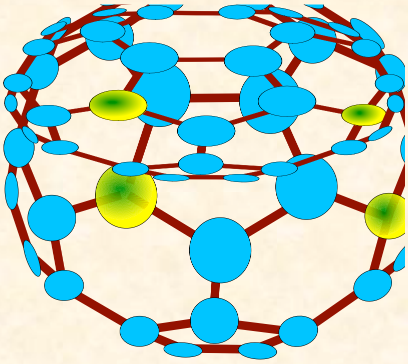
$$\eta \sim \frac{\sqrt{mk_BT}}{d^2}$$

To have $\frac{\eta}{s} < \frac{\hbar}{4\pi k_B}$ with C_{60}

need $N_f > 10^{4000}$ (Dam Son, 2007)

To have $\frac{\eta}{s} \sim 8.8\frac{\hbar}{4\pi k_B}$ need

$N_f \sim 10^{450}$ species



Buckminsterfullerene C_{60}

a.k.a."buckyball"

A.Dobado, F.Llanes-Estrada, hep-th/0703132

T.Cohen, hep-th/0702136

Can we test

$$\eta/s \geq 1/4\pi \pm ?$$

experimentally?

A characteristic feature of systems saturating the bound: strong interactions

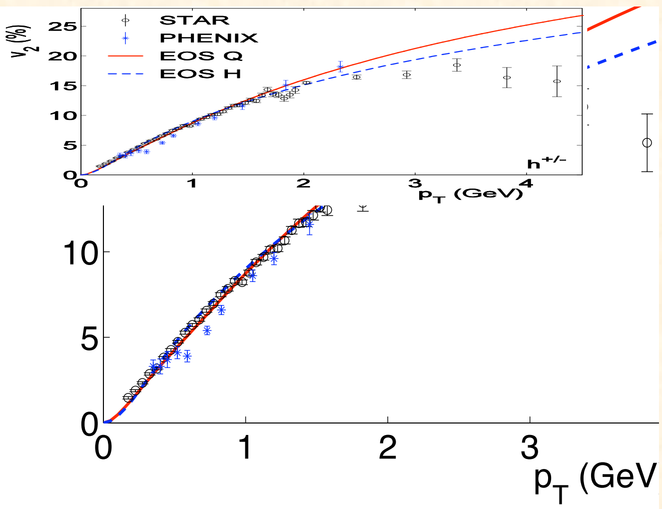
- Heavy ion collisions - experiments at RHIC
- (Indirect) lattice QCD simulations
- Trapped atoms – strongly interacting Fermi systems

QCD kinetics

Viscosity is ONE of the parameters used in the hydro models describing the azimuthal anisotropy of particle distribution at RHIC

$$\frac{d^2N^i}{dp_T d\phi} = N_0^i \left[1 + 2v_2^i(p_T)\cos 2\phi + \cdots\right]$$

$v_2^i(p_T)$ -elliptic flow for particle species "i"



Elliptic flow reproduced for

$$0 < \eta/s \leq 0.3$$

e.g. Baier, Romatschke, nucl-th/0610108

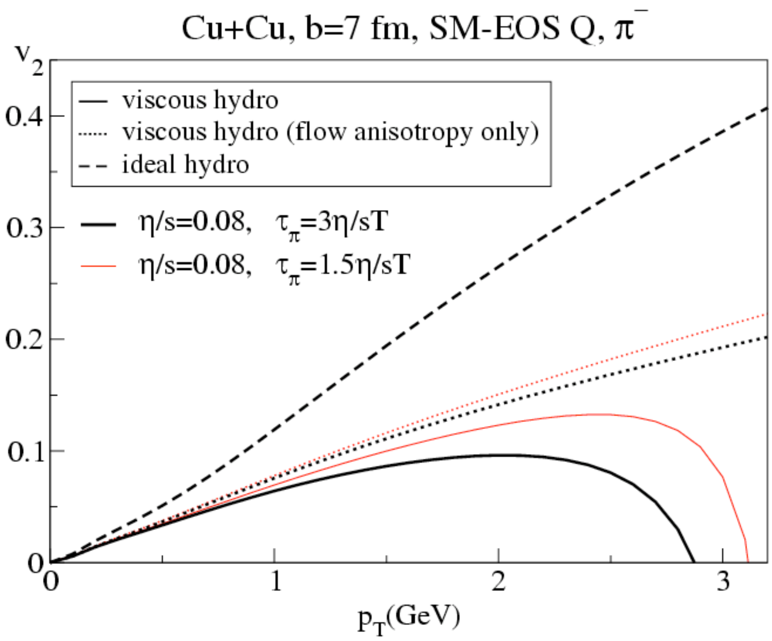
Perturbative QCD:

$$\eta/s\,(T_{\rm RHIC}) \approx 1.6 \sim 1.8$$

Chernai, Kapusta, McLerran, nucl-th/0604032

SYM: $\eta/s \approx 0.09 \sim 0.28$

Elliptic flow from relativistic hydro simulations
(Israel-Stewart formalism)



