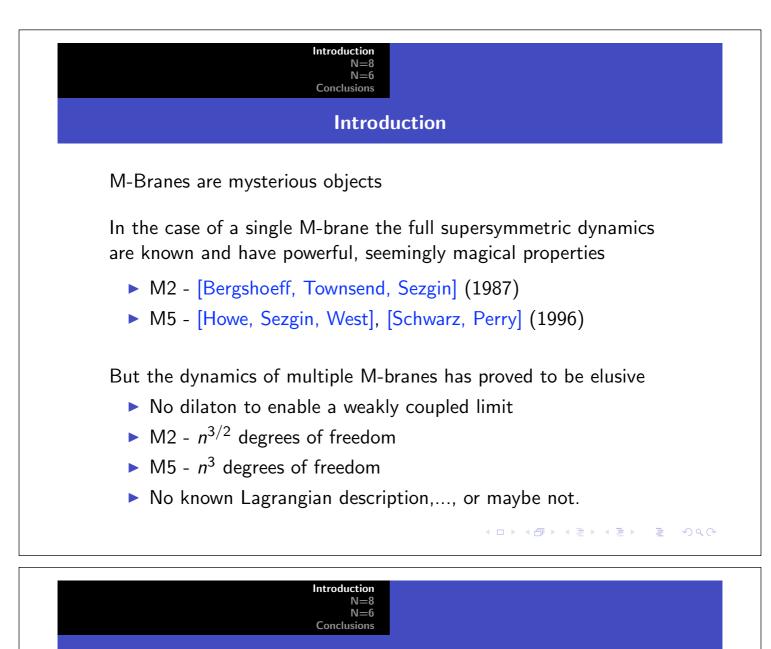


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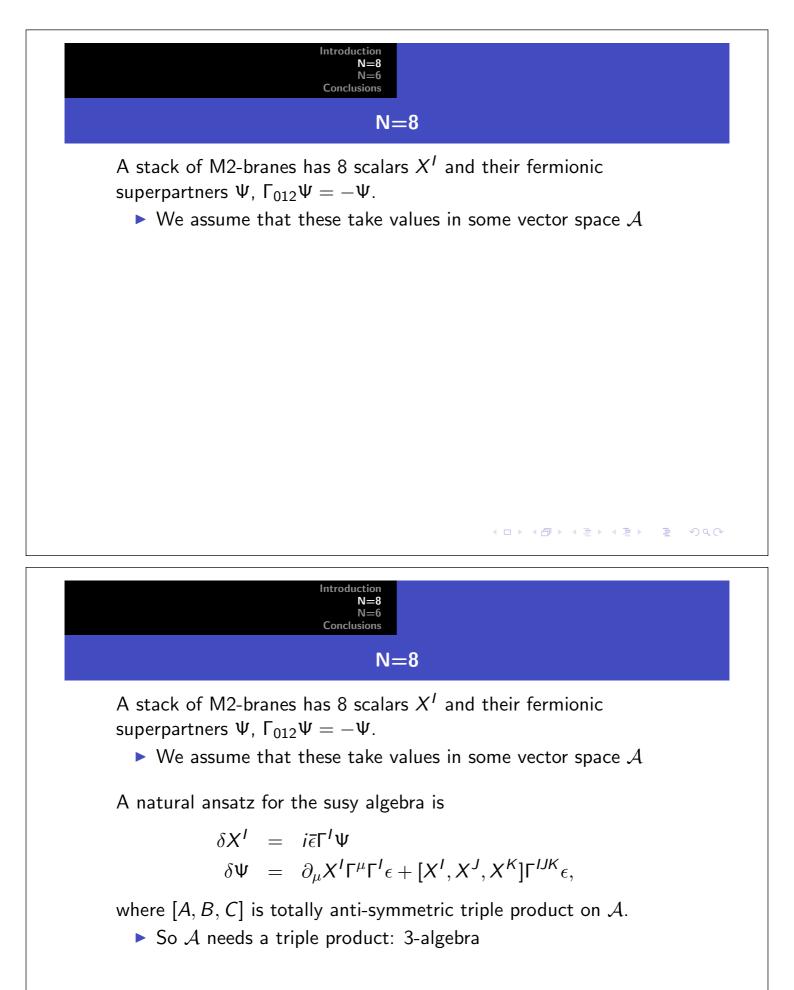
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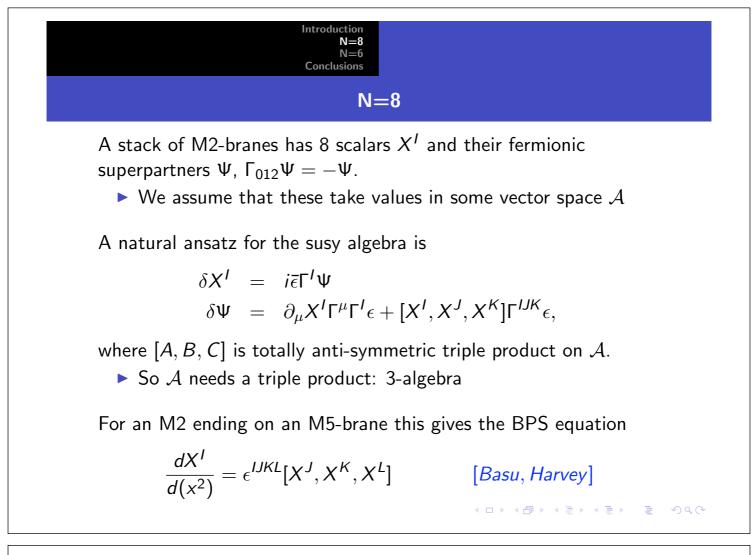
Introduction

