The gauge-string duality and QCD at finite temperature

Steve Gubser
Princeton University

Strings 2008, CERN

August 21, 2008

Contents

1 Near-extremal D3-branes 3
2 Shear viscosity 5
3 Equation of state and bulk viscosity 9
4 The trailing string 14
  4.1 Stochastic forces on heavy quarks 19
5 Jet-splitting? 21
6 Jet quenching 25
7 Falling strings 28
8 Total multiplicity 32
9 Outlook 36
1. Near-extremal D3-branes

The near-extremal D3-brane metric describes $\mathcal{N} = 4$ gauge theory at finite temperature [Gubser et al. 1996] (also unpublished work of Strominger):

$$
    ds^2 = H^{-1/2} (-d\tau^2 + d\vec{x}^2) + H^{1/2} \left( \frac{dr^2}{h} + r^2 d\Omega_5^2 \right)
$$

(1)

$$
    H = 1 + \frac{L^4}{r^4} \quad \text{and} \quad h = 1 - \frac{r_0^4}{r^4}.
$$

In the now-familiar strong coupling limit of AdS/CFT [Maldacena 1998; Gubser et al. 1998a; Witten 1998]

$$
    \frac{L^8}{G_{10}} = \frac{2N^2}{\pi^4} \gg 1 \quad \text{and} \quad \frac{L^4}{\alpha'^2} = \lambda \equiv g_{YM}^2N \gg 1
$$

(2)

One finds free energy density [Gubser et al. 1998b]

$$
    f(\lambda) = \frac{F}{V} = \left( \frac{3}{4} + \frac{15\zeta(3)}{8\lambda^{3/2}} + \ldots \right) f_{\text{free}}
$$

(3)

where $f_{\text{free}} = -\frac{\pi^2}{6}(N^2 - 1)T^4$ for $SU(N)$ super-Yang-Mills.

At weak coupling [Fotopoulos and Taylor 1999; Vazquez-Mozo 1999; Kim and Rey 2000; Nieto and Tytgat 1999],

$$
    f(\lambda) = \left( 1 - \frac{3}{2\pi^2} \lambda + \frac{\sqrt{2} + 3}{\pi^3} \lambda^{3/2} \ldots \right) f_{\text{free}}
$$

(4)

The most modern treatment I know of is by [Blaizot et al. 2006]: (3) and (4) uniquely fix a (4,4) Padé estimate,

$$
    \frac{f}{f_{\text{free}}} = \frac{1 + \alpha \lambda^{1/2} + \beta \lambda + \gamma \lambda^{3/2}}{1 + \bar{\alpha} \lambda^{1/2} + \bar{\beta} \lambda + \bar{\gamma} \lambda^{3/2}}
$$

(5)

Comparison with a hard thermal loop calculation of $s/s_{\text{free}}$ (roughly, two-loop perturbation theory supplemented by a self-consistent gap equation for thermal masses) does pretty well out to $\lambda \sim 4$.

HTL (green) calculations of entropy in $\mathcal{N} = 4$ [Blaizot et al. 2006].
2. Shear viscosity

Neglecting loop and stringy corrections to two-derivative gravity, a broad set of black branes have [Policastro et al. 2001; Buchel and Liu 2004; Kovtun et al. 2005]

\[ \frac{\eta}{s} = \frac{1}{4\pi} ; \]

(6)

and D3-branes in particular have [Buchel et al. 2005]

\[ \frac{\eta}{s} = \frac{1}{4\pi} \left( 1 + \frac{135\zeta(3)}{8\lambda^{3/2}} + \ldots \right). \]

(7)

Loop corrections may lead to violations [Kats and Petrov 2007; Brigante et al. 2008] of the conjectured bound \( \eta/s \geq 1/4\pi \).

\( \eta \) is a key input for relativistic hydrodynamics:

\[
T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} - P^{\mu\alpha} P^{\nu\beta} \left[ \eta \left( \nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3} g_{\alpha\beta} \nabla_\lambda u^\lambda \right) + \zeta g_{\alpha\beta} \nabla_\lambda u^\lambda \right]
\]

where \( P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu \).

