Heterotic Standard Models

Ron Donagi (Penn)

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Strings 08 @ CERN

Heterotic Standard Models
The High Country region of the string landscape

- Goal: Study string vacua which reproduce the MSSM (or close cousins thereof) at low energies
  - String landscape is huge, but High Country region may be much smaller
- Questions:
  - How many such vacua?
  - Do they have common properties (predictions)?
  - Constraints coming from string UV completion?
- Crucial: Must require global consistency of the string vacuum

A particular corner of the string landscape:

$E_8 \times E_8$ heterotic string on $\mathbb{R}^{3,1} \times X$ with gauge instanton V, where X is a smooth compact Calabi-Yau threefold

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Outline

References

References:

BD1 [Bouchard, D] **An SU(5) heterotic standard model**
Phys. Lett. B633 2006

BCD [Bouchard, Cvetič, D] **Tri-linear couplings ...**
Nuclear Phys. B745 2006

BD2 [Bouchard, D] **On a class of non-simply connected CY3s**
Comm.Numb.Theor.Phys.2 2008

BD3 [Bouchard, D] **On heterotic model constraints**
hep-th 0804-2096

DW [D, Wendland] **On orbifolds and free fermion constructions**
hep-th 0808-xxxx

Bak [Bak] **Penn thesis**

BBDG [Bak, Bouchard, D, Gross] **In Prep**

Outline

SUSY heterotic vacua

Data:

- X: smooth compact Calabi-Yau threefold
- $V \longrightarrow X$: hol. vector bundle with structure group $G \subset E_8$

Consistency constraints:

- V is polystable w.r.t a Kähler class [DUY: connection soln to HYM]B
- $c_2(X) - c_2(V) = [M5]$ [anomaly cancellation with $M5$-branes]B

Phenomenological requirements:

- Commutant H of G in E_8 is low-energy GUT group
- $\pi_1(X) = F$ to break H to MSSM gauge group with discrete Wilson line
- Various extra phenomenological constraints:
  - $c_1(V) = 0$ [$G = SU(n)$]B
  - $c_3(V) = \pm 6$ [3 generations]B
  - $H^1(V), H^2(V), H^1(\wedge^2 V), H^2(\wedge^2 V), \ldots$ [Particle spectrum]B
  - Triple products of cohomology groups, ... [Tri-linear couplings]B

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Summary and examples

Heterotic vacuum:

1. Non-simply connected Calabi-Yau threefold X
2. Polystable bundle $V \to X$ satisfying a lot of constraints

Examples:

- $\pi_1(X) = \mathbb{Z}_2$, $G = SU(5)$
  - $SU(5)$ GUT
  - $SU(5) \xrightarrow{\mathbb{Z}_2} SU(3) \times SU(2) \times U(1)$
- $\pi_1(X) = \mathbb{Z}_6$ or $(\mathbb{Z}_3)^2$, $G = SU(4)$
  - $SO(10)$ GUT
  - $SO(10) \xrightarrow{\mathbb{Z}_6 \text{ or } (\mathbb{Z}_3)^2} SU(3) \times SU(2) \times U(1) \times U(1)$

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Outline

1st step: Constructing non-simply connected CY 3-folds X

Consider a smooth simply connected Calabi-Yau threefold $\tilde{X}$ admitting a group F of automorphisms acting freely on $\tilde{X}$

$\rightarrow X = \tilde{X}/F$ is a smooth Calabi-Yau threefold with $\pi_1(X) = F$

- $\tilde{X}$ is a smooth fiber product of two rational elliptic surfaces [Schoen]B
  - We classified all possible finite groups F acting freely on $\tilde{X}$ [BD2]B
- $\tilde{X}$ is the small resolution of a particular complete intersection of four quadrics in $\mathbb{P}^7$ [Gross]B
  - free $(\mathbb{Z}_8)^2$ action [Gross]B
  - 2 non-Abelian groups of order 64 act freely [Borisov-Hua]B
- Hypersurfaces/complete intersections in toric threefolds, ...