N=8

N=6

Conclusions



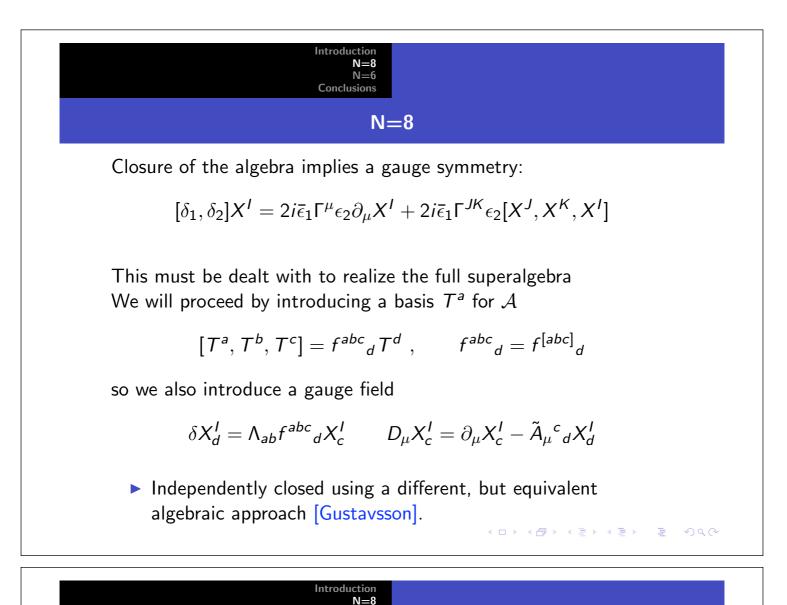


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Closure of the algebra implies a gauge symmetry:

 $[\delta_1, \delta_2] X' = 2i\bar{\epsilon}_1 \Gamma^{\mu} \epsilon_2 \partial_{\mu} X' + 2i\bar{\epsilon}_1 \Gamma^{JK} \epsilon_2 [X^J, X^K, X']$ 

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Full superalgebra takes the form  $SX^{I} = -\frac{1}{2} \Gamma^{I} \mu$ 

$$\delta X_{d} = I \epsilon \Gamma \Psi_{d}$$
  

$$\delta \Psi_{d} = D_{\mu} X_{d}^{I} \Gamma^{\mu} \Gamma^{I} \epsilon - \frac{1}{6} X_{a}^{I} X_{b}^{J} X_{c}^{K} f^{abc}{}_{d} \Gamma^{IJK} \epsilon$$
  

$$\delta \tilde{A}_{\mu}{}^{c}{}_{d} = i \epsilon \Gamma_{\mu} \Gamma_{I} X_{a}^{I} \Psi_{b} f^{abc}{}_{d},$$

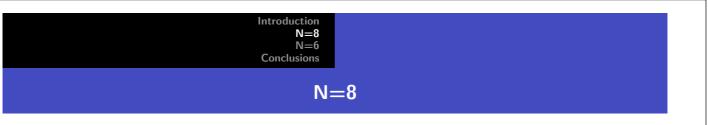
N=8

Indeed this closes (on-shell) if  $f^{abcd}$  satisfies the fundamental identity:

N=6 Conclusions

$$f^{efg}{}_b f^{cba}{}_d + f^{fea}{}_b f^{cbg}{}_d + f^{gaf}{}_b f^{ceb}{}_d + f^{age}{}_b f^{cfb}{}_d = 0.$$

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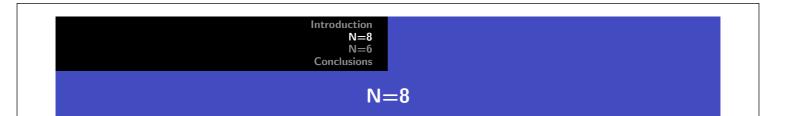
Full superalgebra takes the form

$$\begin{split} \delta X_d^I &= i \bar{\epsilon} \Gamma^I \Psi_d \\ \delta \Psi_d &= D_\mu X_d^I \Gamma^\mu \Gamma^I \epsilon - \frac{1}{6} X_a^I X_b^J X_c^K f^{abc}{}_d \Gamma^{IJK} \epsilon \\ \delta \tilde{A}_\mu{}^c{}_d &= i \bar{\epsilon} \Gamma_\mu \Gamma_I X_a^I \Psi_b f^{abc}{}_d, \end{split}$$

Indeed this closes (on-shell) if  $f^{abcd}$  satisfies the fundamental identity:

$$f^{efg}{}_b f^{cba}{}_d + f^{fea}{}_b f^{cbg}{}_d + f^{gaf}{}_b f^{ceb}{}_d + f^{age}{}_b f^{cfb}{}_d = 0.$$

This ensures that the gauge symmetries generated by the triple product are those of a Lie-algebra



The invariant Lagrangian is a Chern-Simons theory:

$$\mathcal{L} = -\frac{1}{2} (D_{\mu} X^{al}) (D^{\mu} X^{l}_{a}) + \frac{i}{2} \bar{\Psi}^{a} \Gamma^{\mu} D_{\mu} \Psi_{a} + \frac{i}{4} \bar{\Psi}_{b} \Gamma_{IJ} X^{l}_{c} X^{J}_{d} \Psi_{a} f^{abcd} + \frac{1}{2} \varepsilon^{\mu\nu\lambda} (f^{abcd} A_{\mu ab} \partial_{\nu} A_{\lambda cd} + \frac{2}{3} f^{cda}{}_{g} f^{efgb} A_{\mu ab} A_{\nu cd} A_{\lambda ef}) - \frac{1}{12} \text{Tr} ([X^{I}, X^{J}, X^{K}])^{2}$$

 $\blacktriangleright$  Tr is an invariant trace (or inner-product) on  ${\cal A}$ 

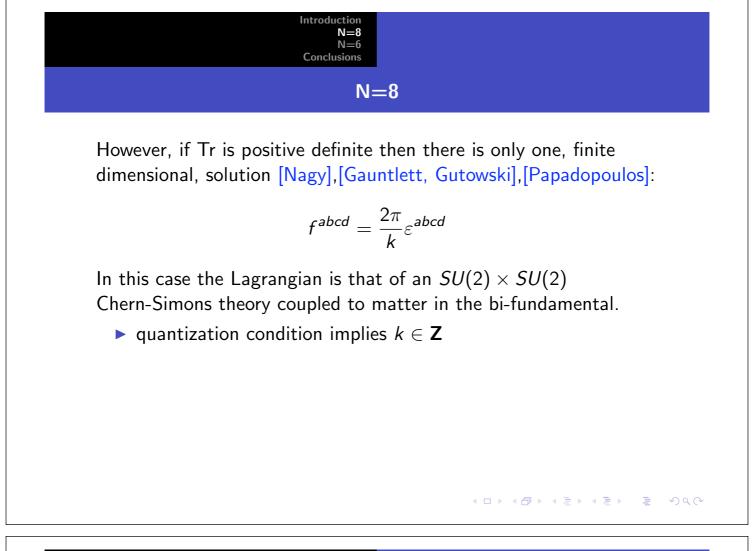
• gauge invariance implies  $f^{abcd} = f^{[abcd]}$ 

$$\blacktriangleright \tilde{A}_{\mu}{}^{c}{}_{d} = f^{abc}{}_{d}A_{\mu ab}$$

This Lagrangian has all the expected symmetries of multiple M2-branes:  $\mathcal{N} = 8$  supersymmetry, SO(8) R-symmetry and Parity.