H.Song and U.Heinz, 0712.3715 [nucl-th]

Experimental and theoretical motivation

- Heavy ion collision program at RHIC, LHC (2000-2008-2020 ??)
- Studies of hot and dense nuclear matter
- Abundance of experimental results, poor theoretical understanding:

- the collision apparently creates a fireball of "quark-gluon fluid"

- need to understand both thermodynamics and kinetics

-in particular, need theoretical predictions for parameters entering equations of relativistic hydrodynamics – viscosity etc – computed from the underlying microscopic theory (thermal QCD)

-this is difficult since the fireball is a strongly interacting nuclear fluid, not a dilute gas

Predictions of the second-order conformal hydrodynamics

Sound dispersion: $$\omega_{1,2} = \pm c_s\, q - i\Gamma\, q^2 \pm \frac{\Gamma}{c_s}\left(c_s^2 \tau_\Pi - \frac{\Gamma}{2}\right) q^3 + O(q^4)$$

$$\Gamma = \frac{d-2}{d-1}\frac{\eta}{\varepsilon + P}$$

Kubo: $$G_R^{xy,xy}(\omega, q) = P - i\,\eta\,\omega + \eta\,\tau_\Pi\,\omega^2 - \frac{\kappa}{2}\left[(d-3)\omega^2 + q^2\right]$$

New transport coefficients in $\mathcal{N} = 4$ SYM

Sound dispersion: $$\omega = \pm\frac{1}{\sqrt{3}} q - \frac{i}{6\pi T} q^2 + \frac{3 - 2\ln 2}{24\pi^2\sqrt{3}T^2} q^3 + \cdots$$

Kubo:

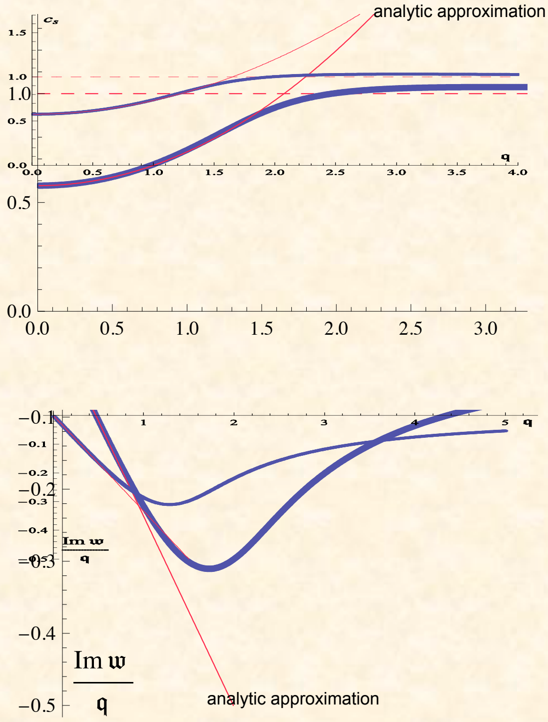
$$G_R^{xy,xy}(\omega, q) = -\frac{\pi^2 N_c^2 T^4}{4}\left[iw - w^2 + k^2 + w^2 \ln 2 - \frac{1}{2}\right] + O(w^3, wk^2)$$

$$w = \omega/2\pi T\,, \quad k = q/2\pi T$$

$$P = \frac{\pi^2}{8} N_c^2 T^4\,, \quad \eta = \frac{\pi}{8} N_c^2 T^3\,, \quad \tau_\Pi = \frac{2 - \ln 2}{2\pi T}\,, \quad \kappa = \frac{\eta}{\pi T}$$