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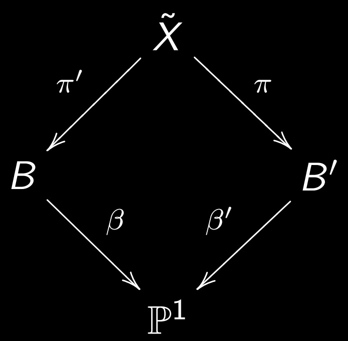
Free quotients of Schoen's threefolds

Let B and B' be RES, and $\tilde{X} = B \times_{\mathbb{P}^1} B'$ a smooth fiber product:



Idea: Consider special B and B' s.t. $\tilde{X}$ admits a free group of automorphisms $F_{\tilde{X}}$.

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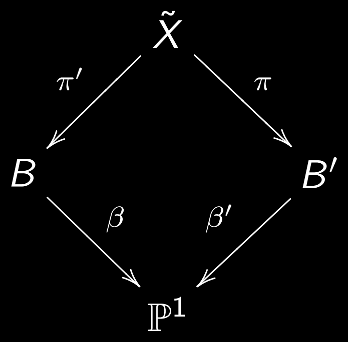

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Free quotients of Schoen's threefolds

- Automorphisms $\tau_{\tilde{X}} : \tilde{X} \to \tilde{X}$ have the form $\tau_{\tilde{X}} = \tau_B \times_{\mathbb{P}^1} \tau_{B'}$
- Classification of $(\tilde{X}, F_{\tilde{X}})$ reduces to classification of (B, F_B), for suitable groups of automorphisms F_B

We produced such a classification, and we obtained a large class of $\tilde{X}$ with $F_{\tilde{X}}$ one of the following: [BD2]B

$$(\mathbb{Z}_3)^2, \quad \mathbb{Z}_4 \times \mathbb{Z}_2, \quad \mathbb{Z}_6, \quad \mathbb{Z}_5,$$
$$\mathbb{Z}_4, \quad (\mathbb{Z}_2)^2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_2$$

2nd step: Constructing stable vector bundles $V \longrightarrow X$

- Fourier-Mukai transform [FMW, D]B
  - Use dual Fourier-Mukai data to construct the bundle
  - Needs X to be fibered (usually torus-fibered, but can be generalized)
  - Pros: Easy to prove stability from FM data [FMW]B
  - Cons: If start with $\tilde{V} \to \tilde{X}$, invariance under $F_{\tilde{X}}$ hard to prove
- Serre construction by extension

$$0 \longrightarrow V_1 \longrightarrow V \longrightarrow V_2 \longrightarrow 0$$

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- To satisfy phenomenological constraints, may need combination of both methods [DOPW]B
- Other methods: monads [AHL]B , Hecke transforms, . . .

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Best (and only) model so far [BD1, BCD]B

- The manifold: $\tilde{X} = B \times_{\mathbb{P}^1} B'$, with special B and B' such that $F_{\tilde{X}} \simeq \mathbb{Z}_2$ acts freely on $\tilde{X}$
- The bundle: $SU(5)$, $\mathbb{Z}_2$-invariant, stable bundle $\tilde{V} \to \tilde{X}$ constructed by

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  where V_3 and V_2 are rank 3 and 2 bundles on $\tilde{X}$ constructed using Fourier-Mukai transform
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Phenomenology of this model

- MSSM gauge group $SU(3) \times SU(2) \times U(1)$ with no extra $U(1)$'s
- Precisely the MSSM massless spectrum with no exotic particles, up to moduli fields
- Semi-realistic tri-linear couplings at tree level
- R-parity is conserved at tree level (proton is stable)
- Higgs μ-terms and (possible) neutrino mass terms

To be addressed

- SUSY breaking? (hidden sector)
- Moduli stabilization?
- Higher order corrections?
- More phenomenology needed