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However, if Tr is positive definite then there is only one, finite dimensional, solution [Nagy],[Gauntlett, Gutowski],[Papadopoulos]:

$$f^{abcd} = \frac{2\pi}{k} \varepsilon^{abcd}$$

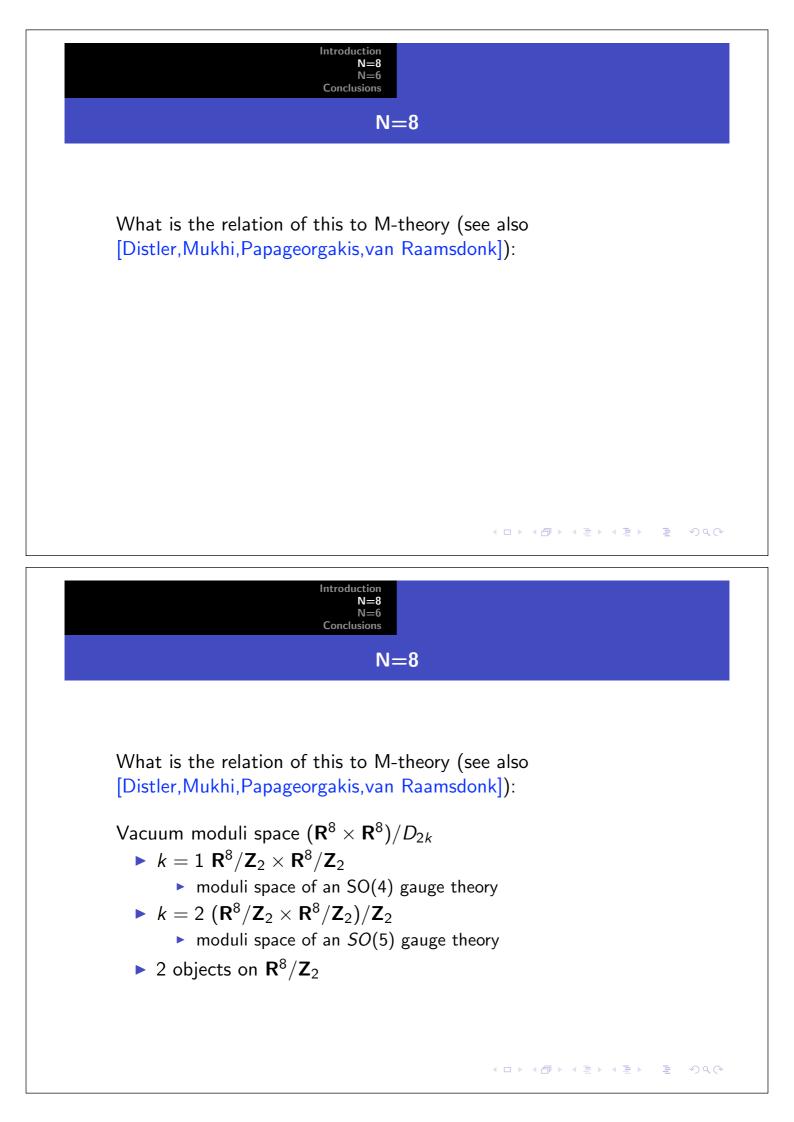
In this case the Lagrangian is that of an  $SU(2) \times SU(2)$ Chern-Simons theory coupled to matter in the bi-fundamental.