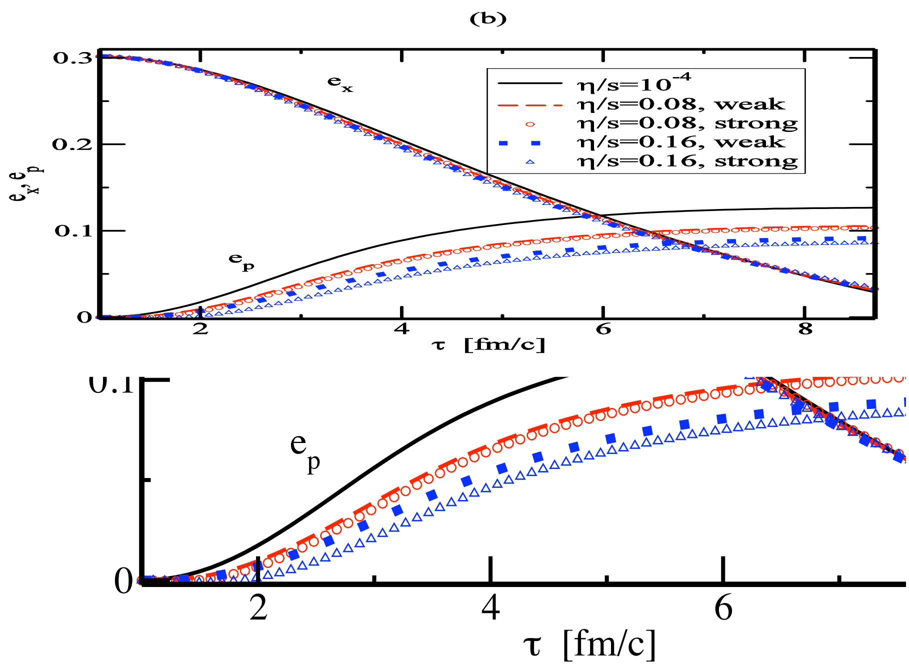
Sound dispersion in $4d$ $\mathcal{N}=4$ SYM

$\mathrm{Re}(\omega)/q$



q

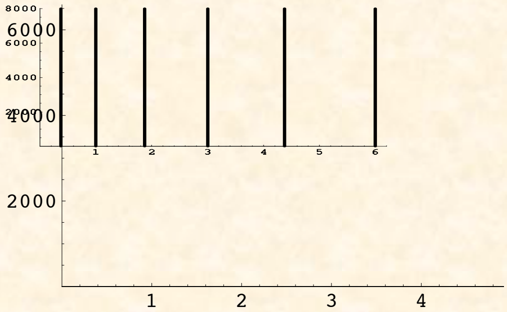
The effect of the second-order coefficients



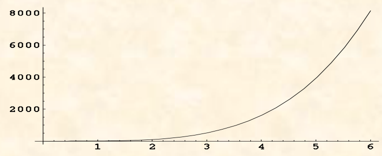
Luzum and Romatschke, 0804.4015 [nuc-th]

Spectral function and quasiparticles in finite-temperature "AdS + IR cutoff" model

$T < T_c$



$T > T_c$



$$\chi(\omega) \sim N_c^2 \sum_{n=0}^{\infty} \omega_n^2 \, \rho(\omega_n) \, \delta(\omega - \omega_n)$$

$$\chi(\omega) = \frac{N_c^2}{16\pi} \frac{\omega^2 \sinh(\omega/2T)}{\cosh(\omega/2T) - \cos(\omega/2T)}$$

$\mathcal{N} = 4$ SYM

Computing transport coefficients from dual gravity – various methods

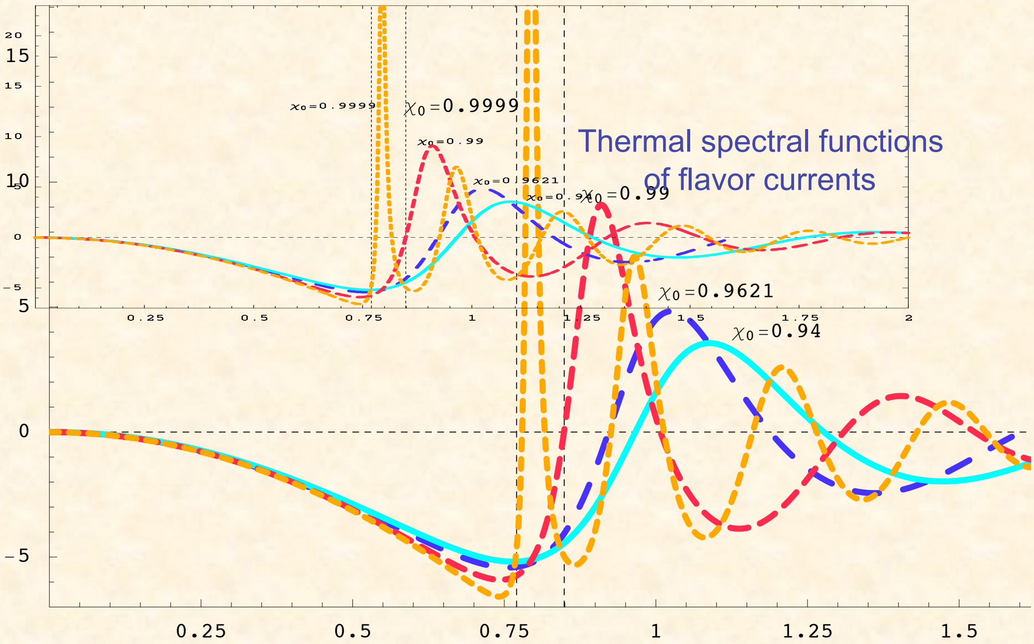
1. Green-Kubo formulas (+ retarded correlator from gravity)