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Other models

- Buchmuller, Hamaguchi, Lebedev, Ratz (and many others)
  $\mathbb{Z}/6$-orbifolds
- Braun, He, Ovrut, Pantev
  $\pi_1(X) = \mathbb{Z}/3 \times \mathbb{Z}/3$, V unstable
- Faraggi, NAHE: Free Fermionic models
  $(\mathbb{Z}/2)^n$ orbifolds "non-geometric"
- D, Faraggi: not within a particular class of geometric orbifolds
- DW: not a geometric orbifold

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Constructing other realistic models

- Using methods similar to [BD1], we tried to construct realistic bundles on fiber product $\tilde{X}$ with $F_{\tilde{X}} \simeq \mathbb{Z}_6$
  $\longrightarrow$ No realistic bundle [BD3]B
  - Main insight: strong tension between inequalities coming from anomaly cancellation and stability
- Work in progress: physical bundles on Gross' threefold with $\pi_1(X) = (\mathbb{Z}_8)^2$ [BBDG]B
  - We constructed bundles phenomenologically viable at the topological level (up to a few subtleties that remain to be checked), using Fourier-Mukai transform on Abelian surface fibrations, and Hecke transforms
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Relaxing the constraints (. . .)

Recall: strong tension between anomaly cancellation and stability

- In principle, one can "forget" about anomaly cancellation
  - non-SUSY vacua with $M5$- and anti-$M5$-branes [B,BBO]B
  - SUSY broken at the compactification scale :-(

- We get infinite families of such non-SUSY vacua with exactly the phenomenological properties above [BD3]B
- Also get infinite families of models on $\tilde{X}$ with $\pi_1(\tilde{X}) = \mathbb{Z}_6$
- One such model on $\tilde{X}$ with $\pi_1(\tilde{X}) = (\mathbb{Z}_3)^2$ [BHOP]B , perhaps more

- Such infinite familes considered by Acharya-DouglasB in landscape study
  $\rightarrow$ phenomenological cutoff on scale of SUSY breaking

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# of orbifolds T^6/G

$T^6 = E_1 \times E_2 \times E_3$
$0 \to G_S \to G \to G_T^0 \to 0$ where G_S is shifts and G_T^0 twists.
$G_T^0 = \mathbb{Z}/2 \times \mathbb{Z}/2$ acts $(x, y, z) \mapsto (\pm x, \pm y, \pm z)$ with even number of sign changes
After some reduction: $\exists G_T \subset G$, $G_T \xrightarrow{\sim} G_T^0$
Four inequivalent types of G_T.
Use reduction procedure to classify. For each model we calculate:

- Hodge Numbers (via orbifold cohomology)
- Fundamental groups (Wilson lines)
- Some geometry

Effect of discrete torsion, no new Hodge numbers, except mirror-symmetry like interchange $H^{1,1} \iff H^{2,1}$. (Compare: Mirage torsion: Ploger, Ramos-Sanchez, Ratz, Vaudrevange.)

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- Hodge Numbers (via orbifold cohomology)
- Fundamental groups (Wilson lines)
- Some geometry

Effect of discrete torsion, no new Hodge numbers, except mirror-symmetry like interchange $H^{1,1} \iff H^{2,1}$. (Compare: Mirage torsion: Ploger, Ramos-Sanchez, Ratz, Vaudrevange.)

of orbifolds T^6/G

$T^6 = E_1 \times E_2 \times E_3$

$0 \to G_S \to G \to G_T^0 \to 0$ where G_S is shifts and G_T^0 twists.