(8)

Lattice simulations of pure glue [Meyer 2007] indicate

\[ \left[ \frac{\eta}{s} \right]_{\text{best}} = 0.134 \approx \frac{5/3}{4\pi} \quad \frac{\eta}{s} \lesssim 1 \quad @ \ 90\% \ \text{CL} \]

(9)

This is hard work for the lattice because viscosities arise from real-time correlators:

\[
\eta \left( \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{kl} \right) + \zeta \delta_{ij} \delta_{kl} = -\lim_{\omega \to 0} \frac{1}{\omega} \text{Im} \ G_{i,j,k,l}^R(\omega)
\]

(10)

\[
G_{i,j,k,l}^R(\omega) \equiv -i \int d^3x \int d\tau e^{i\omega \tau} \langle [T_{ij}(t, \vec{x}), T_{kl}(0, 0)] \rangle
\]

whereas lattice provides direct access only to Euclidean correlators:

\[
G^E(\omega_n) = \int_0^\beta d\tau \int d^3x \ e^{i\omega_n \tau} \langle T_E(\mathcal{O}(\tau, \vec{x}), \mathcal{O}(0)) \rangle \quad \omega_n = \frac{2\pi n}{\beta}
\]

(11)

To get \( G^R(\omega) \) for real \( \omega \) starting from lattice data, some assumptions about spectral density \( \rho(\omega) \) have to be made.
Elliptic flow in heavy ion collisions puts bounds on $\eta$. Here’s the relevant geometry:

*Side view of an off-center gold-gold collision. The reaction plane is the plane of the page $b$ as a vector is approximately determined for each event.*

$\gamma \approx 100$ at RHIC, 2800 at LHC.

Cartoon of elliptic flow. From [Baker 2001]. Uneven pressure gradients lead to anisotropic expansion.

Experimental measure of elliptic flow is $d$-wave coefficient in an expansion of azimuthal distribution of particles (here $y = \tanh^{-1} p_z/E$ is rapidity):

$$
\frac{dN}{p_T d\phi d\eta} = \frac{dN}{p_T d\phi d\eta} [1 + 2v_2 \cos 2\phi + \ldots] \quad (12)
$$

Viscosity dependence of $v_2$ was studied e.g. in [Teaney 2003] in terms of $\Gamma_s/\tau_0$, where

$$
\Gamma_s = \frac{4\eta}{3T} \quad \text{sound attenuation length}
$$

$$
\tau_0 T \approx 1 \quad \text{characteristic expansion} \quad (13)
$$

$$
\frac{\Gamma_s}{\tau_0} = 0.1 \quad \leftrightarrow \quad \frac{\eta}{s} \approx \frac{1}{4\pi}
$$

But... Ideal hydro, $\Gamma_s = 0$, was “designed” to agree with data in this study.

Upshot: data favors the range

$$
0 \leq \frac{\eta}{s} \lesssim 0.2 \approx \frac{5/2}{4\pi} \quad . \quad (14)
$$
3. Equation of state and bulk viscosity

QCD is significantly non-conformal near $T_c$, and confinement is a smooth cross-over, not a phase transition.

Lattice results for the equation of state of QCD. From [Karsch 2002], $\epsilon_{SB}$ is the energy density for free quarks and gluons. The 20% deficit in $\epsilon/\epsilon_{SB}$ is suggestive of strong coupling.

- $T_c \approx 170$ MeV.
- RHIC operates at $T \approx 280$ MeV.
- LHC will operate at $T \approx 600$ MeV.