2. Poles of the retarded correlators

3. Lowest quasinormal frequency of the dual background

4. The membrane paradigm

Holographic models with fundamental fermions



Additional parameter m_F/T makes life more interesting…

R.Myers, A.S., R.Thomson, 0706.0162 [hep-th]

Example: stress-energy tensor correlator in $4d\ \mathcal{N}=4$ SYM in the limit $N_c \to \infty\,, \quad g^2_{YM} N_c \to \infty$

Zero temperature, Euclid: $G_E(k) = \frac{N_c^2 k_E^4}{32\pi^2} \ln k_E^2$

Finite temperature, Mink:

$$< T_{tt}(-\omega,-q), T_{tt}(\omega,q) >^{\text{ret}} = \frac{3N_c^2\pi^2T^4q^2}{2(\omega^2 - q^2/3 + i\omega q^2/3\pi T)} + \cdots$$

(in the limit $\omega/T \ll 1\,, \quad q/T \ll 1$)

The pole
(or the lowest quasinormal freq.)

$$\omega = \pm\frac{1}{\sqrt{3}}q - \frac{i}{6\pi T}q^2 + \frac{3-2\ln 2}{24\pi^2\sqrt{3}T^2}q^3 + \cdots$$

Compare with hydro: $\omega = \pm v_s q - \frac{i}{2sT}\left(\zeta + \frac{4}{3}\eta\right) q^2 + \cdots$

In CFT: $v_s = \frac{1}{\sqrt{3}}\,, \quad \zeta = 0$

$\Rightarrow \quad \eta = \pi N_c^2 T^3/8$

Also, $s = \pi^2 N_c^2 T^3/2$ (Gubser, Klebanov, Peet, 1996)

Example 2 (continued): stress-energy tensor correlator in

$4d\ \mathcal{N}=4$ SYM in the limit $N_c \to \infty\,, \quad g^2_{YM} N_c \to \infty$

Zero temperature, Euclid: $G_E(k) = \frac{N_c^2 k_E^4}{32\pi^2} \ln k_E^2$

Finite temperature, Mink:

$$< T_{tx}(-\omega,-q), T_{tx}(\omega,q) >^{\text{ret}} \sim \frac{N_c^2 T^4 \omega^2}{\omega - iq^2/4\pi T} + \cdots$$

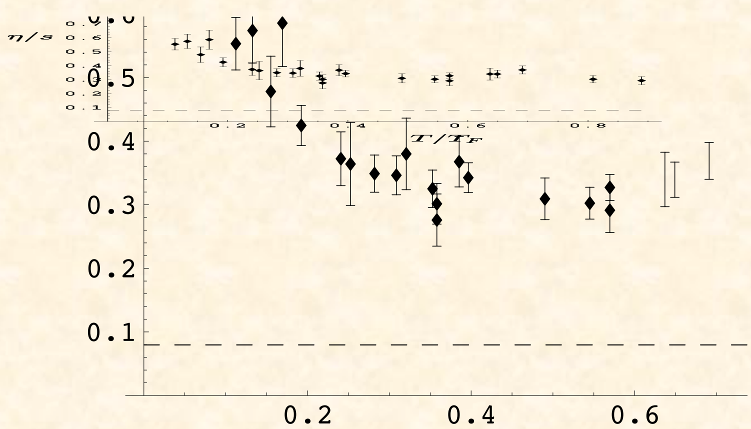
(in the limit $\omega/T \ll 1\,,\ \ q/T \ll 1$)

The pole
(or the lowest quasinormal freq.) $\omega = -\frac{i}{4\pi T} q^2 - \frac{i(1-\ln 2)}{32\pi^3 T^3} q^4 + \cdots$

