Aug '08 ①

Really important questions

1. Foundations of gauge/string duality

2. „Geometry" at „∞ curvature

3. Gauge/strings and de Sitter

4. Cosmological constant

②

Properties and paradoxes of the de Sitter.

1. From $S \Rightarrow dS$

$S)$ $\vec{n}^2 + n_0^2 = 1$

$dS)$ $\vec{n}^2 - n_0^2 = 1$ $\qquad n_0 \Rightarrow i n_0$

The propagator

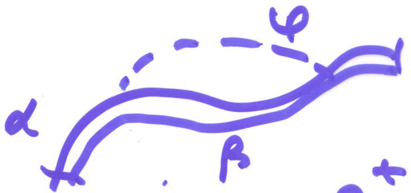
$\langle \varphi(n_1)\,\varphi(n_2) \rangle$

$= \dfrac{1}{\sin \pi \nu} P_\nu(-(n_1 n_2))$

or $\frac{1}{\sin \pi \nu} C_\nu^{d/2}(-n_1 n_2)$ if

$D = d+1$ / (Chernikov Tagirov '67, Bunch-Davies ...

Geodesic observer /Unruh detector/:



$$W_{\alpha\to\beta} \propto \int_{-\infty}^{+\infty} ds\, e^{-i(\varepsilon_\beta-\varepsilon_\alpha)s} \langle \varphi(u(s))\,\varphi(u(0))\rangle$$

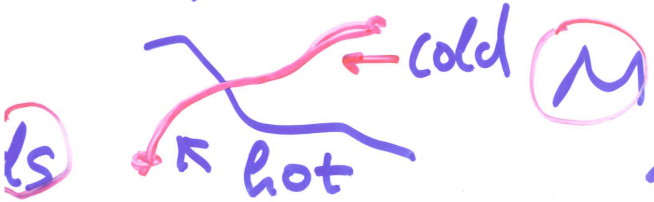
The B.-D. Green function

$$G \propto A e^{-imL} + B e^{imL}$$

⇒ energy can be taken from (dS).

(L – geodesic distance)

Paradox: patch of dS and Minkowsky



"Equivalence principle violated"

2d Model

$$\mathcal{L} = \psi_-^*(\partial_+ - h_{++}\partial_-)\psi_- + \psi_+^*\partial_-\psi_+ + m(\psi_+^*\psi_- + c.c.)$$

dS space $h_{++} = R(x^-)^2$

/analogous gauge model:

$$\mathcal{L} = \psi_-^*(\partial_+ + iA_+)\psi_- + \dots$$

$A_+ = E x^-$ /

Bosonize

$$\mathcal{L} = (\partial\varphi)^2 + m^2(1-\cos\varphi) + R(x^-)^2(\partial_-\varphi)^2 \quad \text{(grav)}$$

$$\mathcal{L} = (\partial\varphi)^2 + m^2(1-\cos\varphi) + E x^-\partial_-\varphi \quad \text{(gauge)}$$

⑤

L.C. quantization

$$P_+ = \int dx^- (1-\cos\varphi) -$$
$$- \int dx^- (x^-)^2 R \cdot (\partial_-\varphi)^2$$

(gravity)

$$P_+ = \int dx^- (1-\cos\varphi)$$
$$- \int dx^- E\varphi$$

Both unbounded below

Back-reaction cures this by screening E & R: $\langle T_{\mu\nu}\rangle - \Lambda g_{\mu\nu} = 0$

Nature abhors (positive) curvature

⑥

Stability of the curved space (in general)

How to quantize?

Proposal

In the stable space-time we should use Feynman's principle, defining

$$G_F(x,x') = \sum_{(P_{xx'})} e^{-imL(P_{xx'})}$$

$$G_F(x,x') \underset{x-x'\to\infty}{\sim} e^{-imL(x,x')}$$

This is NOT the case in Bunch-Davies vacuum.

(7)

Now we formulate the eternity condition:

Feynman = Schwinger - - Keldysh

Define

$G_{++} = G_F(x, x', M^2 - i0)$

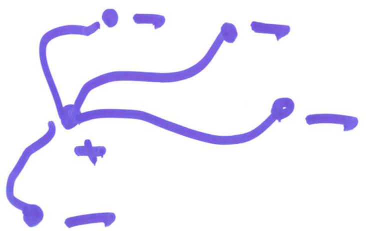
$G_{+-} = G(x + i\eta, x', M^2)$

(η - time-like, & infinitesimal)

$G_{-+} = G(x - i\eta, x', M^2)$

Stability condition

No spiders



$= 0$

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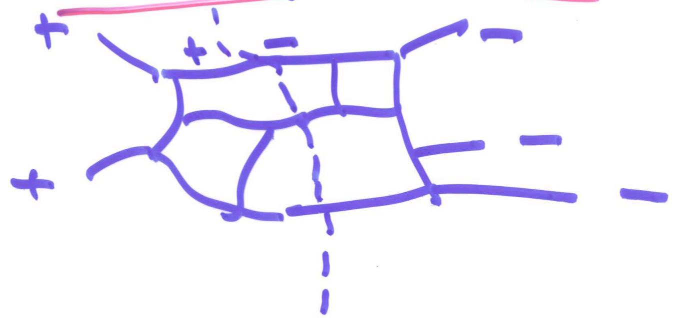
No spiders ⇒ usual cutting rules:



Also stability requires

$\mathrm{Im}\, G_F(x, x) = 0$

This implies that

${}_{out}\langle 0|0\rangle_{in} = e^{iW}$; $\mathrm{Im}\, W = 0$.