In a bottom-up approach [Gubser and Nellore 2008], we can reproduce the lattice eos using

$$\mathcal{L} = \frac{1}{2\kappa_5^2} \left[ R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right].$$

$V(\phi)$ can be adjusted to match dependence of

$$c_s^2 = \frac{dp}{d\epsilon}$$

on $T$. Then adjust $\kappa_5^2$ to get desired $\epsilon/T^4$ at some high scale (say 3 GeV). Here’s a quasi-realistic choice:

$$V(\phi) = \frac{-12 \cosh \gamma \phi + b \phi^2}{L^2} \quad \gamma = 0.606, \quad b = 2.057.$$  

Authors of [Gursoy and Kiritsis 2008; Gursoy et al. 2008ab] took same starting point (15) further: an appropriate $V(\phi)$, with $V \sim -\phi^2 e^{\sqrt{3}/4}$, gives a Hawking-Page transition to confinement, logarithmic RG in UV, and glueball with $m^2 \sim n$, as in linear confinement.
Once conformal invariance is broken, we can investigate bulk viscosity \cite{Gubser2008cb}, following a number of earlier works, e.g. \cite{Parnachev2005, Buchel2007}:

\[
\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \Im \int d^3x \, dt \, e^{i\omega t} \theta(t) \left\langle [T^\mu \_\nu(t, \vec{x}), T^\nu \_\nu(0, 0)] \right\rangle .
\]  \hspace{1cm} (18)

Shear viscosity relates to absorption probability for an $h_{12}$ graviton. Bulk viscosity relates to absorption of a mixture of the $h_{ii}$ graviton and the scalar $\phi$.

\[
ds^2 = e^{2A(r)} \left( -h(r) dt^2 + d\vec{x}^2 \right) + e^{2B(r)} \frac{dr^2}{h(r)} \hspace{1cm} \phi = \phi(r).
\]  \hspace{1cm} (19)

In a gauge where $\delta \phi = 0$, let’s set $h_{11} = e^{-2A} \delta g_{11} = e^{-2A} \delta g_{22} = e^{-2A} \delta g_{33}$. Then

\[
h^\mu_{11} = \left( -\frac{1}{3A'} - 4A' + 3B' - \frac{h'}{h} \right) h_{11}' + \left( -\frac{e^{-2A+2B}}{h^2} \omega^2 + \frac{h'}{6hA'} - \frac{h'B'}{h} \right) h_{11}.
\]  \hspace{1cm} (20)

- Type I: smooth crossover, like (17).
- Type II: nearly second order, $c_s^2 \to 0$ at $T_c$.
- Type III: No BH below $T_c$, like \cite{Gursoy2008b}.

- Sharper behavior of $c_s^2$ gives sharper $\zeta / s$.
- Large $\zeta$ at $T_c$ is hard to arrange with a reasonably realistic EOS.
- Poses a challenge for “soft statistical hadronization” proposal of \cite{Karsch2007}.
Is bulk viscosity experimentally relevant?

**Interesting proposal** of Kharzeev and collaborators [Kharzeev and Tuchin 2007; Karsch et al. 2007]: bulk viscosity is a strong correction to hydro at $T = T_c$ leading to last-instant entropy production accompanying freezeout:

![Diagram of expansion](image)

*If $\zeta$ is large, much entropy / many soft particles are produced as thermal medium expands. This depiction is in imitation of a figure in [Kharzeev].*

Bottom-up calculations in AdS suggest that it’s hard to get $\zeta/s > 0.1$ with quasi-realistic eos. If that’s right, then expansion-induced entropy is probably not so significant.

4. **The trailing string**

A heavy external quark moving at speed $v$ experiences a drag force [Herzog et al. 2006; Gubser 2006a] (see also [Casalderrey-Solana and Teaney 2006]):

$$\frac{dp}{dt} = -\frac{\pi \sqrt{\lambda}}{2} T^2 \frac{v}{\sqrt{1 - v^2}}. \quad (21)$$

(21) arises in a simple way: a fundamental string trails out behind the quark into $AdS_5$-Schwarzschild, pulling back upon it.

![Diagram of static and drag forces](image)

*Static force versus drag force. In both cases, the classical shape of the string is known analytically.*
Mass is formally infinite, but if we use instead a finite heavy quark mass $M$, find
\[ \frac{dp}{dt} = -\frac{p}{\tau_Q} \]
where \[ \tau_Q = \frac{2}{\pi \sqrt{\lambda T^2}} \] (22).