Compare with hydro: $\omega = -\frac{i\,\eta}{sT} q^2 + \cdots$

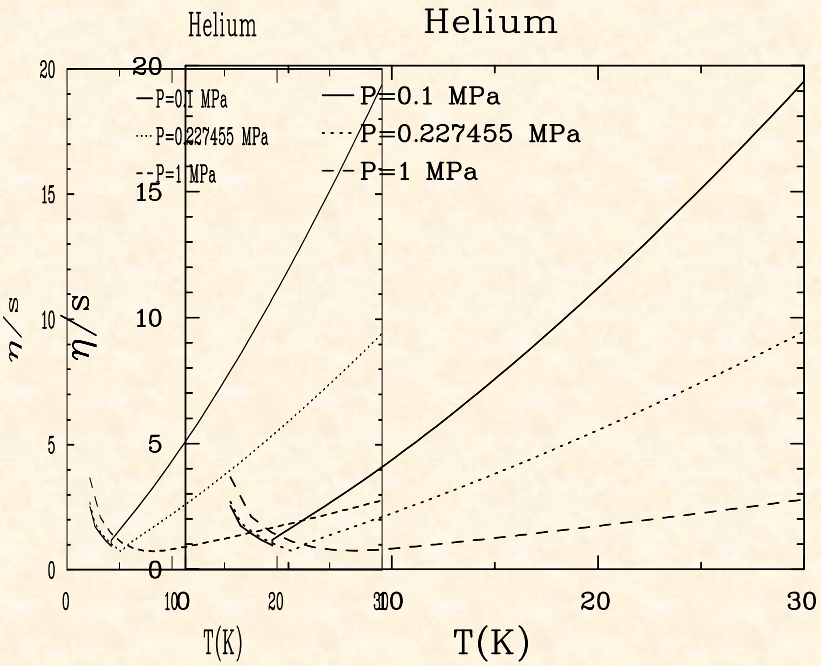
$s = \pi^2 N_c^2 T^3/2 \qquad \Rightarrow \quad \eta = \pi N_c^2 T^3/8$

Viscosity-entropy ratio of a trapped Fermi gas



$\eta/s \sim 4.2$ in units of $\frac{\hbar}{4\pi k_B}$ T.Schafer, cond-mat/0701251

(based on experimental results by Duke U. group, J.E.Thomas et al., 2005-06)



$(\eta/s)_{\rm min} \sim 8.8$ in units of $\dfrac{\hbar}{4\pi k_B}$

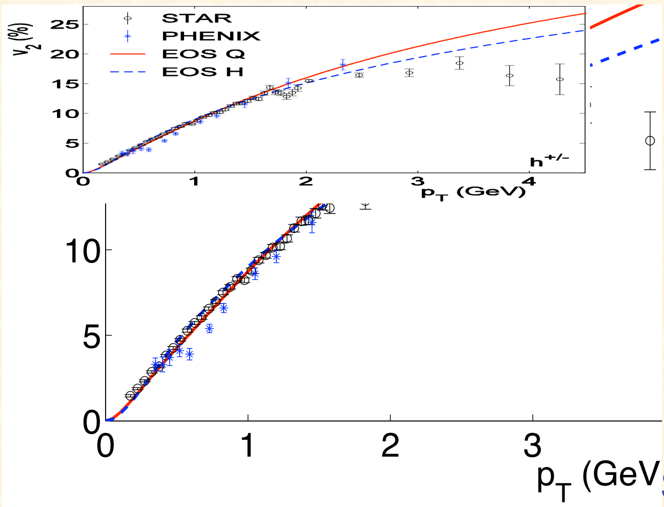
Chernai, Kapusta, McLerran, nucl-th/0604032

Viscosity "measurements" at RHIC

Viscosity is ONE of the parameters used in the hydro models describing the azimuthal anisotropy of particle distribution

$$\frac{d^2N^i}{dp_T d\phi} = N_0^i \left[1 + 2v_2^i(p_T)\cos 2\phi + \cdots\right]$$

$v_2^i(p_T)$ -elliptic flow for particle species "i"



Elliptic flow reproduced for

$$0 < \eta/s \leq 3.8 \times \frac{\hbar}{4\pi k_B}$$

Is it so? Heinz and Song, 2007

Perturbative QCD:

$$\eta/s\,(T_{\rm RHIC}) \approx (20 \sim 23) \times \frac{\hbar}{4\pi k_B}$$

Chernai, Kapusta, McLerran, nucl-th/0604032

SYM: $\eta/s \approx (1.1 \sim 3.5) \times \dfrac{\hbar}{4\pi k_B}$

Two-point correlation function of stress-energy tensor

Field theory

Zero temperature: $\langle T_{\mu\nu} T_{\alpha\beta} \rangle_{T=0} = \Pi_{\mu\nu,\alpha\beta} F(k^2) + Q_{\mu\nu,\alpha\beta} G(k^2)$