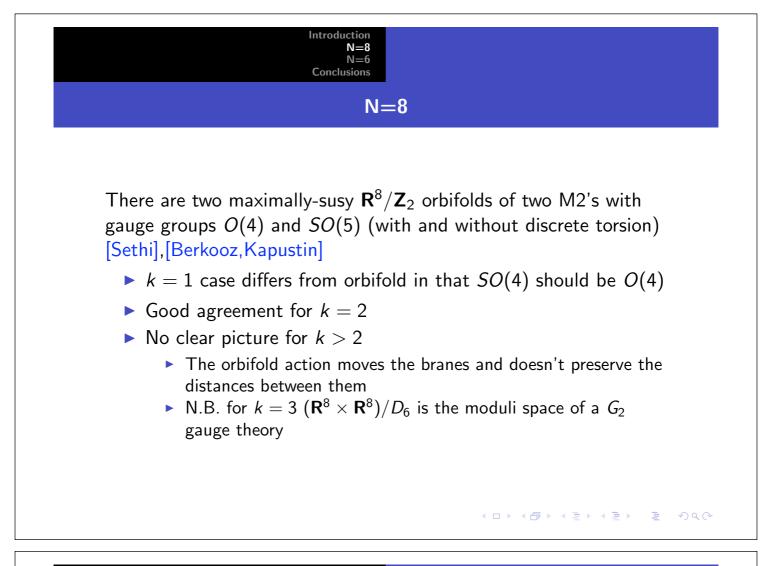
• quantization condition implies  $k \in \mathbf{Z}$ 

Infinite dimensional examples can be constructed by considering the Nambu bracket on an 'auxiliary' three-manifold  $\Sigma$ .

 More recently been identified with a single M5 on R<sup>3</sup> × Σ [Ho,Matsuo] [Banados, Townsend]

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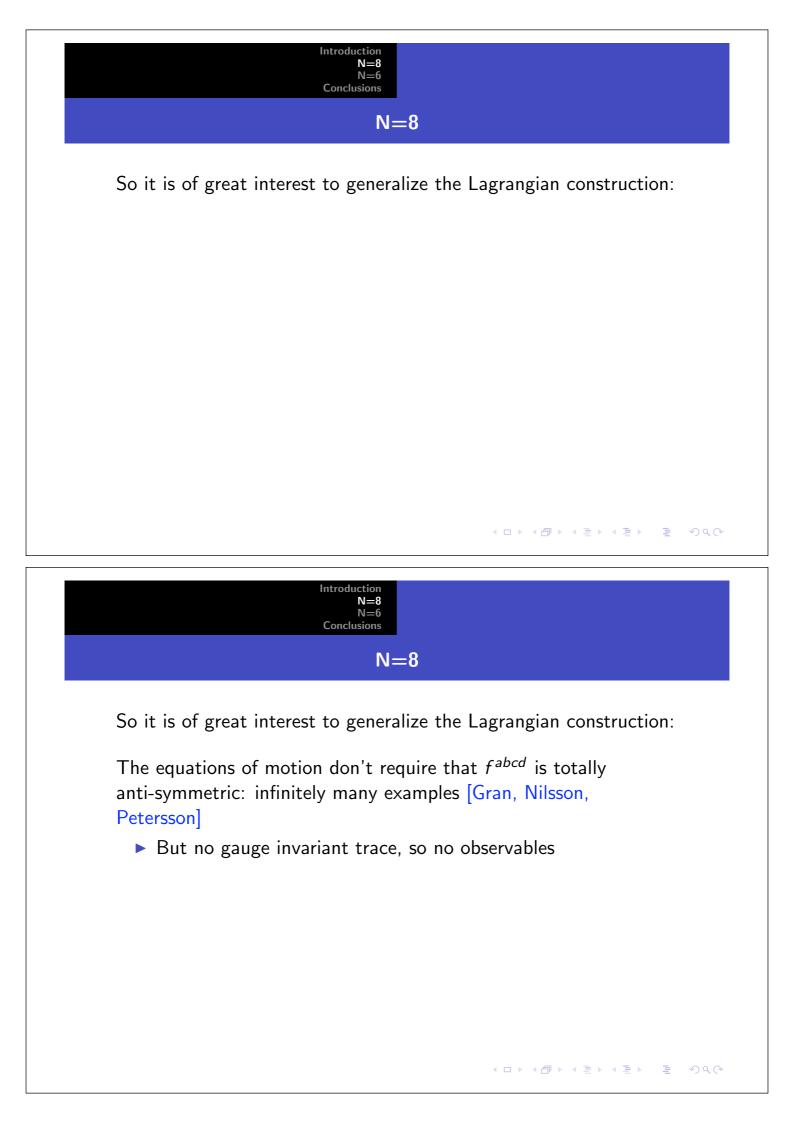
It is interesting to note that on the Coulomb branch one finds the classical mass formula

