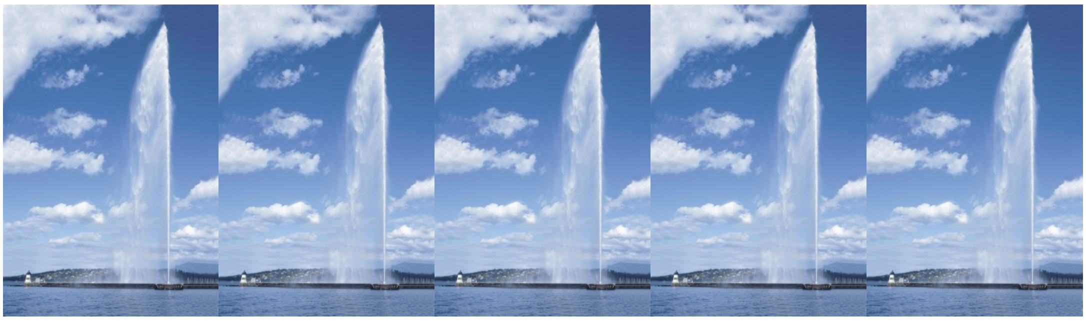
Multiple Membrane Dynamics



Sunil Mukhi
Tata Institute of Fundamental Research, Mumbai

Strings 2008, Geneva, August 19, 2008

- Based on:

  "M2 to D2",
  SM and Costis Papageorgakis,
  arXiv:0803.3218 [hep-th], JHEP 0805:085 (2008).

  " M2-branes on M-folds",
  Jacques Distler, SM, Costis Papageorgakis and Mark van Raamsdonk,
  arXiv:0804.1256 [hep-th], JHEP 0805:038 (2008).

  " D2 to D2",
  Bobby Ezhuthachan, SM and Costis Papageorgakis,
  arXiv:0806.1639 [hep-th], JHEP 0807:041, (2008).

  Mohsen Alishahiha and SM, to appear

Outline

Motivation and background

The Higgs mechanism

Lorentzian 3-algebras

Higher-order corrections for Lorentzian 3-algebras

Conclusions

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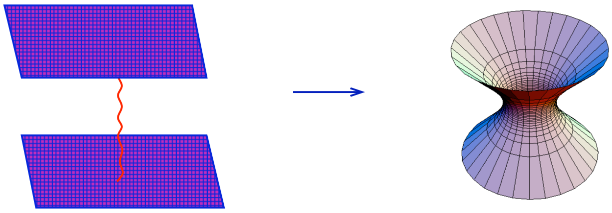

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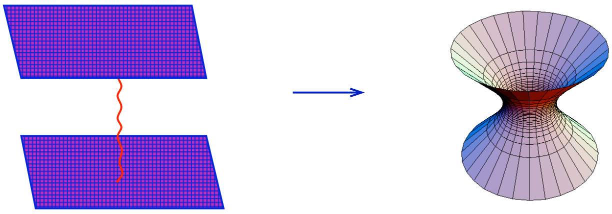

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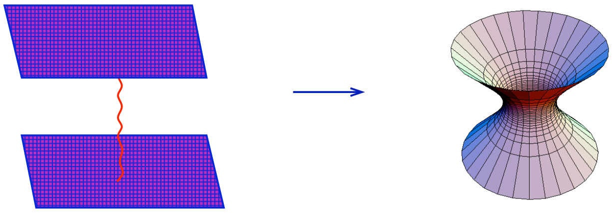

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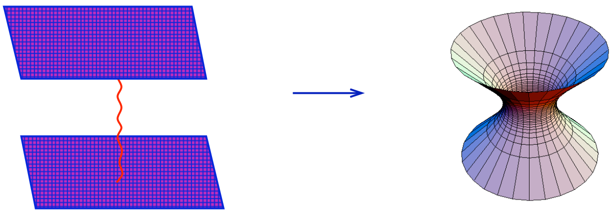

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- And yet, despite ~ 200 recent papers – and two Strings 2008 talks – on the subject, we don't exactly know what the multiple membrane theory is.

- Even the French aristocracy doesn't seem to know...

- Of course, there is one description that is clearly right and has manifest $\mathcal{N} = 8$ supersymmetry (but not manifest conformal symmetry):

$$\lim_{g_{YM} \to \infty} \frac{1}{g_{YM}^2} \mathcal{L}_{SYM}$$

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- Let us look at the Lagrangians that have been proposed to describe this limit.