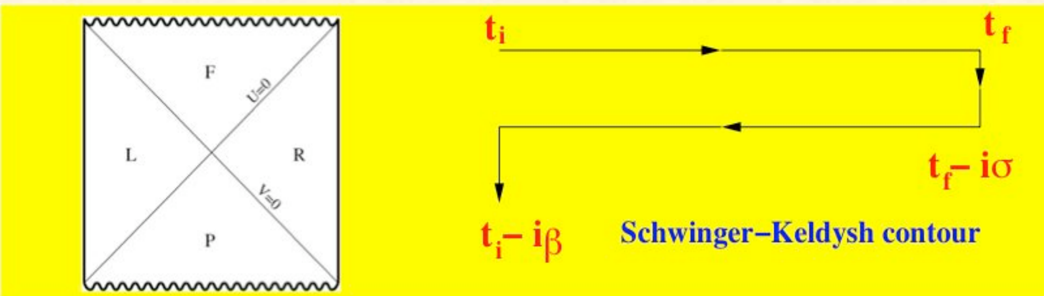
Finite temperature:
$$\langle T_{\mu\nu} T_{\alpha\beta} \rangle_T = S^{(1)}_{\mu\nu,\alpha\beta} G_1(\omega, q) + S^{(2)}_{\mu\nu,\alpha\beta} G_2(\omega, q) + S^{(3)}_{\mu\nu,\alpha\beta} G_3(\omega, q) + S^{(4)}_{\mu\nu,\alpha\beta} G_4 + S^{(5)}_{\mu\nu,\alpha\beta} G_5$$

Dual gravity

- Five gauge-invariant combinations Z_1, Z_2, Z_3, Z_4, Z_5 of $h_{\mu\nu}$ and other fields determine G_1, G_2, G_3, G_4, G_5
- Z_1, Z_2, Z_3, Z_4, Z_5 obey a system of coupled ODEs
- Their (quasinormal) spectrum determines singularities of the correlator

Computing finite-temperature correlation functions from gravity

- Need to solve 5d e.o.m. of the dual fields propagating in asymptotically AdS space
- Can compute Minkowski-space 4d correlators
- Gravity maps into real-time finite-temperature formalism (Son and A.S., 2001; Herzog and Son, 2002)

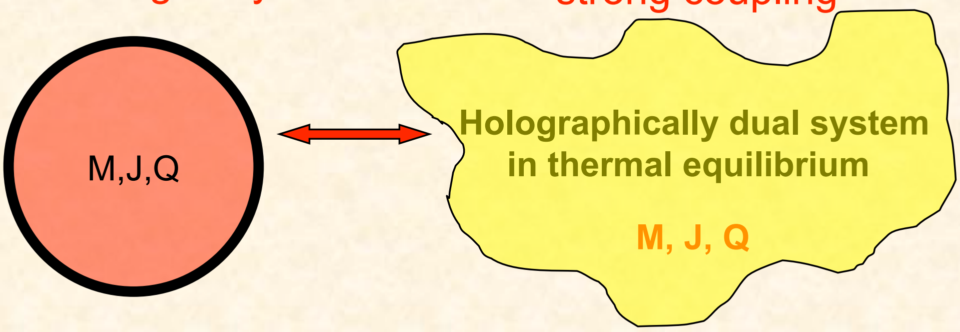


Outlook

- Gravity dual description of thermalization ?
- Beyond hydrodynamics: AdS/CFT vs Israel-Stewart
  Baier, Romatschke, Stephanov, Son, A.S., to appear
- Understanding 1/N corrections
- Phonino
- The role of membrane paradigme?

10-dim gravity

4-dim gauge theory – large N, strong coupling



T_{Hawking} $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T S

Gravitational fluctuations $\Longleftrightarrow$ Deviations from equilibrium

$g^{(0)}_{\mu\nu} + h_{\mu\nu}$ $\Longleftrightarrow$????