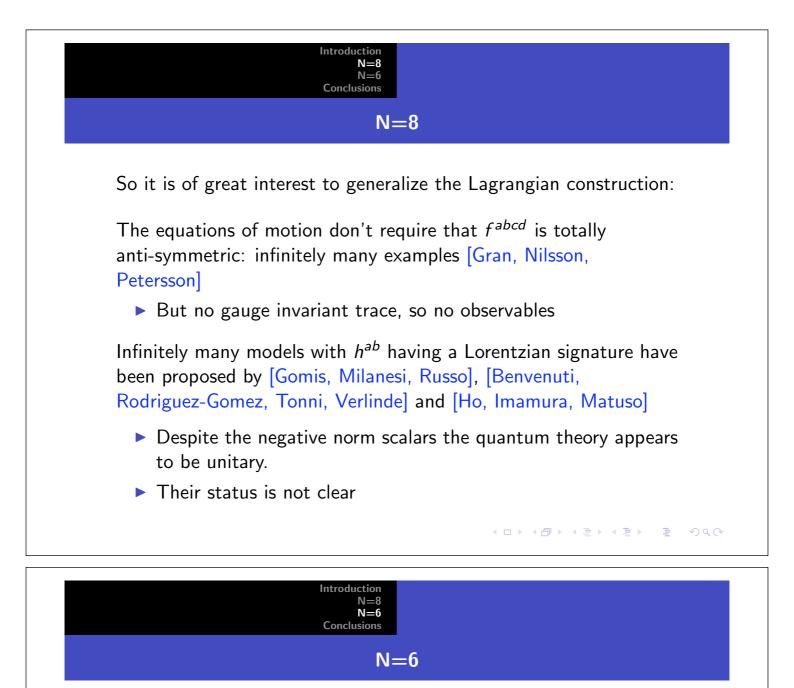
$$M = \frac{2\pi}{k}$$
 (area of a triangle with vertices on an M2)

- Tempting clue to microscopic states in M-theory analogous to stretched open strings
  - N.B. orbifold action preserves M
- hints towards an origin of  $N^3$ .
- note that there is an enhanced gauge symmetry whenever the M2s are collinear

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 strongly coupled (*c.f.* origin of **R**<sup>7</sup> Coulomb branch in D2-branes)

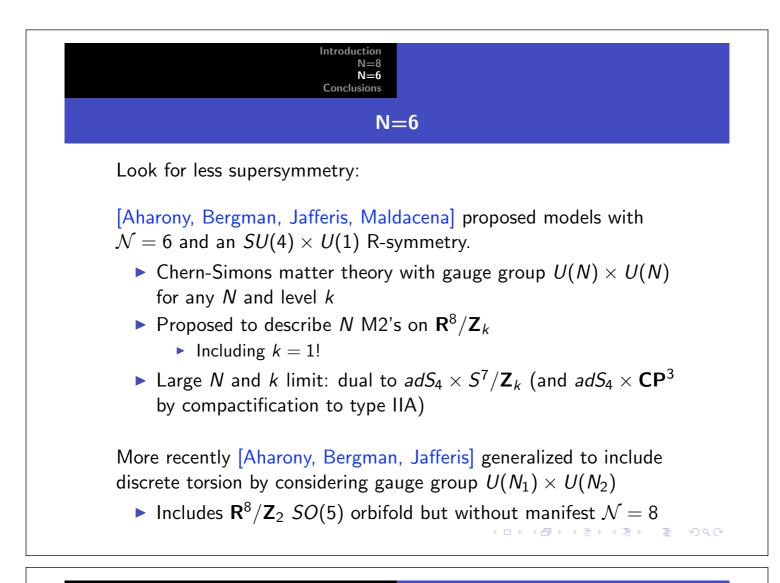


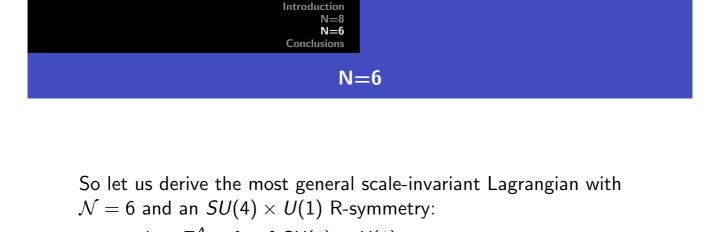


Look for less supersymmetry:

[Aharony, Bergman, Jafferis, Maldacena] proposed models with  $\mathcal{N} = 6$  and an  $SU(4) \times U(1)$  R-symmetry.

