Lagrangians for Multiple M2's

Neil Lambert
King's College London

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and D. Tong 0804.1114

Introduction

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- M2 - [Bergshoeff, Townsend, Sezgin] (1987)
- M5 - [Howe, Sezgin, West], [Schwarz, Perry] (1996)

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But the dynamics of multiple M-branes has proved to be elusive

- No dilaton to enable a weakly coupled limit
- M2 - $n^{3/2}$ degrees of freedom
- M5 - n^3 degrees of freedom
- No known Lagrangian description

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- We assume that these take values in some vector space $\mathcal{A}$

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A natural ansatz for the susy algebra is

$$\begin{aligned} \delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\ \delta\Psi &= \partial_\mu X^I\Gamma^\mu\Gamma^I\epsilon + [X^I, X^J, X^K]\Gamma^{IJK}\epsilon, \end{aligned}$$

where $[A, B, C]$ is totally anti-symmetric triple product on $\mathcal{A}$.

- So $\mathcal{A}$ needs a triple product: 3-algebra

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For an M2 ending on an M5-brane this gives the BPS equation

$$\frac{dX^I}{d(x^2)} = \epsilon^{IJKL}[X^J, X^K, X^L] \qquad [\textit{Basu, Harvey}]$$

N=8

Closure of the algebra implies a gauge symmetry:

$$[\delta_1, \delta_2]X^I = 2i\bar{\epsilon}_1\Gamma^\mu\epsilon_2\partial_\mu X^I + 2i\bar{\epsilon}_1\Gamma^{JK}\epsilon_2[X^J, X^K, X^I]$$

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This must be dealt with to realize the full superalgebra
We will proceed by introducing a basis T^a for $\mathcal{A}$

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d \ , \qquad f^{abc}{}_d = f^{[abc]}{}_d$$

so we also introduce a gauge field

$$\delta X^I_d = \Lambda_{ab} f^{abc}{}_d X^I_c \qquad D_\mu X^I_c = \partial_\mu X^I_c - \tilde{A}_\mu{}^c{}_d X^I_d$$

- Independently closed using a different, but equivalent algebraic approach [Gustavsson].

N=8

Full superalgebra takes the form

$$\begin{aligned}
\delta X^I_d &= i\bar{\epsilon}\Gamma^I\Psi_d \\
\delta\Psi_d &= D_\mu X^I_d\Gamma^\mu\Gamma^I\epsilon - \frac{1}{6}X^I_a X^J_b X^K_c f^{abc}{}_d\Gamma^{IJK}\epsilon \\
\delta\tilde{A}_\mu{}^c{}_d &= i\bar{\epsilon}\Gamma_\mu\Gamma_I X^I_a\Psi_b f^{abc}{}_d,
\end{aligned}$$

Indeed this closes (on-shell) if f^{abcd} satisfies the fundamental identity:

$$f^{efg}{}_b f^{cba}{}_d + f^{fea}{}_b f^{cbg}{}_d + f^{gaf}{}_b f^{ceb}{}_d + f^{age}{}_b f^{cfb}{}_d = 0.$$

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This ensures that the gauge symmetries generated by the triple product are those of a Lie-algebra

N=8

The invariant Lagrangian is a Chern-Simons theory:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}(D_\mu X^{aI})(D^\mu X^I_a) + \frac{i}{2}\bar{\Psi}^a\Gamma^\mu D_\mu\Psi_a + \frac{i}{4}\bar{\Psi}_b\Gamma_{IJ}X^I_c X^J_d\Psi_a f^{abcd} \\ & +\frac{1}{2}\varepsilon^{\mu\nu\lambda}(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}) \\ & -\frac{1}{12}\mathrm{Tr}([X^I, X^J, X^K])^2 \end{aligned}$$

- Tr is an invariant trace (or inner-product) on $\mathcal{A}$
- gauge invariance implies $f^{abcd} = f^{[abcd]}$
- $\tilde{A}_\mu{}^c{}_d = f^{abc}{}_d A_{\mu ab}$

This Lagrangian has all the expected symmetries of multiple M2-branes: $\mathcal{N} = 8$ supersymmetry, $SO(8)$ R-symmetry and Parity.

