Based on:

“M2 to D2”,
SM and Costis Papageorgakis,

“M2-branes on M-folds”,
Jacques Distler, SM, Costis Papageorgakis and Mark van Raamsdonk,

“D2 to D2”,
Bobby Ezhuthachan, SM and Costis Papageorgakis,

Mohsen Alishahiha and SM, to appear
Motivation

▶ We understand the field theory on multiple D-branes rather well, but the one on multiple M-branes not so well.
Motivation

- We understand the field theory on multiple D-branes rather well, but the one on multiple M-branes not so well.
- The latter should hold the key to M-theory:

- While there is an obstacle (due to (anti) self-dual 2-forms) to writing the $M5$-brane field theory, there is no obstacle for $M2$-branes as far as we know.
We understand the field theory on multiple D-branes rather well, but the one on multiple M-branes not so well. The latter should hold the key to M-theory:

While there is an obstacle (due to (anti) self-dual 2-forms) to writing the $M_5$-brane field theory, there is no obstacle for $M_2$-branes as far as we know.

And yet, despite $\sim 200$ recent papers – and two Strings 2008 talks – on the subject, we don’t exactly know what the multiple membrane theory is.

Even the French aristocracy doesn’t seem to know...
Of course, there is one description that is clearly right and has manifest $\mathcal{N} = 8$ supersymmetry (but not manifest conformal symmetry):

$$\lim_{g_{YM} \to \infty} \frac{1}{2} L_{SYM}$$
Of course, there is one description that is clearly right and has manifest $\mathcal{N} = 8$ supersymmetry (but not manifest conformal symmetry):

$$\lim_{g_{YM} \to \infty} \frac{1}{g_{YM}^2} \mathcal{L}_{SYM}$$

The question is whether this conformal IR fixed point has an explicit Lagrangian description wherein all the symmetries are manifest.

This includes a global $SO(8)_R$ symmetry describing rotations of the space transverse to the membranes – enhanced from the $SO(7)$ of SYM.
➤ Of course, there is one description that is clearly right and has manifest $\mathcal{N} = 8$ supersymmetry (but not manifest conformal symmetry):

$$\lim_{g_{YM} \to \infty} \frac{1}{g_{YM}^2} \mathcal{L}_{SYM}$$

➤ The question is whether this conformal IR fixed point has an explicit Lagrangian description wherein all the symmetries are manifest.

➤ This includes a global $SO(8)_R$ symmetry describing rotations of the space transverse to the membranes – enhanced from the $SO(7)$ of SYM.

➤ Let us look at the Lagrangians that have been proposed to describe this limit.

➤ Euclidean 3-algebra [Bagger-Lambert, Gustavsson]: Labelled by integer $k$. Algebra is $SU(2) \times SU(2)$.

$\Rightarrow$ Argued to describe a pair of $M2$ branes at $Z_k$ singularity. But no generalisation to $> 2$ branes.
Euclidean 3-algebra [Bagger-Lambert, Gustavsson]: Labelled by integer $k$. Algebra is $SU(2) \times SU(2)$.

$\Rightarrow$ Argued to describe a pair of $M2$ branes at $Z_k$ singularity. But no generalisation to $> 2$ branes.

Lorentzian 3-algebra [Gomis-Milanesi-Russo, Benvenuti-Rodriguez-Gomez-Tonni-Verlinde, Bandres-Lipstein-Schwarz, Gomis-Rodriguez-Gomez-van Raamsdonk-Verlinde]: Based on arbitrary Lie algebras, have $\mathcal{N} = 8$ superconformal invariance.

$\Rightarrow$ Certainly correspond to $D2$-branes, and perhaps to $M2$-branes. Status of latter unclear at the moment.

ABJM theories [Aharony-Bergman-Jafferis-Maldacena]: Labelled by algebra $G \times G'$ and integer $k$, with $\mathcal{N} = 6$ superconformal invariance. Is actually a “relaxed” 3-algebra.

$\Rightarrow$ Describe multiple M2-branes at orbifold singularities. But the $k = 1$ theory is missing two manifest supersymmetries and decoupling of CM mode not visible.
These theories all have 8 scalars and 8 fermions.