- Chern-Simons matter theory with gauge group U(N) × U(N) for any N and level k
- Proposed to describe *N* M2's on  $\mathbf{R}^8/\mathbf{Z}_k$ 
  - Including k = 1!
- Large N and k limit: dual to adS<sub>4</sub> × S<sup>7</sup>/Z<sub>k</sub> (and adS<sub>4</sub> × CP<sup>3</sup> by compactification to type IIA)





- ▶ scalars  $Z_a^A \in \mathbf{4}_1$  of  $SU(4) \times U(1)$
- fermions  $\psi_{Aa} \in \mathbf{\bar{4}}_1$  of  $SU(4) \times U(1)$
- ▶ susys  $\epsilon_{AB} \in \mathbf{6}_0$  of  $SU(4) \times U(1)$ 
  - $\bullet \ (\epsilon_{AB})^* = \epsilon^{AB} = \frac{1}{2} \varepsilon^{ABCD} \epsilon_{CD}$

Complex conjugation raises/lowers and A-index and flips the U(1) charge

## Introduction N=8 N=6 Conclusions

## N=6

Starting from the most general form for the susy's one finds

$$\begin{split} \delta Z_{d}^{A} &= i \overline{\epsilon}^{AB} \psi_{Bd} \\ \delta \psi_{Bd} &= \gamma^{\mu} D_{\mu} Z_{d}^{A} \epsilon_{AB} + f^{abc}{}_{d} Z_{a}^{C} Z_{b}^{A} Z_{Cc} \epsilon_{AB} + f^{abc}{}_{d} Z_{a}^{C} Z_{b}^{D} Z_{Bc} \epsilon_{CD} \\ \delta \widetilde{A}_{\mu}{}^{c}{}_{d} &= -i \overline{\epsilon}_{AB} \gamma_{\mu} Z_{a}^{A} \psi_{b}^{B} f^{cab}{}_{d} + i \overline{\epsilon}^{AB} \gamma_{\mu} Z_{Ab} \psi_{Ba} f^{cba}{}_{d} \end{split}$$

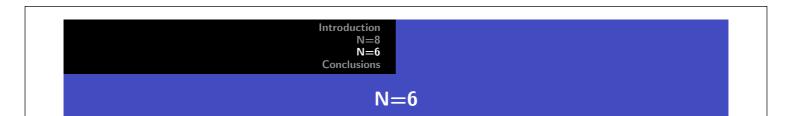
Provided that

$$f^{efg}{}_b f^{cba}{}_d + f^{fea}{}_b f^{cbg}{}_d + f^{*gaf}{}_b f^{ceb}{}_d + f^{*age}{}_b f^{cfb}{}_d = 0$$

 $\mathsf{and}$ 

$$f^{abcd} = -f^{bacd} = -f^{abdc} = f^{*cdab}$$

- *i.e.* f<sup>abcd</sup> need not be real nor totally anti-symmetric.
  - Everything is determined by specifying a triple product [X, Y; Z] on A