- Euclidean 3-algebra [Bagger-Lambert, Gustavsson]: Labelled by integer k. Algebra is $SU(2) \times SU(2)$.

  $\Rightarrow$ Argued to describe a pair of $M2$ branes at Z_k singularity. But no generalisation to > 2 branes.

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- ABJM theories [Aharony-Bergman-Jafferis-Maldacena]: Labelled by algebra $G \times G'$ and integer k, with $\mathcal{N} = 6$ superconformal invariance. Is actually a "relaxed" 3-algebra.

  $\Rightarrow$ Describe multiple M2-branes at orbifold singularities. But the $k = 1$ theory is missing two manifest supersymmetries and decoupling of CM mode not visible.

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- Thus the basic classification is:

(i) Euclidean signature 3-algebras, which are $G \times G$ Chern-Simons theories:

$$k\ \mathrm{tr}\left(\boldsymbol{A} \wedge d\boldsymbol{A} + \tfrac{2}{3}\boldsymbol{A} \wedge \boldsymbol{A} \wedge \boldsymbol{A} - \tilde{\boldsymbol{A}} \wedge d\tilde{\boldsymbol{A}} - \tfrac{2}{3}\tilde{\boldsymbol{A}} \wedge \tilde{\boldsymbol{A}} \wedge \tilde{\boldsymbol{A}}\right)$$

BLG : $G = SU(2)$

ABJM : $G = SU(N)$ or $U(N)$, any N (+ other choices)

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(ii) Lorentzian signature 3-algebras, which are $\boldsymbol{B} \wedge \boldsymbol{F}$ theories based on any Lie algebra.

scalars, fermions are singlet + adjoint, e.g. $X^I_+, \boldsymbol{X}^I$

- Both classes make use of the triple product X^{IJK}:

Euclidean : $X^{IJK} \sim X^I X^{J\dagger} X^K, \quad X^I$ bi-fundamental

Lorentzian : $X^{IJK} \sim X^I_+[\boldsymbol{X}^J, \boldsymbol{X}^K] +$ cyclic

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- The potential is:

$$V(X) \sim (\boldsymbol{X}^{IJK})^2$$

therefore sextic.

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- However it's also maximally superconformal, which should give us a lot of power in dealing with it.
- In this talk I'll deal with some things we have understood about the desired theory.

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- If we give a vev v to one component of the bi-fundamental fields, then at energies below this vev, the Lagrangian becomes:

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- This is an unusual result. In SYM with gauge group G, when we give a vev to one component of an adjoint scalar, at low energy the Lagrangian becomes:

$$\frac{1}{g_{\mathrm{YM}}^2} L_{SYM}^{(G)}\Big|_{vev\ v} = \frac{1}{g_{\mathrm{YM}}^2} L_{SYM}^{(G'\subset G)}$$

where G' is the subgroup that commutes with the vev.

- Let's give a quick derivation of this novel Higgs mechanism, first for $k = 1$:

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\begin{aligned}
L_{CS} &= \mathrm{tr}\left(\boldsymbol{A} \wedge d\boldsymbol{A} + \tfrac{2}{3}\boldsymbol{A} \wedge \boldsymbol{A} \wedge \boldsymbol{A} - \tilde{\boldsymbol{A}} \wedge d\tilde{\boldsymbol{A}} - \tfrac{2}{3}\tilde{\boldsymbol{A}} \wedge \tilde{\boldsymbol{A}} \wedge \tilde{\boldsymbol{A}}\right) \\
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- Thus, $\boldsymbol{A}_-$ is massive – but not dynamical. Integrating it out gives us:

$$-\frac{1}{4v^2}(\boldsymbol{F}_+)_{\mu\nu}(\boldsymbol{F}_+)^{\mu\nu} + \mathcal{O}\left(\frac{1}{v^3}\right)$$

  so $\boldsymbol{A}_+$ becomes dynamical.

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- The RHS is by definition the theory on $M2$-branes! So this is more like a "proof" that the original Chern-Simons theory really is the theory on $M2$-branes.

- However once we introduce the Chern-Simons level k then the analysis is different [Distler-SM-Papageorgakis-van Raamsdonk]:

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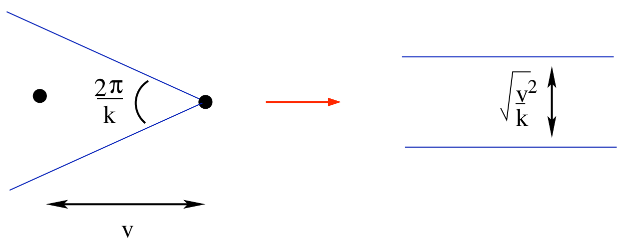
- So this time we have compactified the theory! How can that be?