And they have non-dynamical (Chern-Simons-like) gauge fields.
These theories all have 8 scalars and 8 fermions. And they have non-dynamical (Chern-Simons-like) gauge fields. Thus the basic classification is:

(i) Euclidean signature 3-algebras, which are $G \times G$ Chern-Simons theories:

$$k \: \text{tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A - \tilde{A} \wedge d\tilde{A} - \frac{2}{3} \tilde{A} \wedge \tilde{A} \wedge \tilde{A} \right)$$

BLG : $G = SU(2)$

ABJM : $G = SU(N)$ or $U(N)$, any $N$ (+ other choices)

both : scalars, fermions are bi-fundamental, e.g. $X^I_{a\dot{a}}$

(ii) Lorentzian signature 3-algebras, which are $B \wedge F$ theories based on any Lie algebra.

scalars, fermions are singlet + adjoint, e.g. $X^I_+, X^I$

Both classes make use of the triple product $X^{IJK}$:

Euclidean : $X^{IJK} \sim X^I X^J \tilde{X}^K$, $X^I$ bi-fundamental

Lorentzian : $X^{IJK} \sim X^I_+ [X^J, X^K] + \text{cyclic}$

$X^I_+ = \text{singlet}, X^J = \text{adjoint}$
Both classes make use of the triple product $X^{IJK}$:

- **Euclidean**: $X^{IJK} \sim X^I X^J X^K$, $X^I$ bi-fundamental
- **Lorentzian**: $X^{IJK} \sim X_+^I [X^J, X^K] + \text{cyclic}$
  $X_+^I = \text{singlet}, X^J = \text{adjoint}$

The potential is:

$$V(X) \sim (X^{IJK})^2$$

therefore **sextic**.

One might have expected a simple and unique description for the theory on $N$ M2-branes in flat spacetime.
One might have expected a simple and unique description for the theory on $\mathcal{N} \text{ M2-branes}$ in flat spacetime.

It is basically an analogue of 4d $\mathcal{N} = 4$ super-Yang-Mills!

The only “excuse” we have for not doing better is that the theory we seek will be strongly coupled. So it’s not even clear what the classical action means.
One might have expected a simple and unique description for the theory on $\mathcal{N} M2$-branes in flat spacetime. It is basically an analogue of 4d $\mathcal{N} = 4$ super-Yang-Mills! The only “excuse” we have for not doing better is that the theory we seek will be strongly coupled. So it’s not even clear what the classical action means. However it’s also maximally superconformal, which should give us a lot of power in dealing with it.

In this talk I’ll deal with some things we have understood about the desired theory.
The Higgs mechanism

For the $G \times G$ Chern-Simons class of theories, the following holds true [SM-Papageorgakis].
For the $G \times G$ Chern-Simons class of theories, the following holds true [SM-Papageorgakis].

If we give a vev $v$ to one component of the bi-fundamental fields, then at energies below this vev, the Lagrangian becomes:

$$L_{CS}^{(G \times G)}|_{vev\ v} = \frac{1}{v^2}L^{(G)}_{SYM} + \mathcal{O}\left(\frac{1}{v^3}\right)$$

and the $G$ gauge field has become dynamical!

This is an unusual result. In SYM with gauge group $G$, when we give a vev to one component of an adjoint scalar, at low energy the Lagrangian becomes:

$$\frac{1}{g_{YM}^2}L^{(G)}_{SYM}|_{vev\ v} = \frac{1}{g_{YM}^2}L^{(G' \subset G)}_{SYM}$$

where $G'$ is the subgroup that commutes with the vev.
Let’s give a quick derivation of this novel Higgs mechanism, first for $k = 1$:

$$L_{CS} = \text{tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A - \tilde{A} \wedge d\tilde{A} - \frac{2}{3} \tilde{A} \wedge \tilde{A} \wedge \tilde{A} \right)$$

$$= \text{tr} \left( A_- \wedge F_+ + \frac{1}{6} A_- \wedge A_- \wedge A_- \right)$$

where $A_\pm = A \pm \tilde{A}$, $F_+ = dA_+ + \frac{1}{2} A_+ \wedge A_+$.

Also the covariant derivative on a scalar field is:

$$D_\mu X = \partial_\mu X - A_\mu X + X \tilde{A}_\mu$$
Let’s give a quick derivation of this novel Higgs mechanism, first for $k = 1$:

$$L_{CS} = \text{tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A - \tilde{A} \wedge d\tilde{A} - \frac{2}{3} \tilde{A} \wedge \tilde{A} \wedge \tilde{A} \right)$$

$$= \text{tr} \left( A_+ \wedge F_+ + \frac{1}{6} A_- \wedge A_- \wedge A_- \right)$$

where $A_{\pm} = A \pm \tilde{A}$, $F_+ = dA_+ + \frac{1}{2} A_+ \wedge A_+$. 

Also the covariant derivative on a scalar field is:

$$D_\mu X = \partial_\mu X - A_\mu X + X \tilde{A}\mu$$

If $\langle X \rangle = v$ then:

$$-(D_\mu X)^2 \sim -v^2 (A_-)_\mu (A_-)^\mu + \cdots$$

Thus, $A_-$ is massive – but not dynamical. Integrating it out gives us:

$$- \frac{1}{4v^2} (F_+)^{\mu\nu} (F_+)^{\mu\nu} + \mathcal{O} \left( \frac{1}{v^3} \right)$$

so $A_+$ becomes dynamical.
One can check that the bi-fundamental $X^I$ reduces to an adjoint under $A_+$. The rest of $\mathcal{N} = 8$ SYM assembles itself correctly.