So characteristic stopping length / time is $\tau_Q$.

To get a numerical value for $\tau_Q$, I favor comparing $\mathcal{N} = 4$ SYM to QCD at fixed energy density rather than temperature. $SU(3)$ SYM has about $3 \times$ the number of degrees of freedom as QCD, and I expect $\tau_Q$ to decrease with number of dof’s.

To fix $\lambda$, I favor [Gubser 2006c] using the following effective measure of $\alpha_s$:
\[ \alpha_{\bar{q}q}(r, T) \equiv \frac{3}{4} r^2 \partial^2 F_{\bar{q}q} / \partial r^2 \]
$F_{\bar{q}q}$ is excess free energy from heavy $q$-$\bar{q}$ pair. (23)

\[ \alpha_{\text{SYM}}(T = 0) = \frac{3}{4} r^2 \partial^2 V_{\bar{q}q} / \partial r^2 = \frac{\sqrt{\lambda}}{\Gamma(1/4)} \frac{3\pi^2}{(1/4)!}. \] (24)

To fix $\lambda \approx 5.5$, compare to lattice at largest $r$ where U-shape dominates.

\[ \begin{array}{c}
\text{a) } T_{\text{SYM}} = 190 \text{ MeV} \\
\end{array} \]

Static quark force for $\mathcal{N} = 4$ SYM (yellow band) versus $N_f = 2$ lattice results from [Kaczmarek and Zantow 2005].

- $\epsilon_{\text{SYM}} = \epsilon_{\text{QCD}}$ means $T_{\text{SYM}} = T_{\text{QCD}}/3^{1/4}$. I took $T_{\text{QCD}} \approx 250$ MeV here.
- An alternative perspective can be found in [Sin and Zahed 2007].

The match is conspicuously imperfect! At least we fix $\lambda$ from a leading-order effect. Matching Debye length in large $r$ tail gives even smaller $\lambda$ [Bak et al. 2007].

A sensible alternative is $T_{\text{QCD}} = T_{\text{SYM}}$ with $\lambda \approx 6\pi$ from setting $\frac{g_{\text{YM}}^2}{4\pi} = \alpha_s \approx 0.5$. Always, $N = 3$. 

Two string theory configurations contributing to $F_{\bar{q}q}$. Only U-shape is fully understood. But see [Bak et al. 2007] for recent work on exchange diagram.
Using my preferred comparison scheme, $\tau_c \approx 2\text{ fm}/c$ for charm at RHIC; also $\tau_b/\tau_c = m_b/m_c$. So charm equilibrates, and $b$ does so only partially.

$R_{AA}$ and $v_2$ for heavy quarks. $p_T$ is for a non-photonic electron. From [Adare et al. 2006].

- Crudely, $R_{AA}(p_T)$ is the % of charm quarks escaping at a given transverse momentum.
- But $p_T$ shown is for $e^\pm$ decay product, so roughly double it to get $p_T$ of $c$.
- Smaller $R_{AA}$ and bigger $v_2$ go together.
- van Hees curves have $\tau_c \approx 4.5$ fm.

**Upshot:** Data favors larger $\tau_c$, but not much larger, than string theory analysis. For an alternative viewpoint, see e.g. [Teaney 2008]; also, beware $b$ contribution.

---

Tagging $b$’s and $c$’s should be possible after detector upgrades at RHIC, and at LHC. A distinctive difference [Horowitz and Gyulessy 2007] between pQCD and AdS/CFT predictions from RHIC to LHC energies may come from

$$R^{cb}_{AA} \equiv \frac{R^{b}_{AA}}{R^{c}_{AA}} \sim \begin{cases} \frac{t_{\text{bottom}}}{t_{\text{charm}}} \approx \frac{m_{\text{charm}}}{m_{\text{bottom}}} & \text{for AdS/CFT} \\ 1 - p_{cb}/p_T & \text{for pQCD, } p_{cb} \propto \hat{q}L^2 \end{cases}$$

pQCD predictions for $R^{b}_{AA}$ separate clearly from AdS/CFT because assumptions about initial conditions cancel out. But beware uncertainty on the limits of validity of AdS/CFT.