- We proposed this should be understood as deconstruction for an orbifold C^4/Z_k:



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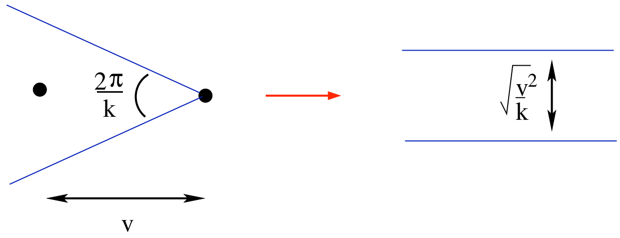

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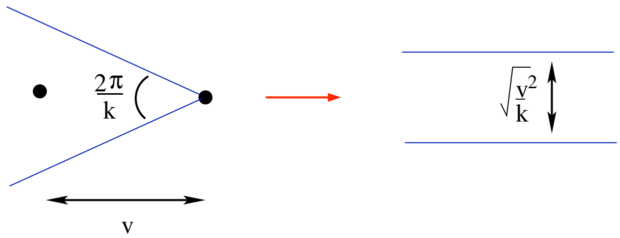

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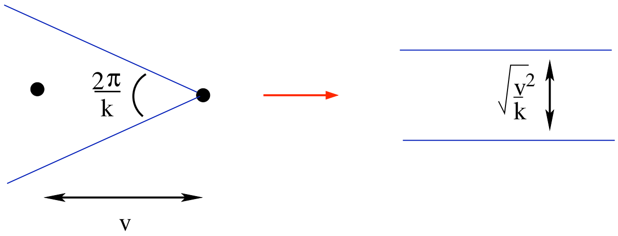

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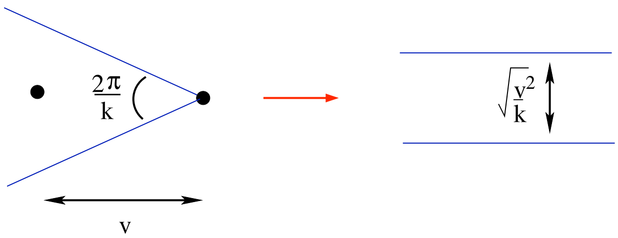

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- Instead, as ABJM found, it's the BLG field theory that needs to be modified to have $\mathcal{N}=6$.
- One lesson we learn is that for large k we are in the regime of weakly coupled string theory.
- A lot can be done in that regime, but for understanding the basics of M2-branes, that is not where we want to be.

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Lorentzian 3-algebras

- The Lorentzian 3-algebra theories have the following Lagrangian:

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\begin{aligned}
L^{(G)}_{L3A} \;=\; & \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I \\
& -\tfrac{1}{12}\left(X^I_+[\boldsymbol{X}^J,\boldsymbol{X}^K] + X^J_+[\boldsymbol{X}^K,\boldsymbol{X}^I] + X^K_+[\boldsymbol{X}^I,\boldsymbol{X}^J]\right)^2\Big) \\
& + \;(C^{\mu\, I} - \partial^\mu X^I_-)\partial_\mu X^I_+ + L_{\text{gauge fixing}} + L_{\text{fermions}}
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where

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$$L^{(G)}_{L3A} = \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I - \tfrac{1}{12}\left(X^I_+[\boldsymbol{X}^J,\boldsymbol{X}^K] + X^J_+[\boldsymbol{X}^K,\boldsymbol{X}^I] + X^K_+[\boldsymbol{X}^I,\boldsymbol{X}^J]\right)^2\Big) + (C^{\mu\,I} - \partial^\mu X^I_-)\partial_\mu X^I_+ + L_{\text{gauge fixing}} + L_{\text{fermions}}$$

where

$$\hat{D}_\mu \boldsymbol{X}^I \equiv \partial_\mu \boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - \boldsymbol{B}_\mu X^I_+$$

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- They have $SO(8)$ global symmetry acting on the indices $I, J, K \in 1, 2, \cdots, 8$.
- The equation of motion of the auxiliary gauge field C_μ^I implies that $X_+ =$ constant.

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- This leads one to suspect that the theory is a re-formulation of SYM.
- In fact it can be derived [Ezhuthachan-SM-Papageorgakis] starting from $\mathcal{N} = 8$ SYM.