$G_T^0 = \mathbb{Z}/2 \times \mathbb{Z}/2$ acts $(x, y, z) \mapsto (\pm x, \pm y, \pm z)$ with even number of sign changes

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We list the automorphism groups by rank. For each group G we list its twist group G_T, its shift part G_S (if non-empty), the Hodge numbers $h^{1,1}, h^{2,1}$ of a small resolution of X/G, the fundamental group $\pi_1(X/G)$, and the list of contributing sectors and their contribution. For the fundamental groups we use the abbreviations:

A : the extension of $\mathbb{Z}_2$ by $\mathbb{Z}^2$ (so $H_1(X) = (\mathbb{Z}_2)^3$)

B : any extension of $(\mathbb{Z}_2)^2$ by $\mathbb{Z}^6$ (with various possible $H_1(X)$)

C : $\mathbb{Z}_2$

D : $(\mathbb{Z}_2)^2$

A shift element is denoted by a triple $(\epsilon_1, \epsilon_2, \epsilon_3)$, where $\epsilon_i \in E_i$ is a point of order 2, abbreviated as one of $0, 1, \tau, \tau 1 := 1 + \tau$. A twist element is denoted by a triple $(\epsilon_1\delta_1, \epsilon_2\delta_2, \epsilon_3\delta_3)$, where $\epsilon_i \in E_i$ is as above and $\delta_i \in \{\pm\}$ indicates the pure twist part. A two-entry contribution (a, b) adds a units to $h^{1,1}$ and b units to $h^{2,1}$. When $b = 0$ we abbreviate (a, b) to the single entry contribution a.

	G_T	sectors	contribution	$(h^{1,1}, h^{2,1})$	π_1
Rank 0:					
$(0-1)$	$(0+, 0-, 0-), (0-, 0+, 0-)$			$(51, 3)$	0
		$0+, 0-, 0-$	16		
		$0-, 0+, 0-$	16		
		$0-, 0-, 0+$	16		
$(0-2)$	$(0+, 0-, 0-), (0-, 0+, 1-)$			$(19, 19)$	0
		$0+, 0-, 0-$	$8, 8$		
		$0-, 0+, 1-$	$8, 8$		
$(0-3)$	$(0+, 0-, 0-), (0-, 1+, 1-)$			$(11, 11)$	A
		$0+, 0-, 0-$	$8, 8$		
$(0-4)$	$(1+, 0-, 0-), (0-, 1+, 1-)$			$(3, 3)$	B

Note: These are the four types of groups G_T.

	G_T	G_S sectors	contribution	$(h^{1,1}, h^{2,1})$	π_1
Rank 1:					
$(1-1)$	$(0+, 0-, 0-), (0-, 0+, 0-)$	(τ, τ, τ)		$(27, 3)$	C
		$0+, 0-, 0-$	8		
		$0-, 0+, 0-$	8		
		$0-, 0-, 0+$	8		
$(1-2)$	$(0+, 0-, 0-), (0-, 0+, \tau-)$	(τ, τ, τ)		$(15, 15)$	0
		$0+, 0-, 0-$	$4, 4$		
		$0-, 0+, \tau-$	$4, 4$		
		$\tau-, \tau-, 0+$	$4, 4$		
$(1-3)$	$(0+, 0-, 0-), (0-, 0+, 1-)$	(τ, τ, τ)		$(11, 11)$	C
		$0+, 0-, 0-$	$4, 4$		
		$0-, 0+, 1-$	$4, 4$		
$(1-4)$	$(0+, 0-, 0-), (0-, 1+, 1-)$	(τ, τ, τ)		$(7, 7)$	A
		$0+, 0-, 0-$	$4, 4$		
$(1-5)$	$(1+, 0-, 0-), (0-, 1+, 1-)$	(τ, τ, τ)		$(3, 3)$	B
$(1-6)$	$(0+, 0-, 0-), (0-, 0+, 0-)$	$(\tau, \tau, 0)$		$(31, 7)$	0
		$0+, 0-, 0-$	8		
		$0-, 0+, 0-$	8		
		$0-, 0-, 0+$	8		
		$\tau-, \tau-, 0+$	$4, 4$		
$(1-7)$	$(0+, 0-, 0-), (0-, 0+, 1-)$	$(\tau, \tau, 0)$		$(11, 11)$	C
		$0+, 0-, 0-$	$4, 4$		
		$0-, 0+, 1-$	$4, 4$		
$(1-8)$	$(0+, 0-, 0-), (0-, 1+, 0-)$	$(\tau, \tau, 0)$		$(15, 15)$	0
		$0+, 0-, 0-$	$4, 4$		
		$0-, 1-, 0+$	$4, 4$		
		$\tau-, \tau1-, 0+$	$4, 4$		
$(1-9)$	$(0+, 0-, 0-), (0-, 1+, 1-)$	$(\tau, \tau, 0)$		$(7, 7)$	A
		$0+, 0-, 0-$	$4, 4$		
$(1-10)$	$(1+, 0-, 0-), (0-, 1+, 0-)$	$(\tau, \tau, 0)$		$(11, 11)$	A
		$1-, 1-, 0+$	$4, 4$		
		$\tau1-, \tau1-, 0+$	$4, 4$		
$(1-11)$	$(1+, 0-, 0-), (0-, 1+, 1-)$	$(\tau, \tau, 0)$		$(3, 3)$	B