But how should we physically interpret this?

$$L_{CS}^{G \times G} \bigg|_{vev} = \frac{1}{v^2} L_{SYM}^{G} + \mathcal{O}\left(\frac{1}{v^3}\right)$$
One can check that the bi-fundamental $X^I$ reduces to an adjoint under $A_+$. The rest of $\mathcal{N} = 8$ SYM assembles itself correctly.

But how should we physically interpret this?

$$L^{G\times G}_{CS} \bigg|_{vev} = \frac{1}{v^2} L^G_{SYM} + \mathcal{O} \left( \frac{1}{v^3} \right)$$

It seems like the $M2$ is becoming a $D2$ with YM coupling $v$.

Have we somehow compactified the theory? No.
One can check that the bi-fundamental $X^I$ reduces to an adjoint under $A_+$. The rest of $\mathcal{N} = 8$ SYM assembles itself correctly.

But how should we physically interpret this?

$$L_{CS}^{G \times G} \bigg|_{vev\ v} = \frac{1}{v^2} L_{SYM}^G + \mathcal{O} \left( \frac{1}{v^3} \right)$$

It seems like the $M2$ is becoming a $D2$ with YM coupling $v$.

Have we somehow compactified the theory? No.

For any finite $v$, there are corrections to the SYM. These decouple only as $v \to \infty$. So at best we can say that:

$$L_{CS}^{G \times G} \bigg|_{vev\ v \to \infty} = \lim_{v \to \infty} \frac{1}{v^2} L_{SYM}^G$$

The RHS is by definition the theory on $M2$-branes! So this is more like a “proof” that the original Chern-Simons theory really is the theory on $M2$-branes.
However once we introduce the Chern-Simons level $k$: then the analysis is different [Distler-SM-Papageorgakis-van Raamsdonk]:

$$L_{CS}^{G \times G} \bigg|_{vev} = \frac{k}{v^2} L_{SYM}^G + \mathcal{O} \left( \frac{k}{v^3} \right)$$

If we take $k \to \infty, v \to \infty$ with $v^2/k = g_{YM}$ fixed, then in this limit the RHS actually becomes:

$$\frac{1}{2g_{YM}} L_{SYM}^G$$

and this is definitely the Lagrangian for $D2$ branes at finite coupling.
► However once we introduce the Chern-Simons level $k$: then the analysis is different [Distler-SM-Papageorgakis-van Raamsdonk]:

$$L^G_{CS} \big|_{\text{vev}} = \frac{k}{v^2} L^G_{SYM} + O\left(\frac{k}{v^3}\right)$$

► If we take $k \rightarrow \infty, v \rightarrow \infty$ with $v^2/k = g_{YM}$ fixed, then in this limit the RHS actually becomes:

$$\frac{1}{g_{YM}^2} L_{SYM}$$

and this is definitely the Lagrangian for $D2$ branes at finite coupling.

► So this time we have compactified the theory! How can that be?

► We proposed this should be understood as deconstruction for an orbifold $C^4/Z_k$:

![Diagram of deconstruction](image)
We proposed this should be understood as deconstruction for an orbifold $C^4/Z_k$:

In our paper we observed that the orbifold $C^4/Z_k$ has $\mathcal{N} = 6$ supersymmetry and $SU(4)$ $R$-symmetry. We thought this might be enhanced to $\mathcal{N} = 8$ for some unknown reason.

Instead, as ABJM found, it’s the BLG field theory that needs to be modified to have $\mathcal{N} = 6$. 
We proposed this should be understood as deconstruction for an orbifold $C^4/Z_k$:

In our paper we observed that the orbifold $C^4/Z_k$ has $\mathcal{N} = 6$ supersymmetry and $SU(4)$ $R$-symmetry. We thought this might be enhanced to $\mathcal{N} = 8$ for some unknown reason.

Instead, as ABJM found, it’s the BLG field theory that needs to be modified to have $\mathcal{N} = 6$.

One lesson we learn is that for large $k$ we are in the regime of weakly coupled string theory.