Related studies by Brasoveanu and d’Enteria are in progress.
4.1. Stochastic forces on heavy quarks

Drag force is not the whole story: in a Langevin description [Casalderrey-Solana and Teaney 2006; Gubser 2006b; Casalderrey-Solana and Teaney 2007]

\[ \frac{d\vec{p}}{dt} = -\eta \vec{p} + \vec{F}(t) \quad \eta = \frac{\pi \sqrt{\lambda T^2}}{2m} \] (26)

where \( \vec{F} \) is a stochastic force: if \( \vec{p} \) is in the \( \hat{1} \) direction, then

\[ \langle F_1(t_1)F_1(t_2) \rangle \approx \kappa_L \delta(t_1 - t_2), \quad \kappa_L = \pi \sqrt{\lambda} \frac{T^3}{(1 - v^2)^{5/4}} \] (27)

\[ \langle F_i(t_1)F_j(t_2) \rangle \approx \kappa_T \delta_{ij} \delta(t_1 - t_2), \quad \kappa_T = \pi \sqrt{\lambda} \frac{T^3}{\sqrt{4/1 - v^2}} \]

String theory value for \( \kappa_L \) exceeds Einstein relation except near \( v = 0 \):

\[ \kappa_L = \frac{1}{(1 - v^2)^{3/4}} 2TE\eta, \] (28)

hinting that Langevin description doesn’t capture all the physics.

Also: correlation time in \( \vec{F}(t) \) diverges as \( 1/\sqrt{1 - v^2} \).

\[ ds_5^2 = \frac{L^2 \pi^2 T^2}{y^2} \left[ -(1 - y^4)dt^2 + d\vec{x}^2 + \frac{1}{\pi^2 T^2} \frac{dy^2}{1 - y^4} \right]. \] (29)

Consider observers who stay at fixed \( y \) while holding onto the trailing string:

- \( d\tau^2 > 0 \) if \( y > y_v \equiv \sqrt{1 - v^2} \): “outside” the worldsheet black hole.
- \( d\tau^2 < 0 \) if \( y < y_v \): “inside” the worldsheet black hole. The observer can’t stay at fixed \( y \), but slides down the string.

Something roughly like Hawking radiation must emanate from the worldsheet horizon, leading to stochastic \( \vec{F}(t) \). Actual computations directly access \( \langle F_i(t_1)F_j(t_2) \rangle \).
5. Jet-splitting?

A hard process occurring near the edge of the medium produces a near-side “trigger” jet (red). The away-side parton interacts strongly with the medium. From [Jacak 2006].

Jet reconstruction is impractical, so make histograms of azimuthal separation between two energetic hadrons.

With appropriate $p_T$ cuts, observe a double-hump structure on away-side: “jet-splitting.” [Jia 2007].

More inclusive cuts fill in the region around $\Delta \phi = \pi$: “jet-broadening” [Adams et al. 2005].

A string theory calculation has been done for heavy quarks: [Gubser et al. 2007; Chesler and Yaffe 2007] and refs therein.

A heavy quark trails a string behind it. The string couples to gravitons dual to $\langle T_{mn} \rangle$ in the gauge theory.

Calculate $h_{mn}$ using linearized Einstein equations.

One big calculation gives $\langle T^{0m} \rangle$ over a broad range of scales; high $k$ asymptotics pioneered in [Yarom 2007] turn out to be especially interesting.

Render all quantities dimensionless:

$$\vec{X} = \pi T \vec{x}, \quad S_i(\vec{X}) = \sqrt{1 - \eta^2} \left( \frac{\langle T^{0i} \rangle(0, \vec{x}) - T_{\text{Coulomb}}^{0i}(0, \vec{x})}{(\pi T)^4 \sqrt{\lambda}} \right).$$ (30)
Rescaled, subtracted Poynting vector generated by a quark in an infinite, static medium. Green shows the Mach angle, and blue shows the parabolic boundary of the diffusion wake. For $T \approx 318\text{ MeV}$, $|\vec{x}| = 5$ is a distance 1 fm from the quark. From [Gubser et al. 2007].