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- Start with $\mathcal{N} = 8$ SYM in (2+1)d. Introducing two new adjoint fields $\boldsymbol{B}_\mu, \boldsymbol{\phi}$, the dNS duality transformation is:

$$-\frac{1}{4g_{\mathsf{YM}}^2}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}_{\mu\nu} \to \frac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu\boldsymbol{F}_{\nu\lambda} - \frac{1}{2}\left(D_\mu\boldsymbol{\phi} - g_{\mathsf{YM}}\boldsymbol{B}_\mu\right)^2$$

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Note that D_μ is the covariant derivative with respect to the original gauge field $\boldsymbol{A}$.
- In addition to the gauge symmetry G, the new action has a noncompact abelian gauge symmetry:

$$\delta\boldsymbol{\phi} = g_{\mathsf{YM}}\boldsymbol{M}\ , \qquad \delta\boldsymbol{B}_\mu = D_\mu\boldsymbol{M}$$

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  where $\boldsymbol{M}(x)$ is an arbitrary matrix in the adjoint of G.
- To prove the duality, use this symmetry to set $\boldsymbol{\phi}=0$. Then integrating out $\boldsymbol{B}_\mu$ gives the usual YM kinetic term for $\boldsymbol{F}_{\mu\nu}$.

- The dNS-duality transformed $\mathcal{N}=8$ SYM is:

$$\begin{aligned} L &= \mathrm{tr}\Big(\frac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu\boldsymbol{F}_{\nu\lambda} - \frac{1}{2}\big(D_\mu\boldsymbol{\phi} - g_{YM}\boldsymbol{B}_\mu\big)^2 \\ &\quad - \frac{1}{2}D_\mu\boldsymbol{X}^i D^\mu\boldsymbol{X}^i - \frac{g_{YM}^2}{4}[\boldsymbol{X}^i,\boldsymbol{X}^j]^2 + \text{fermions}\Big) \end{aligned}$$

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- Rename $\boldsymbol{\phi} \to \boldsymbol{X}^8$. Then the scalar kinetic terms are:

$$-\tfrac{1}{2}\hat{D}_\mu\boldsymbol{X}^I\hat{D}^\mu\boldsymbol{X}^I = -\tfrac{1}{2}\left(\partial_\mu\boldsymbol{X}^I - [\boldsymbol{A}_\mu,\boldsymbol{X}^I] - g^I_{\mathrm{YM}}\boldsymbol{B}_\mu\right)^2$$

where $g^I_{\mathrm{YM}} = (0,\ldots,0,g_{\mathrm{YM}})$.

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  where $g^I_{\mathsf{YM}} = (0,\ldots,0,g_{\mathsf{YM}})$.
- Next, we can allow g^I_{YM} to be an arbitrary 8-vector.

- The action is now $SO(8)$-invariant if we rotate both the fields X^I and the coupling-constant vector g^I_{YM}:

$$\begin{aligned} L &= \operatorname{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I \\ &- \tfrac{1}{12}\left(g^I_{\mathsf{YM}}[\boldsymbol{X}^J,\boldsymbol{X}^K] + g^J_{\mathsf{YM}}[\boldsymbol{X}^K,\boldsymbol{X}^I] + g^K_{\mathsf{YM}}[\boldsymbol{X}^I,\boldsymbol{X}^J]\right)^2\Big) \end{aligned}$$

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- The final step is to introduce an 8-vector of new (gauge-singlet) scalars X^I_+ and replace:

$$g^I_{\mathsf{YM}} \to X^I_+(x)$$

- This is legitimate if and only if $X^I_+(x)$ has an equation of motion that renders it constant. Then on-shell we can recover the original theory by writing $\langle X^I_+\rangle = g^I_{\mathsf{YM}}$.

- Constancy of X^I_+ is imposed by introducing a new set of abelian gauge fields and scalars: C^I_μ, X^I_- and adding the following term:

$$L_C = (C^\mu_I - \partial X^I_-)\partial_\mu X^I_+$$

- Constancy of X_+^I is imposed by introducing a new set of abelian gauge fields and scalars: C_μ^I, X_-^I and adding the following term:

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$$\delta X_-^I = \lambda^I, \quad \delta C_\mu^I = \partial_\mu \lambda^I$$

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which will remove the negative-norm states associated to C_μ^I.