N=8

However, if Tr is positive definite then there is only one, finite dimensional, solution [Nagy],[Gauntlett, Gutowski],[Papadopoulos]:

$$f^{abcd} = \frac{2\pi}{k}\varepsilon^{abcd}$$

In this case the Lagrangian is that of an $SU(2) \times SU(2)$ Chern-Simons theory coupled to matter in the bi-fundamental.

- quantization condition implies $k \in \mathbf{Z}$

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Infinite dimensional examples can be constructed by considering the Nambu bracket on an 'auxiliary' three-manifold Σ.

- More recently been identified with a single M5 on $\mathbf{R}^3 \times \Sigma$ [Ho,Matsuo] [Banados, Townsend]

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What is the relation of this to M-theory (see also [Distler,Mukhi,Papageorgakis,van Raamsdonk]):

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Vacuum moduli space $(\mathbf{R}^8 \times \mathbf{R}^8)/D_{2k}$

- $k = 1$ $\mathbf{R}^8/\mathbf{Z}_2 \times \mathbf{R}^8/\mathbf{Z}_2$
  - moduli space of an SO(4) gauge theory
- $k = 2$ $(\mathbf{R}^8/\mathbf{Z}_2 \times \mathbf{R}^8/\mathbf{Z}_2)/\mathbf{Z}_2$
  - moduli space of an $SO(5)$ gauge theory
- 2 objects on $\mathbf{R}^8/\mathbf{Z}_2$

N=8

There are two maximally-susy $\mathbf{R}^8/\mathbf{Z}_2$ orbifolds of two M2's with gauge groups $O(4)$ and $SO(5)$ (with and without discrete torsion) [Sethi],[Berkooz,Kapustin]

- $k = 1$ case differs from orbifold in that $SO(4)$ should be $O(4)$
- Good agreement for $k = 2$
- No clear picture for $k > 2$
  - The orbifold action moves the branes and doesn't preserve the distances between them
  - N.B. for $k = 3$ $(\mathbf{R}^8 \times \mathbf{R}^8)/D_6$ is the moduli space of a G_2 gauge theory

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It is interesting to note that on the Coulomb branch one finds the classical mass formula

$$M = \frac{2\pi}{k}\,(\text{area of a triangle with vertices on an M2})$$

- Tempting clue to microscopic states in M-theory analogous to stretched open strings
  - N.B. orbifold action preserves M
- hints towards an origin of N^3.
- note that there is an enhanced gauge symmetry whenever the M2s are collinear
  - strongly coupled (*c.f.* origin of $\mathbf{R}^7$ Coulomb branch in D2-branes)

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The equations of motion don't require that f^{abcd} is totally anti-symmetric: infinitely many examples [Gran, Nilsson, Petersson]

- But no gauge invariant trace, so no observables

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- But no gauge invariant trace, so no observables

Infinitely many models with h^{ab} having a Lorentzian signature have been proposed by [Gomis, Milanesi, Russo], [Benvenuti, Rodriguez-Gomez, Tonni, Verlinde] and [Ho, Imamura, Matuso]

- Despite the negative norm scalars the quantum theory appears to be unitary.
- Their status is not clear

N=6

Look for less supersymmetry:

[Aharony, Bergman, Jafferis, Maldacena] proposed models with $\mathcal{N} = 6$ and an $SU(4) \times U(1)$ R-symmetry.