	G_T	G_S sectors	contribution	$(h^{1,1}, h^{2,1})$	π_1
Rank 2:					
$(2-1)$	$(0+, 0-, 0-), (0-, 0+, 0-)$	$(1, 1, 1), (\tau, \tau, \tau)$		$(15, 3)$	D
		$0+, 0-, 0-$	4		
		$0-, 0+, 0-$	4		
		$0-, 0-, 0+$	4		
$(2-2)$	$(0+, 0-, 0-), (0-, 0+, 1-)$	$(1, 1, 1), (\tau, \tau, \tau)$		$(9, 9)$	C
		$0+, 0-, 0-$	$2, 2$		
		$0-, 0+, 1-$	$2, 2$		
		$1-, 1-, 0+$	$2, 2$		
$(2-3)$	$(0+, 0-, 0-), (0-, 0+, 0-)$	$(1, 1, 1), (\tau, \tau, 0)$		$(17, 5)$	C
		$0+, 0-, 0-$	4		
		$0-, 0+, 0-$	4		
		$0-, 0-, 0+$	4		
		$\tau-, \tau-, 0+$	$2, 2$		

$(2-4)$	$(0+,0-,0-),(0-,0+,1-)$	$(1,1,1),(\tau,\tau,0)$ $0+,0-,0-$ $0-,0+,1-$ $1-,1-,0+$ $\tau 1-,\tau 1-,0+$	 $2,2$ $2,2$ $2,2$ $2,2$	$(11,11)$	0
$(2-5)$	$(0+,0-,0-),(0-,0+,\tau-)$	$(1,1,1),(\tau,\tau,0)$ $0+,0-,0-$ $0-,0+,\tau-$	 $2,2$ $2,2$	$(7,7)$	D
$(2-6)$	$(0+,0-,0-),(0-,0+,0-)$	$(1,1,1),(\tau,1,0)$ $0+,0-,0-$ $0-,0+,0-$ $0-,0-,0+$ $\tau 1-,0+,1-$ $\tau-,1-,0+$	 4 4 4 $2,2$ $2,2$	$(19,7)$	0
$(2-7)$	$(0+,0-,0-),(0-,0+,\tau-)$	$(1,1,1),(\tau,1,0)$ $0+,0-,0-$ $0-,0+,\tau-$ $\tau 1-,0+,\tau 1-$	 $2,2$ $2,2$ $2,2$	$(9,9)$	C
$(2-8)$	$(0+,0-,0-),(0-,\tau+,\tau-)$	$(1,1,1),(\tau,1,0)$ $0+,0-,0-$	 $2,2$	$(5,5)$	A
$(2-9)$	$(0+,0-,0-),(0-,0+,0-)$	$(0,1,1),(1,0,1)$ $0+,0-,0-$ $0-,0+,0-$ $0-,0-,0+$ $0+,1-,1-$ $1-,0+,1-$ $1-,1-,0+$	 4 4 4 4 4 4	$(27,3)$	0
$\&G_T$	G_S	sectors	$(h^{1,1},h^{2,1})$ contribution	π_1	
$(2-10)$	$(0+,0-,0-),(0-,0+,\tau-)$	$(0,1,1),(1,0,1)$ $0+,0-,0-$ $0+,1-,1-$ $0-,0+,\tau-$ $1-,0+,\tau 1-$	 $2,2$ $2,2$ $2,2$ $2,2$	$(11,11)$	0