A lot can be done in that regime, but for understanding the basics of M2-branes, that is not where we want to be.
Lorentzian 3-algebras

The Lorentzian 3-algebra theories have the following Lagrangian:

\[
L_{L3A}'^{(G)} = \text{tr} \left( \frac{1}{2} \epsilon^{\mu \nu \lambda} B_{\mu} F_{\nu \lambda} - \frac{1}{2} \hat{D}_{\mu} X^I \hat{D}^\mu X^I 
- \frac{1}{12} \left( X_I^J [X^J, X^K] + X_I^K [X^K, X^I] + X_I^K [X^I, X^J] \right)^2 \right) + (C^I - \partial^I X^I) \partial_{\mu} X^I_+ + L_{\text{gauge fixing}} + L_{\text{fermions}}
\]

where

\[
\hat{D}_{\mu} X^I \equiv \partial_{\mu} X^I - [A_{\mu}, X^I] - B_{\mu} X^I_+
\]
The Lorentzian $3$-algebra theories have the following Lagrangian:

\[
L_{L3A}^{(G)} = \text{tr} \left( \frac{1}{2} \epsilon^{\mu \nu \lambda} B_{\mu} F_{\nu \lambda} - \frac{1}{2} \hat{D}_\mu X^I \hat{D}_\mu X^I \\
- \frac{1}{12} \left( X^I_+ [X^J_-, X^K_+] + X^J_+ [X^K_-, X^I_-] + X^K_+ [X^I_-, X^J_-] \right)^2 \right) \\
+ \left( C^\mu I - \partial^\mu X^I_- \right) \partial_\mu X^I_+ + L_{\text{gauge fixing}} + L_{\text{fermions}}
\]

where

\[
\hat{D}_\mu X^I \equiv \partial_\mu X^I - [A_\mu, X^I] - B_\mu X^I_+
\]

These theories have no parameter $k$.

They have $SO(8)$ global symmetry acting on the indices $I, J, K \in 1, 2, \cdots, 8$. 
Lorentzian 3-algebras

The Lorentzian 3-algebra theories have the following Lagrangian:

\[
L^{(G)}_{L3A} = \text{tr} \left( \frac{1}{2} \epsilon^{\mu \nu \lambda} B_\mu F_{\nu \lambda} - \frac{1}{2} \hat{D}_\mu X^I \hat{D}^\mu X^I \\
- \frac{1}{12} \left( X_+^I [X^J, X^K] + X_+^J [X^K, X^I] + X_+^K [X^I, X^J] \right)^2 \right) \\
+ \left( C^\mu I - \partial^\mu X_+^I \right) \partial_\mu X_+^I + L_{\text{gauge fixing}} + L_{\text{fermions}}
\]

where

\[
\hat{D}_\mu X^I \equiv \partial_\mu X^I - [A_\mu, X^I] - B_\mu X_+^I
\]

These theories have no parameter \( k \).

They have \( SO(8) \) global symmetry acting on the indices \( I, J, K \in 1, 2, \ldots , 8 \).

The equation of motion of the auxiliary gauge field \( C^I_\mu \) implies that \( X_+ = \text{constant} \).

Our Higgs mechanism works in these theories, but it works too well! [Ho-Imamura-Matsuo]
Our Higgs mechanism works in these theories, but it works too well! [Ho-Imamura-Matsuo]

On giving a vev to the singlet field $X^I_+$, say:

$$\langle X^8_+ \rangle = v$$

one finds:

$$L^{(G)}_{L3A} \bigg|_{\text{vev } v} = \frac{1}{v^2} L^{(G)}_{SYM} (\text{+ no corrections})$$

This leads one to suspect that the theory is a re-formulation of SYM.
Our Higgs mechanism works in these theories, but it works too well! [Ho-Imamura-Matsuo]

On giving a vev to the singlet field $X^I_\perp$, say:

$$\langle X^8_\perp \rangle = v$$

one finds:

$$L_{L3A}^{(G)} \bigg|_{vev} = \frac{1}{v^2} L_{SYM}^{(G)} \quad (+ \text{ no corrections})$$

This leads one to suspect that the theory is a re-formulation of SYM.

In fact it can be derived [Ezhuthachan-SM-Papageorgakis] starting from $\mathcal{N} = 8$ SYM.

The procedure involves a non-Abelian (dNS) duality [deWit-Nicolai-Samtleben] on the (2+1)d gauge field.
The procedure involves a non-Abelian (dNS) duality \cite{deWit-Nicolai-Samtleben} on the (2+1)d gauge field.

Start with $\mathcal{N} = 8$ SYM in (2+1)d. Introducing two new adjoint fields $B_\mu, \phi$, the dNS duality transformation is:

$$-rac{1}{4g_{YM}^2} F^{\mu\nu} F_{\mu\nu} \rightarrow \frac{1}{2} \epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda} - \frac{1}{2} \left( D_\mu \phi - g_{YM} B_\mu \right)^2$$

Note that $D_\mu$ is the covariant derivative with respect to the original gauge field $A$.

In addition to the gauge symmetry $G$, the new action has a noncompact abelian gauge symmetry:

$$\delta \phi = g_{YM} M, \quad \delta B_\mu = D_\mu M$$

where $M(x)$ is an arbitrary matrix in the adjoint of $G$. 
The procedure involves a non-Abelian (dNS) duality on the (2+1)d gauge field.