- Chern-Simons matter theory with gauge group $U(N) \times U(N)$ for any N and level k
- Proposed to describe N M2's on $\mathbf{R}^8/\mathbf{Z}_k$
  - Including $k = 1$!
- Large N and k limit: dual to $adS_4 \times S^7/\mathbf{Z}_k$ (and $adS_4 \times \mathbf{CP}^3$ by compactification to type IIA)

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More recently [Aharony, Bergman, Jafferis] generalized to include discrete torsion by considering gauge group $U(N_1) \times U(N_2)$

- Includes $\mathbf{R}^8/\mathbf{Z}_2$ $SO(5)$ orbifold but without manifest $\mathcal{N} = 8$

N=6

So let us derive the most general scale-invariant Lagrangian with $\mathcal{N} = 6$ and an $SU(4) \times U(1)$ R-symmetry:

- scalars $Z^A_a \in \mathbf{4}_1$ of $SU(4) \times U(1)$
- fermions $\psi_{Aa} \in \bar{\mathbf{4}}_1$ of $SU(4) \times U(1)$
- susys $\epsilon_{AB} \in \mathbf{6}_0$ of $SU(4) \times U(1)$
  - $(\epsilon_{AB})^* = \epsilon^{AB} = \frac{1}{2}\varepsilon^{ABCD}\epsilon_{CD}$

Complex conjugation raises/lowers and A-index and flips the $U(1)$ charge

N=6

Starting from the most general form for the susy's one finds

$$\begin{aligned}
\delta Z_d^A &= i\bar{\epsilon}^{AB}\psi_{Bd} \\
\delta\psi_{Bd} &= \gamma^\mu D_\mu Z_d^A \epsilon_{AB} + f^{abc}{}_d Z_a^C Z_b^A Z_{Cc}\epsilon_{AB} + f^{abc}{}_d Z_a^C Z_b^D Z_{Bc}\epsilon_{CD} \\
\delta\tilde{A}_\mu{}^c{}_d &= -i\bar{\epsilon}_{AB}\gamma_\mu Z_a^A \psi_b^B f^{cab}{}_d + i\bar{\epsilon}^{AB}\gamma_\mu Z_{Ab}\psi_{Ba} f^{cba}{}_d
\end{aligned}$$

Provided that

$$f^{efg}{}_b f^{cba}{}_d + f^{fea}{}_b f^{cbg}{}_d + f^{*gaf}{}_b f^{ceb}{}_d + f^{*age}{}_b f^{cfb}{}_d = 0$$

and

$$f^{abcd} = -f^{bacd} = -f^{abdc} = f^{*cdab}.$$

i.e. f^{abcd} need not be real nor totally anti-symmetric.

- Everything is determined by specifying a triple product $[X, Y; Z]$ on $\mathcal{A}$

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The Lagrangian is

$$\begin{aligned}
\mathcal{L} = &-D_\mu Z_a^A D^\mu Z_A^a - i\bar{\psi}_a^A \gamma^\mu D_\mu \psi_A^a \\
&-if^{abcd}\bar{\psi}_d^A \psi_{Aa} Z_b^B Z_{Bc} + 2if^{abcd}\bar{\psi}_d^A \psi_{Ba} Z_b^B Z_{Ac} \\
&+\frac{i}{2}\varepsilon_{ABCD} f^{abcd}\bar{\psi}_d^A \psi_c^B Z_a^C Z_b^D - \frac{i}{2}\varepsilon^{ABCD} f^{cdab}\bar{\psi}_{Ac}\psi_{Bd} Z_{Ca} Z_{Db} \\
&+\frac{1}{2}\varepsilon^{\mu\nu\lambda}(f^{abcd}A_{\mu cb}\partial_\nu A_{\lambda da} + \frac{2}{3} f^{acd}{}_g f^{gefb} A_{\mu ab} A_{\nu dc} A_{\lambda fe}) \\
&-\frac{2}{3}\Upsilon_{Bd}^{CD}\Upsilon_{CD}^{Bd}.
\end{aligned}$$

with

$$\Upsilon_{Bd}^{CD} = Z_a^C Z_b^D Z_{Bc} f^{abc}{}_d - \frac{1}{2}\delta_B^C Z_a^E Z_b^D Z_{Ec} f^{abc}{}_d + \frac{1}{2}\delta_B^D Z_a^E Z_b^C Z_{Ec} f^{abc}{}_d$$