+ fluctuations of other fields

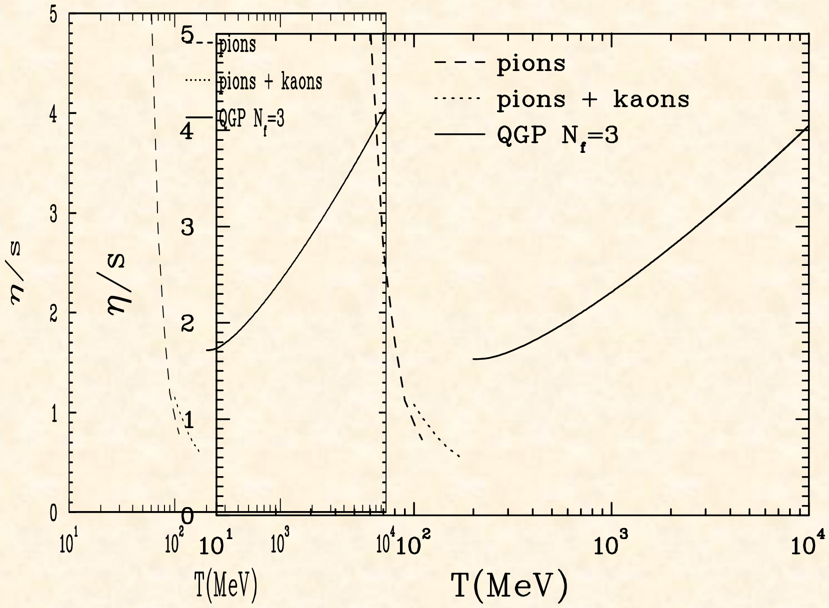
$j_i = -D\partial_i j^0 + \cdots$

$''\Box'' \; h_{\mu\nu} = 0$ and B.C. $\Longleftrightarrow$ $\partial_t j^0 + \partial_i j^i = 0$

$\partial_t j^0 = D\nabla^2 j^0$

Quasinormal spectrum $\Longleftrightarrow$ $\omega = -iDq^2 + \cdots$

QCD



Chernai, Kapusta, McLerran, nucl-th/0604032

Now look at the correlators obtained from gravity

$$\langle T_{00}(k)T_{00}(-k)\rangle = \frac{3N^2\pi^2T^4q^2}{2(\omega^2 - q^2/3 + i\omega q^2/3\pi T)} + \cdots$$

The correlator has poles at $\omega = \pm\frac{q}{\sqrt{3}} - i\frac{q^2}{6\pi T} + \cdots$

The speed of sound coincides with the hydro prediction!

$$\eta = \frac{\pi N^2 T^3}{8}$$

$$s = \frac{\pi^2}{2}N^2T^3$$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Similarly, one can analyze another conserved quantity – energy-momentum tensor:

$$\partial_\mu T^{\mu\nu} = 0$$

This is equivalent to analyzing fluctuations of energy and pressure

$$\langle T^{00} \rangle = \epsilon \qquad \langle T^{ij} \rangle = P\, \delta^{ij}$$

We obtain a dispersion relation for the sound wave:

$$\omega = \pm v_s\, q - \frac{i}{2(\epsilon + P)} \left(\frac{4}{3}\eta + \zeta \right) q^2$$

Universality of η/s

Theorem:

For a thermal gauge theory, the ratio of shear viscosity to entropy density is equal to $1/4\pi$
in the regime described by a dual gravity theory

(e.g. at $g^2_{YM} N_c = \infty \,, N_c = \infty$ in $\mathcal{N} = 4$ SYM)

Remarks:

- Extended to non-zero chemical potential:

  Benincasa, Buchel, Naryshkin, hep-th/0610145

- Extended to models with fundamental fermions in the limit $N_f/N_c \ll 1$

  Mateos, Myers, Thomson, hep-th/0610184

- *String/Gravity dual to QCD is currently unknown*

Three roads to universality of η/s

- **The absorption argument**
  D. Son, P. Kovtun, A.S., hep-th/0405231

- **Direct computation of the correlator in Kubo formula from AdS/CFT** A.Buchel, hep-th/0408095

- **"Membrane paradigm" general formula for diffusion coefficient + interpretation as lowest quasinormal frequency = pole of the shear mode correlator + Buchel-Liu theorem**
  P. Kovtun, D.Son, A.S., hep-th/0309213, A.S., to appear, P.Kovtun, A.S., hep-th/0506184, A.Buchel, J.Liu, hep-th/0311175

Universality of shear viscosity in the regime described by gravity duals

$$ds^2 = f(w)\left(dx^2 + dy^2\right) + g_{\mu\nu}(w)dw^\mu dw^\nu$$

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega}\int dt\, dx e^{i\omega t}\langle\left[T_{xy}(t,x), T_{xy}(0,0)\right]\rangle$$