A phenomenological comparison [Betz et al. 2008] including Cooper-Frye hadronization shows that AdS/CFT does lead to jet-splitting at $p_T \approx 5\text{ GeV}$.

But the reason is unexpected: it’s not the hydro region that does it, it’s the “neck” region with $|x| \lesssim 1\text{ fm}$.

Puzzles / problems remain:

- Pseudo-Mach angle is smaller than data, and gets smaller as $v \to 1$.
- This was for heavy quarks!
- Cooper-Frye isn’t perfect.
- Interpretation of experimental phenomenon isn’t universally agreed upon.
6. Jet quenching

According to pQCD (e.g. [Baier et al. 1997; Zakharov 1997; Wiedemann 2000]), radiative energy loss by light quarks and gluons is

$$\Delta E = \frac{1}{4} \alpha_s C_R \hat{q} (\Delta x)^2,$$

where the jet-quenching parameter describes how fast momentum broadens as a function of path length $\Delta x$:

$$\hat{q} = \frac{\langle p^2 \rangle}{\Delta x}.$$  \hspace{1cm} (32)

Authors including [Kovner and Wiedemann 2003; Liu et al. 2006] prefer a definition in terms of a partially light-like Wilson loop with $L \ll \Delta x$:

$$\langle W^{\text{adjoint}}(C) \rangle \approx \exp \left[ -\frac{1}{4} \hat{q} L^2 \Delta x \right].$$  \hspace{1cm} (33)

A gauge-string calculation of $\langle W^{\text{fundamental}} \rangle$ leads to

$$\hat{q} = \frac{\pi^{3/2} \Gamma(3/4)}{\Gamma(5/4)} \sqrt{\lambda} T^3.$$  \hspace{1cm} (34)

A correction factor $\sqrt{s_{QCD}/s_{SYM}}$ is advocated in [Liu et al. 2007] to correct for fewer degrees of freedom. Including this factor and using $\lambda = 6\pi$, as they prefer, I calculate

$$\hat{q} \approx 2.3 \frac{\text{GeV}^2}{\text{fm}} \quad \text{at } T = 280 \text{ MeV},$$

significantly above pQCD’s $\hat{q} \approx 0.77 \text{ GeV}^2/\text{fm}$ and almost big enough to agree with experiment (more later).

But some puzzles remain:

- Argyres and collaborators criticize the choice of saddle point [Argyres et al. 2007 2008] and find $\log \langle W^A(C) \rangle \sim L$ not $L^2$.
- $\hat{q}$ as defined through Wilson loop may not be directly related to energy loss or momentum diffusion in strongly coupled gauge theories.
- Independent calculations of $\hat{q}_T \equiv \langle p^2 \rangle / \Delta x$ for heavy quarks [Herzog et al. 2006; Casalderrey-Solana and Teaney 2006 2007; Gubser 2006b] lead to larger values than (34): larger by $\sim \sqrt{\gamma}$ as $v \to 1$. 
7. Falling strings

Can we calculate *ab initio* the energy loss of a gluon in strongly coupled $\mathcal{N} = 4$?

We propose [Gubser et al. 2008a] to regard an off-shell gluon as a doubled string with both ends passing through the horizon.

At zero temperature, results of [Alday and Maldacena 2007] show that gluon scattering produces approximately this type of string configuration.

At finite temperature, something funny happens: where the string crosses the horizon, it can’t move! (Infinite red-shifting wrt Killing time $t$.)
A doubled string starts at \( t = 0 \) with some total energy and virtuality, then falls into the horizon over a distance \( \Delta x \).