N=6

An infinite class of 3-algebras can be constructed as follows:

- $\mathcal{A}$ = linear maps between two complex vector spaces V_1 and V_2 with dimensions N_1 and N_2.
- $[X, Y; Z] = XZ^\dagger Y - YZ^\dagger X$

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The gauge symmetry generated by the triple product is

$$\delta X = XM_1 - M_2 X$$

with $M_1 \in u(N_1)$ and $M_2 \in u(N_2)$.

The fundamental identity ensures that

$$\delta[X, Y; Z] = [X, Y; Z]M_1 - M_2[X, Y; Z]$$

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The Lagrangian reduces to the $\mathcal{N} = 6$, $U(N) \times U(N)$ Chern-Simons model of ABJM for $N_1 = N_2$ and also the more general $U(N_1) \times U(N_2)$ models of ABJ.

There are also other possibilities:

- $SU(N_1) \times SU(N_2)$
- $Sp(2N) \times O(2)$ [Hosomichi,3-Lee,Park]
- Classified by [Schnabl and Tachikawa]
- see also papers by [Bandres, Lipstein, Schwarz], [Bergshoeff, Holm, Roest, Samtleben, Serzgin], [Cherkis, Saemann][Nilsson, Palmkvist]

Conclusions

- We constructed a (rather unique!) $\mathcal{N} = 8$ 3D Lagrangian field theory with an SO(8) R-symmetry and Parity
  - New maximally superconformal Chern-Simons gauge theory that is not Yang-Mills
  - Identified with 2 M2-branes on $\mathbf{R}^8/\mathbf{Z}_2$ (at least for $k = 2$)

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- Infinitely many theories with $\mathcal{N} = 6$ and an $SU(4) \times U(1)$ R-symmetry
  - Proposed by ABJM and ABJ as N M2's on $\mathbf{R}^8/\mathbf{Z}_k$, for $k = 1, 2, \ldots$

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- Infinitely many theories with $\mathcal{N} = 6$ and an $SU(4) \times U(1)$ R-symmetry
  - Proposed by ABJM and ABJ as N M2's on $\mathbf{R}^8/\mathbf{Z}_k$, for $k = 1, 2, \ldots$
- Gained some insight into the degrees of freedom of multiple M2-branes?
  - Classically the massive states associated to triangles with vertices on the M2's

Conclusions

There are still many issues to understand:

- What is the role of the $\mathcal{N} = 8$ theory?
- Understand the enhancement of $k = 1, 2$ ABJM to $\mathcal{N} = 8$
- What is the role of the Lorentzian theories?
  - Equivalent to $\mathcal{N} = 8$ super-Yang-Mills? [Gomis, Rodriguez-Gomez,van Raamsdonk, Verlinde], [Bandres, Lipstein, Schwarz]
  - M2/D2's on a cylinder [Banerjee, Sen]
  - scaling limit of ABJM [Honma,Iso,Sumitomo,Umetsu,Zhang]
- Can one see the $n^{\frac{3}{2}}$?
  - e.g. see [de Madeiros,Figueroa-O'Farrill, Mendez-Escobar] and [Chu,Hi,Matsuo,Shiba]

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We have tried to stress the central role of 3-algebras. Why?

- They naturally encode all the information of the theory
  - Classification of $\mathcal{N} \geq 6$ theories is a classification of 3-algebras.
- The dynamics of M2-branes is primarily determined by the scalars and fermions and these don't directly see a Lie-bracket.
- Hopefully they are interesting in their own right and a clue to the microscopic degrees of freedom in M-theory.