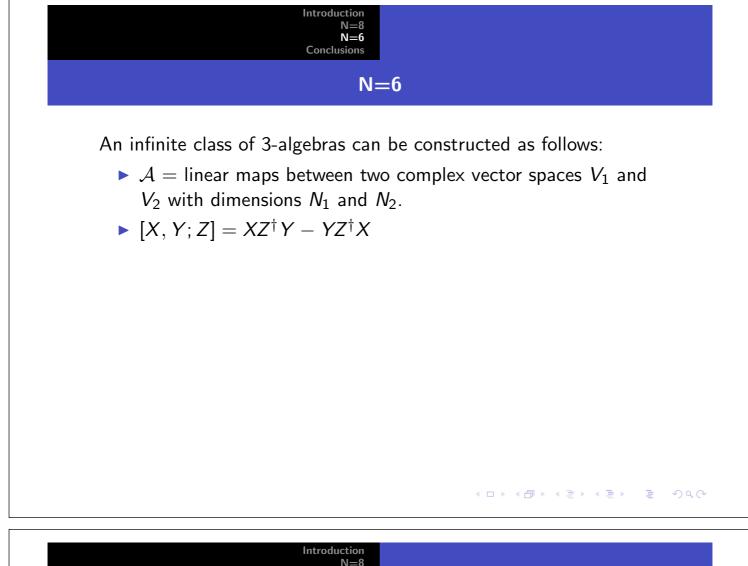
The Lagrangian is

$$\mathcal{L} = -D_{\mu}Z_{a}^{A}D^{\mu}Z_{A}^{a} - i\bar{\psi}_{a}^{A}\gamma^{\mu}D_{\mu}\psi_{A}^{a} -if^{abcd}\bar{\psi}_{d}^{A}\psi_{Aa}Z_{b}^{B}Z_{Bc} + 2if^{abcd}\bar{\psi}_{d}^{A}\psi_{Ba}Z_{b}^{B}Z_{Ac} + \frac{i}{2}\varepsilon_{ABCD}f^{abcd}\bar{\psi}_{d}^{A}\psi_{c}^{B}Z_{a}^{C}Z_{b}^{D} - \frac{i}{2}\varepsilon^{ABCD}f^{cdab}\bar{\psi}_{Ac}\psi_{Bd}Z_{Ca}Z_{Db} + \frac{1}{2}\varepsilon^{\mu\nu\lambda}(f^{abcd}A_{\mu cb}\partial_{\nu}A_{\lambda da} + \frac{2}{3}f^{acd}_{g}f^{gefb}A_{\mu ab}A_{\nu dc}A_{\lambda fe}) - \frac{2}{3}\Upsilon_{Bd}^{CD}\Upsilon_{CD}^{Bd}.$$

with

$$\Upsilon_{Bd}^{CD} = Z_a^C Z_b^D Z_{Bc} f^{abc}_{\ d} - \frac{1}{2} \delta_B^C Z_a^E Z_b^D Z_{Ec} f^{abc}_{\ d} + \frac{1}{2} \delta_B^D Z_a^E Z_b^C Z_{Ec} f^{abc}_{\ d}$$

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N=8 N=6 Conclusions N=6

An infinite class of 3-algebras can be constructed as follows:

- A = linear maps between two complex vector spaces V<sub>1</sub> and V<sub>2</sub> with dimensions N<sub>1</sub> and N<sub>2</sub>.
- $\blacktriangleright [X, Y; Z] = XZ^{\dagger}Y YZ^{\dagger}X$

The gauge symmetry generated by the triple product is

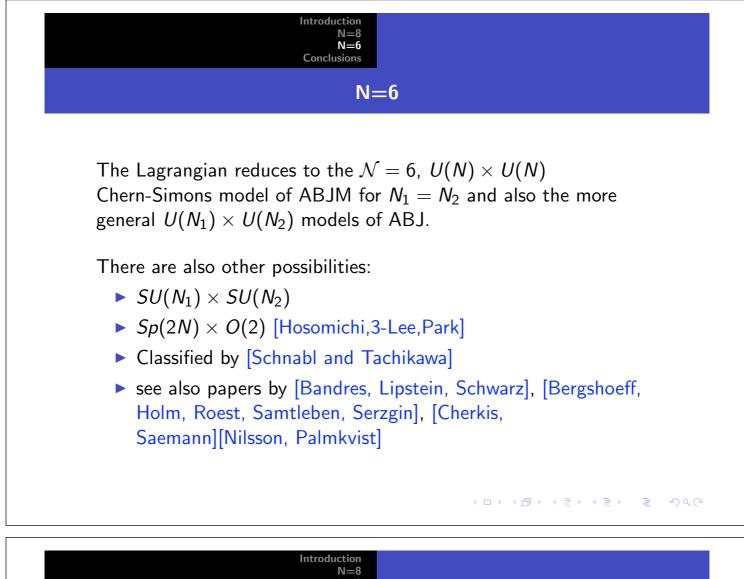
$$\delta X = XM_1 - M_2X$$

with  $M_1 \in u(N_1)$  and  $M_2 \in u(N_2)$ .

The fundamental identity ensures that

 $\delta[X, Y; Z] = [X, Y; Z]M_1 - M_2[X, Y; Z]$ 

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 N=8 N=6 Conclusions
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 ► We constructed a (rather unique!) N = 8 3D Lagrangian field

- theory with an SO(8) R-symmetry and Parity
  - New maximally superconformal Chern-Simons gauge theory that is not Yang-Mills
  - Identified with 2 M2-branes on  $\mathbf{R}^8/\mathbf{Z}_2$  (at least for k=2)

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