$(2-11)$	$(0+,0-,0-),(0-,\tau+,\tau-)$	$(0,1,1),(1,0,1)$ $0+,0-,0-$ $0+,1-,1-$	 $2,2$ $2,2$	$(7,7)$	A
$(2-12)$	$(\tau+,0-,0-),(0-,\tau+,\tau-)$	$(0,1,1),(1,0,1)$		$(3,3)$	B
$(2-13)$	$(0+,0-,0-),(0-,0+,0-)$	$(1,1,0),(\tau,\tau,0)$ $0+,0-,0-$ $0-,0+,0-$ $0-,0-,0+$ $1-,1-,0+$ $\tau-,\tau-,0+$ $\tau 1-,\tau 1-,0+$	 4 4 4 $2,2$ $2,2$ $2,2$	$(21,9)$	0
$(2-14)$	$(0+,0-,0-),(0-,0+,1-)$	$(1,1,0),(\tau,\tau,0)$ $0+,0-,0-$ $0-,0+,1-$	 $2,2$ $2,2$	$(7,7)$	D
Rank 3:					
$(3-1)$	$(0+,0-,0-),(0-,0+,0-)$	$(0,\tau,1)$, $(0,\tau,1),(\tau,1,0)$ $0+,0-,0-$ $0-,0+,0-$ $0-,0-,0+$ $0+,\tau-,1-$ $1-,0+,\tau-$ $\tau-,1-,0+$	 2 2 2 $1,1$ $1,1$ $1,1$	$(12,6)$	0
$(3-2)$	$(0+,0-,0-),(0-,0+,1-)$	$(0,\tau,1)$, $(\tau,1,0),(1,0,\tau)$ $0+,0-,0-$ $0-,0+,1-$ $0-,\tau-,0+$ $0+,\tau-,1-$ $1-,0+,\tau 1-$ $\tau-,\tau 1-,0+$	 $1,1$ 2 2 2 $1,1$ $1,1$	$(12,6)$	0

	G_T	G_S sectors	contribution	$(h^{1,1}, h^{2,1})$	π_1
(3 − 3)	$(0+, 0-, 0-), (0-, 0+, 0-)$	$(1, 1, 0),$		(17, 5)	0
		$(\tau, \tau, 0), (1, \tau, 1)$			
		$0+, 0-, 0-$	2		
		$0-, 0+, 0-$	2		
		$0-, 0-, 0+$	2		
		$0+, \tau 1-, 1-$	2		
		$\tau 1-, 0+, 1-$	2		
		$1-, 1-, 0+$	1, 1		
		$\tau-, \tau-, 0+$	1, 1		
		$\tau 1-, \tau 1-, 0+$	2		
(3 − 4)	$(0+, 0-, 0-), (0-, 0+, \tau-)$	$(1, 1, 0),$		(7, 7)	C
		$(\tau, \tau, 0), (1, \tau, 1)$			
		$0+, 0-, 0-$	1, 1		
		$0-, 0+, \tau-$	1, 1		
		$0+, \tau 1-, 1-$	1, 1		
		$\tau 1-, 0+, \tau 1-$	1, 1		
(3 − 5)	$(0+, 0-, 0-), (0-, 0+, 0-)$	$(0, 1, 1),$		(15, 3)	C
		$(1, 0, 1), (\tau, \tau, \tau)$			
		$0+, 0-, 0-$	2		
		$0-, 0+, 0-$	2		
		$0-, 0-, 0+$	2		
		$0+, 1-, 1-$	2		
		$1-, 0+, 1-$	2		
		$1-, 1-, 0+$	2		
(3 − 6)	$(0+, 0-, 0-), (0-, 0+, \tau-)$	$(0, 1, 1),$		(9, 9)	0
		$(1, 0, 1), (\tau, \tau, \tau)$			
		$0+, 0-, 0-$	1, 1		
		$0-, 0+, \tau-$	1, 1		
		$\tau-, \tau-, 0+$	1, 1		
		$0+, 1-, 1-$	1, 1		
		$1-, 0+, \tau 1-$	1, 1		
		$\tau 1-, \tau 1-, 0+$	1, 1		