In the (dS) case, the Feynman G-function is analytic continuation from AdS: (Not from a sphere)

$G = Q_\nu(n_1 n_2)$

In (dS) (and FRW) the stability for the partial Green function, satisf.

$$(\partial_t^2 - \mu^2 + \mathcal{U}(t))G = \delta()$$

/ $ds^2 = dt^2 - a^2(t)(d\vec{x})^2$ /

implies that the Jost functions ~~as~~ have the property!

$$\int \phi^N(t)\,dt = 0 \quad / \text{zero spider} /$$

(here

$$G_F(t,t') \sim \phi(t_>)\,\chi^*(t_<)$$

$$\phi(t) \to e^{-i\mu t} \qquad t \to -\infty$$

$$\chi \longrightarrow e^{-i\mu t} \qquad t \to +\infty$$

|Novikov's equation for $\mathcal{U}(t)$

In <u>odd</u> dimension (dS) potential $\mathcal{U}(t) \sim \frac{1}{\cosh^2 t}$ is a soliton for the ~~KdV~~ and Novikov eqs.

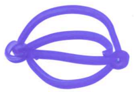
(reflectionless property of $\mathcal{U}(t)$ in odd dim was already noticed by Strominger et al.)

General questions: are there other stable FRW spaces?

Relation to the ~~H~~ Huygens Principle ?

The B.-D. vacuum is unstable not by itself, but by sensitivity to perturbations:

$$\langle E|\, T e^{-i\lambda\int\varphi^4}\, |E\rangle = e^{iW}$$

$$\approx \exp\{-\lambda^2$$  $$\}$$

$$\mathrm{Im}\, W \propto \lambda^2 \cdot (\text{Volume})$$

$$_{out}\langle E|E\rangle_{in} = 0$$

We expect to find at the time t a non-zero occupation numbers $\{f_p(t)\}$.

They satisfy (for small λ) the Boltzmann eqs!

$$\dot f(p) = \int |\gamma(p, p_1, p_2)|^2 \cdot \delta(p+p_1+p_2)$$

$$\{(1+f(p_1))(1+f(p_2))[1+f(p)]$$

$$- f(p_1)\, f(p_2)\, f(p)\} + \dots$$

$$\gamma(p, p_1, p_2) \propto \int \phi\,\phi\,\chi^*$$

(in terms of Jost funct.; for (dS) $\phi \propto H^{(1)}_{i\mu}(p\tau)$)

<u>The key</u>: No energy conservation, hence <u>no detailed balance</u>

$f(p)$ blows up until stopped by back reaction

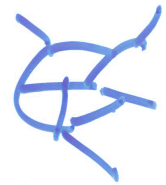
From the different point of view, particle production in (dS) and possible screening of the cosm. const was considered by many people starting from '80. Very incomplete list includes Antoniadis, Mazur, Mottola, Myrhvald, Bros, Epstein, Moschella, Woodard, Tsamis.

Very different version of dS/CFT was suggested by Strominger.

Similar analytic cont. AdS $\to$ dS discussed by Maldacena.

String theory side

Planar diagrams diverge as c^k



(t'Hooft).

Scaling lim: $F = \sum \lambda^k F_k$

$\lambda \to \lambda_{cr}$. they become dense and can be viewed as a world sheet. This is what happens in the matrix models for minimal gravity, but NOT in the usual g/s duality.

We expect λ_{cr} to be complex

As we analytically continue from (AdS) to (dS) /getting the Q-propagator/ we have to continue the gauge theory to complex λ.

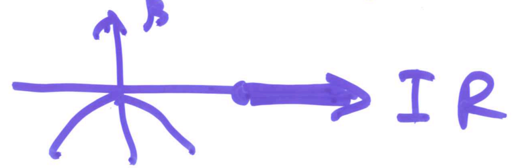
Conjecture $\lambda = \lambda_{cr}$,
de Sitter space is described by the dense Feynman diagrams

The theory is non-unitary
Σ(probabilities) < 1
since the vacuum may disappear.

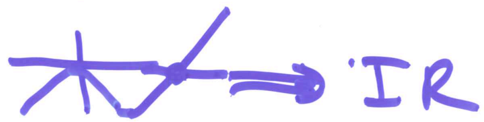
The sigma model

$$S = \frac{1}{2\alpha_0} \int (\partial n)^2 d^2\xi$$

Feynman graphs don't change as we go from sphere $\Rightarrow$ de Sitter
But the theories are different:
(S) – mass gap, due to the compactness of (S) (discrete sp.)
The β – function – asympt. free



(Sph.)



(dS)

Large D expansion

$$\mathcal{L} = (\partial n)^2 + \lambda (n^2 - 1)$$

$$D \alpha_0 \int \frac{d^2 k}{k^2 + \lambda} = 1$$

But this can't be true in (dS). Take 1d sigma Model first. We have on finite circle:

$$D \alpha_0 \sum_n \frac{i}{\omega_n^2 + \lambda + i0} = 1$$

$$\omega_n = 2\pi n / T \qquad \lambda = m^2$$

$$mT = \frac{D \alpha_0 T}{\tanh(mT)}$$

The wrong saddle point

The right one: $m \Rightarrow im$

$$mT = \frac{-D \alpha_0 T}{\tan(mT)}$$

as $T \to \infty$ $\quad mT \to \pi/2$

The mass $m \propto 1/T$.

The same is true in (2d).

The spectrum of dimensions (?)