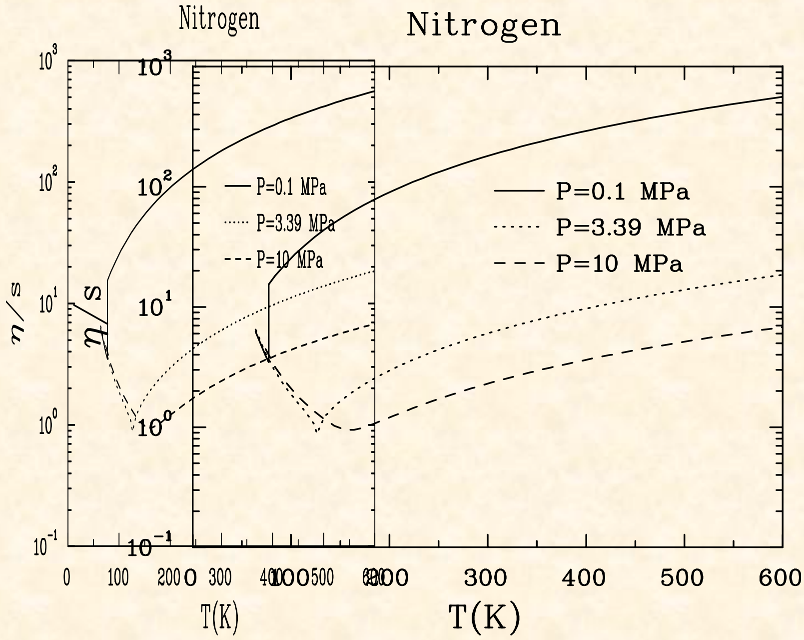
$$\sigma_{abs} = -\frac{16\pi G}{\omega}\,\mathrm{Im}\, G^R(\omega) = \frac{8\pi G}{\omega}\int dt\, dx e^{i\omega t}\langle\left[T_{xy}(t,x), T_{xy}(0,0)\right]\rangle$$

$$\eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

Graviton's component h_y^x obeys equation for a minimally coupled massless scalar. But then $\sigma_{abs}(0) = A_H$.

Since the entropy (density) is $s = A_H/4G$ we get

$$\frac{\eta}{s} = \frac{1}{4\pi}$$



$(\eta/s)_{\text{min}} \sim 23$ in units of $\frac{\hbar}{4\pi k_B}$

Chernai, Kapusta, McLerran, nucl-th/0604032

Predictions of hydrodynamics

Hydrodynamics predicts that the retarded correlator

$$\langle T_{00}(k)\, T_{00}(-k)\rangle$$

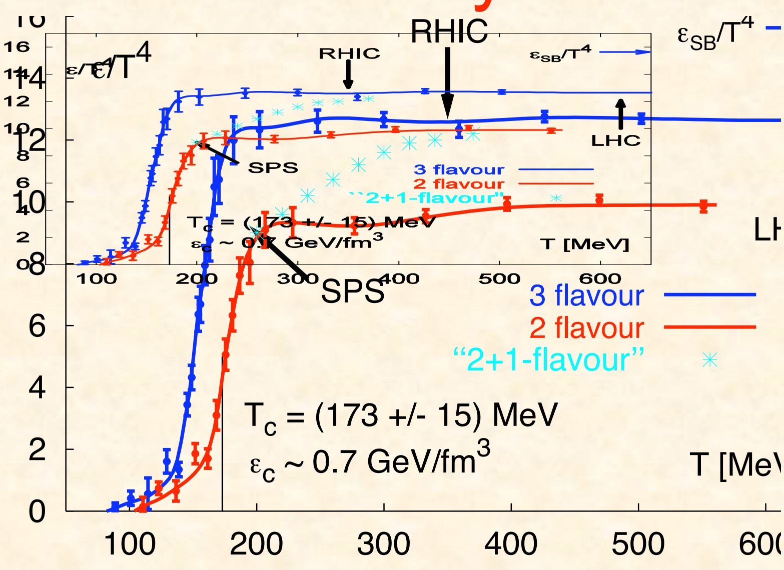
has a “sound wave” pole at

$$\omega = v_s\, q - \frac{i}{2(\epsilon + P)}\left(\frac{4}{3}\eta + \zeta\right) q^2$$

Moreover, in conformal theory $\epsilon = 3P \implies v_s^2 = \frac{\partial P}{\partial \epsilon} = 1/3$

QCD thermodynamics

Energy density vs temperature



QCD deconfinement transition (lattice data)

$\alpha_s(T_{\text{RHIC}}) \sim O(1)$

Computing transport coefficients from "first principles"

Fluctuation-dissipation theory
(Callen, Welton, Green, Kubo)

Kubo formulae allows one to calculate transport coefficients from microscopic models

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

In the regime described by a gravity dual the correlator can be computed using the gauge theory/gravity duality