- Given initial \( E \), what is \( \Delta x \)?
- Answer must depend on virtuality \( \leftrightarrow y_{\text{UV}} \), so what is maximum \( \Delta x \)?
- How do we roughly convert the answer to \( \hat{q} \)?

We made estimates based on assuming the shape of the falling string quickly approaches a segment of the trailing string; confirmed numerically in [Chesler et al. 2008].

For \( E \gg T \), we found \( \Delta \hat{x} \approx \hat{E}^{1/3} \) (see also [Hatta et al. 2008]), where

\[
\hat{x} = \pi T x \quad \hat{E} = \frac{1}{\sqrt{g_{YM}^2 N}} \frac{E}{T}.
\]

This is not too different from pQCD prediction \( \Delta x \propto \sqrt{E/\hat{q}} \). So let’s convert to a rough prediction of \( \hat{q} \):

\[
\hat{q}_{\text{rough}} \equiv \frac{4E}{3\alpha_s(\Delta x)^2}.
\]

Estimates of the jet-quenching parameter, from (37), comparing at fixed energy density, with \( \lambda = 5.5 \). Different symbols correspond to varying assumptions about shape of falling string. From [Gubser et al. 2008a]. LRW is from [Liu et al. 2006] at 280 MeV, for SYM; scaled LRW is for QCD at 280 MeV, including the \( \sqrt{s_{\text{QCD}}/s_{\text{SYM}}} \) factor from [Liu et al. 2007].
The overall picture on jet-quenching is, in my view, somewhat muddled at present:

- **Good** that we’re within $3\sigma$ range, or close.
- **Good** that we can accommodate gluons that start off significantly virtual.
- **Questionable** to compare $\hat{q}$ from falling strings to a value in PQM model, where underlying assumptions are different.
- **Bad** that we don’t understand relation among jet-quenching calculations, plus heavy quark drag / diffusion.
- **Interesting** to consider including fluctuations or graviton response, starting either from [Liu et al. 2006] or [Gubser et al. 2008a].
- **Maybe good** that numerical study [Chesler et al. 2008] shows larger $\Delta x$ (so smaller $\hat{q}$) for falling strings; or was that due to initial conditions?

### 8. Total multiplicity

Central RHIC collision:

$$N_{\text{part}} \approx 2 \times 197 = 394 \quad \text{nucleons in}$$

$$N_{\text{ch}} \approx 5000 \quad \text{charged particles out.}$$

A reasonable estimate of the entropy produced is

$$S \approx 7.5 N_{\text{charged}} \approx 38000 \quad (38)$$

(E.g. consider a gas of free hadrons at $T_c$ and compute $S/N_{\text{charged}}$ starting from partition function.)

How well can we estimate $S$ from the gauge-string duality?

**Strategy** of [Gubser et al. 2008d]:

- Replace QCD by a conformal theory with $\epsilon/T^4 = 11$, as lattice predicts for QCD for $T \gtrsim 1.2T_c$. (Remarkably slow rise thereafter.)

---

Charged tracks measured by STAR in a gold-gold collision [STA]. For multiplicity estimates, see e.g. PHOBOS’s [Back et al. 2005].
• Replace a heavy ion with a boosted “conformal soliton,” dual to a point-sourced gravitational shock wave in $AdS_5$: if $x^- = x^0 - x^3$, then

$$\langle T^- \rangle = \frac{2EL}{\pi [(x^1)^2 + (x^2)^2 + L^2]^3} \delta(x^-),$$

(Power law tails are not a good thing, but at least they’re a big power: $1/x_+^6$.)

A standard but non-rigorous lower bound is

$$S \geq S_{\text{trapped}} \equiv A_{\text{trapped}}/4G_5.$$

Earlier related work is reviewed in [Nastase 2008].

The final result is

$$S_{\text{trapped}} \approx \pi \left( \frac{L^3}{G_5} \right)^{1/3} (2EL)^{2/3} \approx 35000 \left( \frac{\sqrt{s_{NN}}}{200 \text{ GeV}} \right)^{2/3}.$$  

• I set $L = 4.3 \text{ fm}$ to match the rms transverse radius of a gold nucleus.