	G_T	G_S sectors	contribution	$(h^{1,1}, h^{2,1})$	π_1
Rank 4:					
(4 − 1)	$(0+, 0-, 0-), (0-, 0+, 0-)$	$(0, \tau, 1), (\tau, 1, 0),$		(15, 3)	0
		$(1, 0, \tau), (1, 1, 1)$			
		$0+, 0-, 0-$	1		
		$0+, \tau-, 1-$	1		
		$0+, 1-, \tau 1-$	1		
		$0+, \tau 1-, \tau-$	1		
		$0-, 0+, 0-$	1		
		$1-, 0+, \tau-$	1		
		$\tau 1-, 0+, 1-$	1		
		$\tau-, 0+, \tau 1-$	1		
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		$\tau-, 1-, 0+$	1		
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Important Examples

- (0-1): Vafa-Witten (51,3)
- (1-1): (27,3) $\pi_1 = \mathbb{Z}/2$
  (2-9): (27,3) $\pi_1 = 0$
  (2-9) is the NAHE$^+$ model=$(\mathbb{Z}/2)^2$-orbifold of $SO(12)$-torus
  (1-1) is not
- (0-2): Schoen (19,19)
- (1-3): DOPW, BD1 (11,11)
  ⇒FFMs may produce Het Standard Model!
- (1-7), (2-5), (2-14): other free $\mathbb{Z}/2$ and $(\mathbb{Z}/2)^2$ Schoen quotients of [BD2].
- Seven cases of Borcea-Voisin threefolds

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Outline

World record for $\pi_1(X)$

X=Gross threefold [Gross-Popescu, Gross-Pavanelli]
$X \to \mathbb{P}^1$ simply-connected, fibered
Fibers=abelian surfaces (T^4) with polarization of type $(1, 8)$.
Dual fibration: $X^\vee \to \mathbb{P}^1$
$X^\vee \cong X/(\mathbb{Z}/8)^2$, $\pi_1(X^\vee) = (\mathbb{Z}/8)^2$.
Explicitly: $X \to X' \subset \mathbb{P}^7$, X' intersection of 4 quadrics has 64 nodes. $X \to X'$ small resultion.
Advantages:

- Huge $\pi_1 \Rightarrow$ greater phenomenological flexibility
- V on $X^\vee \iff$ spectral data on V
  so don't need invariance

Difficulties:

- Hard to find spectral curves (codim 2)
- Spectral construction needs to combine with Hecke transforms
  $\Rightarrow$ need to check stability

We have: one new example. It seems: many examples.
Construction of spectral curve uses: GW invariants!

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We have: one new example. It seems: many examples.
Construction of spectral curve uses: GW invariants!

World record for $\pi_1(X)$

X=Gross threefold [Gross-Popescu, Gross-Pavanelli]
$X \to \mathbb{P}^1$ simply-connected, fibered
Fibers=abelian surfaces (T^4) with polarization of type $(1, 8)$.
Dual fibration: $X^\vee \to \mathbb{P}^1$
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Explicitly: $X \to X' \subset \mathbb{P}^7$, X' intersection of 4 quadrics has 64 nodes. $X \to X'$ small resultion.
Advantages:

- Huge $\pi_1 \Rightarrow$ greater phenomenological flexibility
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Outline

Summary

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- Perhaps other models in the class of threefolds constructed as quotients of Schoen's threefolds, but not on the $\mathbb{Z}_6$ one, at least with current bundle construction methods
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  - Necessary if we want to study common properties etc.
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