Start with $\mathcal{N} = 8$ SYM in (2+1)d. Introducing two new adjoint fields $B_{\mu}, \phi$, the dNS duality transformation is:

$$-rac{1}{4g_{YM}} F_{\mu\nu} F_{\mu\nu} \rightarrow \frac{1}{2} \epsilon^{\mu\nu\lambda} B_{\mu} F_{\nu\lambda} - \frac{1}{2} (D_{\mu} \phi - g_{YM} B_{\mu})^2$$

Note that $D_{\mu}$ is the covariant derivative with respect to the original gauge field $A$.

In addition to the gauge symmetry $G$, the new action has a noncompact abelian gauge symmetry:

$$\delta \phi = g_{YM} M, \quad \delta B_{\mu} = D_{\mu} M$$

where $M(x)$ is an arbitrary matrix in the adjoint of $G$.

To prove the duality, use this symmetry to set $\phi = 0$. Then integrating out $B_{\mu}$ gives the usual YM kinetic term for $F_{\mu\nu}$.

The dNS-duality transformed $\mathcal{N} = 8$ SYM is:

$$L = \text{tr} \left( \frac{1}{2} \epsilon^{\mu\nu\lambda} B_{\mu} F_{\nu\lambda} - \frac{1}{2} (D_{\mu} \phi - g_{YM} B_{\mu})^2 \right)$$

$$- \frac{1}{2} D_{\mu} X^i D^\mu X^i - \frac{g_{YM}^2}{4} [X^i, X^j]^2 + \text{fermions} \right)$$
The dNS-duality transformed $\mathcal{N} = 8$ SYM is:

\[
L = \text{tr} \left( \frac{1}{2} \epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda} - \frac{1}{2} (D_\mu \phi - g_{YM} B_\mu)^2 \right.
- \frac{1}{2} D_\mu X^i D^\mu X^i - \frac{g_{YM}^2}{4} [X^i, X^j]^2 + \text{fermions} \biggr)
\]

We can now see the $SO(8)$ invariance appearing.

The dNS-duality transformed $\mathcal{N} = 8$ SYM is:

\[
L = \text{tr} \left( \frac{1}{2} \epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda} - \frac{1}{2} (D_\mu \phi - g_{YM} B_\mu)^2 \right.
- \frac{1}{2} D_\mu X^i D^\mu X^i - \frac{g_{YM}^2}{4} [X^i, X^j]^2 + \text{fermions} \biggr)
\]

We can now see the $SO(8)$ invariance appearing.

Rename $\phi \rightarrow X^8$. Then the scalar kinetic terms are:

\[-\frac{1}{2} \hat{D}_\mu X^I \hat{D}^\mu X^I = -\frac{1}{2} (\partial_\mu X^I - [A_\mu, X^I] - g^I_{YM} B_\mu)^2 \]

where $g^I_{YM} = (0, \ldots, 0, g_{YM})$. 
The dNS-duality transformed $\mathcal{N} = 8$ SYM is:

$$L = \text{tr}\left(\frac{1}{2} \epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda} - \frac{1}{2} \left(D_\mu \phi - g_{YM} B_\mu\right)^2 - \frac{1}{2} D_\mu X^i D^\mu X^i - \frac{g^2_{YM}}{4} [X^i, X^j]^2 + \text{fermions}\right)$$

We can now see the $SO(8)$ invariance appearing.

Rename $\phi \rightarrow X^8$. Then the scalar kinetic terms are:

$$-\frac{1}{2} \hat{D}_\mu X^I \hat{D}^\mu X^I = -\frac{1}{2} \left(\partial_\mu X^I - [A_\mu, X^I] - g^I_{YM} B_\mu\right)^2$$

where $g^I_{YM} = (0, \ldots, 0, g_{YM})$.

Next, we can allow $g^I_{YM}$ to be an arbitrary 8-vector.

The action is now $SO(8)$-invariant if we rotate both the fields $X^I$ and the coupling-constant vector $g^I_{YM}$:

$$L = \text{tr}\left(\frac{1}{2} \epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda} - \frac{1}{2} \hat{D}_\mu X^I \hat{D}^\mu X^I - \frac{1}{12} \left(g^I_{YM} [X^J, X^K] + g^J_{YM} [X^K, X^I] + g^K_{YM} [X^I, X^J]\right)^2\right)$$
The action is now $SO(8)$-invariant if we rotate both the fields $X^I$ and the coupling-constant vector $g^I_{YM}$:

\[
L = \text{tr} \left( \frac{1}{2} e^{\mu\nu\lambda} B_{\mu} F_{\nu\lambda} - \frac{1}{2} \hat{D}_{\mu} X^I \hat{D}^{\mu} X^I 
- \frac{1}{12} \left( g^I_{YM} [X^J, X^K] + g^J_{YM} [X^K, X^I] + g^K_{YM} [X^I, X^J] \right)^2 \right)
\]

This is not yet a symmetry, since it rotates the coupling constant.