• $E \approx 19.7 \text{ GeV}$ is beam energy; $\sqrt{s_{NN}} = 200 \text{ GeV}$ is cm energy of a pair of nucleons ($NN$).

$E^{2/3}$ scaling is faster than Landau ($E^{1/2}$) [Landau 1953] and faster than data ($\approx$ Landau).

I think it’s because strong-coupling conformal window covers only a range of scales. A crude solution [Gubser et al.]:

Assume that most entropy is generated within this range, above confinement and below pQCD.
UV cutoff changes scaling from $S_{\text{trapped}} \sim E^{2/3}$ to $E^{1/3}$ at large $E$. So anticipate $N_{\text{charged}} \sim E^{1/3}$. Maybe even for protons?

Roll-over from Landau’s $E^{1/2}$ to slower growth might just be starting at top RHIC energies:

![Total multiplicity per participant as a function of energy. From [Steinberg 2005].](image)

9. **Outlook**

- Gauge-string / Heavy-ion connection is the closest interface we have between modern string theory and modern experiment.

- Many comparisons are successful at a semi-quantitative level. (Many more than I have summarized here...)

- Comparisons are invariably plagued by the difficulty of translating from AdS calculations to real-world QCD.

- We may often be measuring our successes against prevailing interpretations of data rather than data itself.

- At the least, gauge-string calculations show what happens in a truly strongly coupled thermal plasma.

- Insights from AdS/CFT complement pQCD intuitions and may sometimes be closer to capturing the true dynamics.
References


Steven S. Gubser and Abhinav Nellore. “Mimicking the QCD equation of state with a dual black hole”. 2008, 0804.0434.

Steven S. Gubser, Silviu S. Pufu, and Amos Yarom. Forthcoming.


Steven S. Gubser, Daniel R. Gulotta, Silviu S. Pufu, and Fabio D. Rocha. “Gluon energy loss in the
gauge-string duality”. 2008a, 0803.1470.

Steven S. Gubser, Abhinav Nellore, Silviu S. Pufu, and Fabio D. Rocha. “Thermodynamics and bulk
viscosity of approximate black hole duals to finite temperature quantum chromodynamics”. 2008b,
0804.1950.

Steven S. Gubser, Silviu S. Pufu, and Fabio D. Rocha. “Bulk viscosity of strongly coupled plasmas
with holographic duals”. 2008c, 0806.0407.

Steven S. Gubser, Silviu S. Pufu, and Amos Yarom. “Entropy production in collisions of gravitational
shock waves and of heavy ions”. 2008d, 0805.1551.

U. Gursoy and E. Kiritsis. “Exploring improved holographic theories for QCD: Part I”. JHEP, 02:032,

U. Gursoy, E. Kiritsis, and F. Nitti. “Exploring improved holographic theories for QCD: Part II”. JHEP,
02:019, 2008a, 0707.1349. doi: 10.1088/1126-6708/2008/02/019.

Dynamics in Improved Holographic QCD”. 2008b, 0804.0899.


Y. Hatta, E. Iancu, and A. H. Mueller. “Jet evolution in the N=4 SYM plasma at strong coupling”.


Jiangyong Jia. “Mapping out the Jet correlation landscape: Jet quenching and Medium response”. 2007,

Olaf Kaczmarek and Felix Zantow. “Static quark anti-quark interactions in zero and finite temperature
QCD. I: Heavy quark free energies, running coupling and quarkonium binding”. Phys. Rev., D71:
114510, 2005, hep-lat/0503017.

hep-lat/0106019.

Frithjof Karsch, Dmitri Kharzeev, and Kirill Tuchin. “Universal properties of bulk viscosity near the

Yevgeny Kats and Pavel Petrov. “Effect of curvature squared corrections in AdS on the viscosity of the
dual gauge theory”. 2007, 0712.0743.

Dmitri Kharzeev. “QCD-gravity correspondence”. Lectures given at Quark Matter School, Jaipur, India,
February 2008.