The final step is to introduce an 8-vector of new (gauge-singlet) scalars $X_+^I$ and replace:

\[
g^I_{YM} \rightarrow X_+^I(x)
\]
The action is now $SO(8)$-invariant if we rotate both the fields $X^I$ and the coupling-constant vector $g^I_{YM}$:

$$L = \text{tr}\left( \frac{1}{2} e^{\mu\nu\lambda} B_\mu F_{\nu\lambda} - \frac{1}{2} \hat{D}_\mu X^I \hat{D}^\mu X^I \\
- \frac{1}{12} \left( g^I_{YM}[X^J, X^K] + g^J_{YM}[X^K, X^I] + g^K_{YM}[X^I, X^J] \right)^2 \right)$$

This is not yet a symmetry, since it rotates the coupling constant.

The final step is to introduce an 8-vector of new (gauge-singlet) scalars $X^I_+$ and replace:

$$g^I_{YM} \rightarrow X^I_+(x)$$

This is legitimate if and only if $X^I_+(x)$ has an equation of motion that renders it constant. Then on-shell we can recover the original theory by writing $\langle X^I_+ \rangle = g^I_{YM}$.

Constancy of $X^I_+$ is imposed by introducing a new set of abelian gauge fields and scalars: $C^I_\mu, X^I_-$ and adding the following term:

$$L_C = (C^I_\mu - \partial X^I_-) \partial_\mu X^I_+$$
Constancy of $X^I_+$ is imposed by introducing a new set of abelian gauge fields and scalars: $C^I_\mu, X^I_-$ and adding the following term:

$$L_C = (C^\mu_I - \partial X^I_-) \partial_\mu X^I_+$$

This has a shift symmetry

$$\delta X^I_- = \lambda^I, \quad \delta C^I_\mu = \partial_\mu \lambda^I$$

which will remove the negative-norm states associated to $C^I_\mu$.

We have thus ended up with the Lorentzian 3-algebra action [Bandres-Lipstein-Schwarz, Gomis-Rodriguez-Gomez-van Raamsdonk-Verlinde]:

$$L = \text{tr} \left( \frac{1}{2} \epsilon^{\mu \nu \lambda} B_{\mu} F_{\nu \lambda} - \frac{1}{2} \hat{D}_\mu X^I \hat{D}_\mu X^I \
- \frac{1}{12} \left( X^I_+ [X^J, X^K] + X^J_+ [X^K, X^I] + X^K_+ [X^I, X^J] \right)^2 \right) + (C^\mu I - \partial \mu X^I_-) \partial_\mu X^I_+ + L_{\text{gauge-fixing}} + L_{\text{fermions}}$$
The final action has some remarkable properties.

- It has manifest $SO(8)$ invariance as well as $\mathcal{N} = 8$ superconformal invariance.

However, both are spontaneously broken by giving a vev $\langle X^I \rangle = g_I$ and the theory reduces to $\mathcal{N} = 8$ SYM with coupling $|g_{YM}|$.

It will certainly describe M-branes if one can find a way to take $\langle X^I \rangle = \infty$. That has not yet been done.
The final action has some remarkable properties.

It has manifest $SO(8)$ invariance as well as $\mathcal{N} = 8$ superconformal invariance.

However, both are spontaneously broken by giving a vev $\langle X^I_+ \rangle = g^I_{YM}$ and the theory reduces to $\mathcal{N} = 8$ SYM with coupling $|g_{YM}|$.

It will certainly describe M2-branes if one can find a way to take $\langle X^I_+ \rangle = \infty$. That has not yet been done.
One might ask if the non-Abelian duality that we have just performed works when higher order (in $\alpha'$) corrections are included.
One might ask if the non-Abelian duality that we have just performed works when higher order (in $\alpha'$) corrections are included.

For the Abelian case [Duff, Townsend, Schmidhuber] we know that the analogous duality works for the entire DBI action and that fermions and supersymmetry can also be incorporated [Aganagic-Park-Popescu-Schwarz].

Recently we have shown [Alishahiha-SM] that to lowest nontrivial order ($F^4$-type corrections) one can indeed dualise the non-Abelian SYM into an $SO(8)$-invariant form.
One might ask if the non-Abelian duality that we have just performed works when higher order (in $\alpha'$) corrections are included.

For the Abelian case [Duff, Townsend, Schmidhuber] we know that the analogous duality works for the entire DBI action and that fermions and supersymmetry can also be incorporated [Aganagic-Park-Popescu-Schwarz].

Recently we have shown [Alishahiha-SM] that to lowest nontrivial order ($F^4$-type corrections) one can indeed dualise the non-Abelian SYM into an $SO(8)$-invariant form.

Here of course one cannot do all orders in $\alpha'$ because a non-Abelian analogue of DBI is still not known.

However our approach may have a bearing on that unsolved problem.
Let us see how this works. In (2+1)d, the lowest correction to SYM for D2-branes is the sum of the following contributions (here \( X^{ij} = [X^i, X^j] \)):

\[
L_1^{(4)} = \frac{1}{12g_{YM}^4} \left[ F_{\mu\nu} F_{\rho\sigma} F^{\mu\rho} F^{\nu\sigma} + \frac{1}{2} F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu} \right. \\
\left. - \frac{1}{4} F_{\mu\nu} F^{\mu\rho} F_{\rho\sigma} F^{\nu\sigma} - \frac{1}{8} F_{\mu\nu} F_{\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \right]
\]

\[
L_2^{(4)} = \frac{1}{2g_{YM}^2} \left[ F_{\mu\nu} D^i X^j F^{\rho\nu} D_\rho X^i + F_{\mu\nu} D_\rho X^i F^{\mu\rho} D^\nu X^i \\
- 2F_{\mu\rho} F^{\rho\nu} D^i X^j D_\nu X^i - 2F_{\mu\rho} F^{\rho\nu} D_\nu X^i D^\mu X^i \\
- F_{\mu\nu} F^{\mu\rho} D_\rho X^i D_\nu X^i - \frac{1}{2} F_{\mu\nu} D_\rho X^i F_{\rho\sigma} D_\nu X^i \right] \\
- \frac{1}{12} \left( \frac{1}{2} F_{\mu\nu} F^{\mu\nu} X^{ij} X^{ij} + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} X^{ij} \right)
\]

\[
L_3^{(4)} = -\frac{1}{6} \left( D^\mu X^i D^\nu X^j F_{\mu\nu} + D^\nu X^i F_{\mu\nu} D^\mu X^i \\
+ F_{\mu\nu} D^\mu X^i D^\nu X^j \right) X^{ij}
\]

\[
L_4^{(4)} = \frac{1}{12} \left[ D_\mu X^i D_\nu X^j D^\nu X^i D^\mu X^j + D_\mu X^i D_\nu X^j D^\mu X^j D^\nu X^i \\
+ D_\mu X^i D_\nu X^j D^\nu X^j D^\mu X^i - D_\mu X^i D^\mu X^i D_\nu X^j D^\nu X^j \\
- \frac{1}{2} D_\mu X^i D_\nu X^j D^\mu X^i D^\nu X^j \right]
\]

\[
L_5^{(4)} = \frac{g_{YM}^2}{12} \left[ X^{kj} D_\mu X^k X^{ij} D^\nu X^i + X^{ij} D_\mu X^k X^{ik} D^\mu X^j \\
- 2X^{kj} X^{ik} D_\mu X^j D^\mu X^i - 2X^{ki} X^{jk} D_\mu X^j D^\mu X^i \\
- X^{ij} X^{ij} D_\mu X^k D^\mu X^k - \frac{1}{2} X^{ij} D_\mu X^k X^{ij} D^\mu X^k \right]
\]

\[
L_6^{(4)} = \frac{g_{YM}^4}{12} \left[ X^{ij} X^{kl} X^{ik} X^{jl} + \frac{1}{2} X^{ij} X^{jk} X^{kl} X^{li} \\
- \frac{1}{4} X^{ij} X^{ij} X^{kl} X^{kl} - \frac{1}{8} X^{ij} X^{kl} X^{ij} X^{kl} \right]
\]
We have been able to show that this is dual, under the dNS transformation, to:

$$L = \text{tr} \left[ \frac{1}{2} \epsilon^{\mu \nu \rho} B_{\mu} F_{\nu \rho} - \frac{1}{2} \hat{D}_\mu X^I \hat{D}^\mu X^I \right]
+ \frac{1}{12} \left( \hat{D}_\mu X^I \hat{D}_\nu X^J \hat{D}^\mu X^I \hat{D}^\nu X^J + \hat{D}_\mu X^I \hat{D}_\nu X^J \hat{D}^\nu X^J \hat{D}^\mu X^I \hat{D}_\mu X^I - \hat{D}_\mu X^I \hat{D}_\nu X^J \hat{D}_\mu X^I \hat{D}_\nu X^J \hat{D}^\mu X^I \hat{D}^\nu X^J \right)
+ \frac{1}{12} \left( \frac{1}{2} X^{LKJ} \hat{D}_\mu X^K X^{LIJ} \hat{D}^\mu X^I + \frac{1}{2} X^{LIJ} \hat{D}_\mu X^K X^{LJK} \hat{D}^\mu X^J - X^{LKJ} X^{LIJ} \hat{D}_\mu X^K \hat{D}^\mu X^K \hat{D}^\mu X^J - \frac{1}{3} X^{LIJ} X^{LIJ} \hat{D}_\mu X^K \hat{D}^\mu X^K - \frac{1}{6} X^{LIJ} \hat{D}_\mu X^K X^{LIJ} \hat{D}^\mu X^K \right)
- \frac{1}{6} \epsilon_{\mu \nu \rho} \hat{D}^\rho X^I \hat{D}^\mu X^J \hat{D}^\nu X^K X^{IJK} - V(X) \right]$$

In the previous expression,

$$\hat{D}_\mu X^I = \partial_\mu X^I - [A_\mu, X^I] - B_\mu X^I$$
$$X^{IJK} = X^I [X^J, X^K] + X^J [X^K, X^I] + X^K [X^I, X^J]$$
In the previous expression,
\[
\hat{D}_\mu X^I = \partial_\mu X^I - [A_\mu, X^I] - B_\mu X^I \\
\]

Here \( V(X) \) is the potential:
\[
V(X) = \frac{1}{12} X^{IJK} X^{IJK} + \frac{1}{108} \left[ X^{NIJ} X^{NKL} X^{MIK} X^{MJL} \\
+ \frac{1}{2} X^{NIJ} X^{MJK} X^{NKL} X^{MLI} \\
- \frac{1}{4} X^{NIJ} X^{NIJ} X^{MKL} X^{MKL} \\
- \frac{1}{8} X^{NIJ} X^{MKL} X^{NIJ} X^{MKL} \right]
\]

We see that the dual Lagrangian is \( SO(8) \) invariant.

In the previous expression,
\[
\hat{D}_\mu X^I = \partial_\mu X^I - [A_\mu, X^I] - B_\mu X^I \\
\]

Here \( V(X) \) is the potential:
\[
V(X) = \frac{1}{12} X^{IJK} X^{IJK} + \frac{1}{108} \left[ X^{NIJ} X^{NKL} X^{MIK} X^{MJL} \\
+ \frac{1}{2} X^{NIJ} X^{MJK} X^{NKL} X^{MLI} \\
- \frac{1}{4} X^{NIJ} X^{NIJ} X^{MKL} X^{MKL} \\
- \frac{1}{8} X^{NIJ} X^{MKL} X^{NIJ} X^{MKL} \right]
\]

We see that the dual Lagrangian is \( SO(8) \) invariant.
In the previous expression,
\[
\hat{D}_\mu X^I = \partial_\mu X^I - [A_\mu, X^I] - B_\mu X^I
\]
\[
\]

Here \( V(X) \) is the potential:
\[
V(X) = \frac{1}{12} X^{IJK} X^{IJK} + \frac{1}{108} \left[ X^{NIJ} X^{NKL} X^{MIK} X^{MJL} + \frac{1}{2} X^{NIJ} X^{MJK} X^{NKL} X^{MLJ} - \frac{1}{4} X^{NIJ} X^{NIJ} X^{MKL} X^{MKL} - \frac{1}{8} X^{NIJ} X^{MKL} X^{NIJ} X^{MKL} \right]
\]

We see that the dual Lagrangian is \( SO(8) \) invariant.

It's worth noting that this depends crucially on the relative coefficients of various terms in the original Lagrangian.

We see from this that the 3-algebra structure remains intact when higher-derivative corrections are taken into account.
We see from this that the 3-algebra structure remains intact when higher-derivative corrections are taken into account.

We conjecture that $SO(8)$ enhancement holds to all orders in $\alpha'$.

Unfortunately the all-orders corrections are not known for SYM, so we don't have a starting point from which to check this.
Motivation and background

The Higgs mechanism

Lorentzian 3-algebras

Higher-order corrections for Lorentzian 3-algebras

Conclusions

Summary

- Much progress has been made towards finding the multiple membrane field theory representing the IR fixed point of $\mathcal{N} = 8$ SYM.
Much progress has been made towards finding the multiple membrane field theory representing the IR fixed point of $\mathcal{N} = 8$ SYM.

But we don’t seem to be there yet.

The existence of a large-order orbifold (deconstruction) limit provides a way (the only one so far) to relate the membrane theory to $D2$-branes. One would like to understand compactification of transverse or longitudinal directions, as we do for D-branes.
Much progress has been made towards finding the multiple membrane field theory representing the IR fixed point of $\mathcal{N} = 8$ SYM.

But we don't seem to be there yet.

The existence of a large-order orbifold (deconstruction) limit provides a way (the only one so far) to relate the membrane theory to $D2$-branes. One would like to understand compactification of transverse or longitudinal directions, as we do for D-branes.

An interesting mechanism has been identified to dualise the $D2$-brane action into a superconformal, $SO(8)$ invariant one. The result is a Lorentzian 3-algebra and this structure is preserved by $\alpha'$ corrections.

A detailed understanding of multiple membranes should open a new window to M-theory and 11 dimensions.
A detailed understanding of multiple membranes should open a new window to M-theory and 11 dimensions.

...if you were as tiny as a graviton
You could enter these dimensions and go wandering on

and go wandering on.

And they’d find you...