- We have thus ended up with the Lorentzian 3-algebra action [Bandres-Lipstein-Schwarz, Gomis-Rodriguez-Gomez-van Raamsdonk-Verlinde]:

$$\begin{aligned} L = \operatorname{tr}\Big(& \tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}_\mu \boldsymbol{X}^I \\ & - \tfrac{1}{12}\left(X_+^I[\boldsymbol{X}^J,\boldsymbol{X}^K] + X_+^J[\boldsymbol{X}^K,\boldsymbol{X}^I] + X_+^K[\boldsymbol{X}^I,\boldsymbol{X}^J]\right)^2\Big) \\ & + (C^{\mu\, I} - \partial^\mu X_-^I)\partial_\mu X_+^I + L_{\text{gauge-fixing}} + \mathcal{L}_{\text{fermions}} \end{aligned}$$

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- However, both are spontaneously broken by giving a vev $\langle X_+^I \rangle = g_{YM}^I$ and the theory reduces to $\mathcal{N} = 8$ SYM with coupling $|g_{YM}|$.
- It will certainly describe M2-branes if one can find a way to take $\langle X_+^I \rangle = \infty$. That has not yet been done.

Outline

Motivation and background

The Higgs mechanism

Lorentzian 3-algebras

Higher-order corrections for Lorentzian 3-algebras

Conclusions

Higher-order corrections for Lorentzian 3-algebras

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- Here of course one cannot do all orders in α' because a non-Abelian analogue of DBI is still not known.
- However our approach may have a bearing on that unsolved problem.

- Let us see how this works. In (2+1)d, the lowest correction to SYM for D2-branes is the sum of the following contributions (here $\boldsymbol{X}^{ij} = [\boldsymbol{X}^i, \boldsymbol{X}^j]$):

$$
\begin{aligned}
L_1^{(4)} &= \tfrac{1}{12 g_{\mathsf{YM}}^4}\Big[\boldsymbol{F}_{\mu\nu}\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\mu\rho}\boldsymbol{F}^{\nu\sigma} + \tfrac{1}{2}\boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\nu\rho}\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\sigma\mu} \\
&- \tfrac{1}{4}\boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\rho\sigma} - \tfrac{1}{8}\boldsymbol{F}_{\mu\nu}\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}^{\rho\sigma}\Big] \\
L_2^{(4)} &= \tfrac{1}{12 g_{\mathsf{YM}}^2}\Big[\boldsymbol{F}_{\mu\nu}\, D^\mu \boldsymbol{X}^i\, \boldsymbol{F}^{\rho\nu}\, D_\rho \boldsymbol{X}^i + \boldsymbol{F}_{\mu\nu}\, D_\rho \boldsymbol{X}^i\, \boldsymbol{F}^{\mu\rho}\, D^\nu \boldsymbol{X}^i \\
&- 2\boldsymbol{F}_{\mu\rho}\, \boldsymbol{F}^{\rho\nu}\, D^\mu \boldsymbol{X}^i\, D_\nu \boldsymbol{X}^i - 2\boldsymbol{F}_{\mu\rho}\, \boldsymbol{F}^{\rho\nu}\, D_\nu \boldsymbol{X}^i\, D^\mu \boldsymbol{X}^i \\
&- \boldsymbol{F}_{\mu\nu}\, \boldsymbol{F}^{\mu\nu}\, D^\rho \boldsymbol{X}^i\, D_\rho \boldsymbol{X}^i - \tfrac{1}{2}\boldsymbol{F}_{\mu\nu}\, D_\rho \boldsymbol{X}_i\, \boldsymbol{F}_{\mu\nu}\, D_\rho \boldsymbol{X}_i\Big] \\
&- \tfrac{1}{12}\left(\tfrac{1}{2}\boldsymbol{F}_{\mu\nu}\, \boldsymbol{F}^{\mu\nu}\, \boldsymbol{X}^{ij}\, \boldsymbol{X}^{ij} + \tfrac{1}{4}\boldsymbol{F}_{\mu\nu}\, \boldsymbol{X}^{ij}\, \boldsymbol{F}^{\mu\nu}\, \boldsymbol{X}^{ij}\right) \\
L_3^{(4)} &= -\tfrac{1}{6}\Big(D^\mu \boldsymbol{X}^i\, D^\nu \boldsymbol{X}^j \boldsymbol{F}_{\mu\nu} + D^\nu \boldsymbol{X}^j\, \boldsymbol{F}_{\mu\nu}\, D^\mu \boldsymbol{X}^i \\
&+ \boldsymbol{F}_{\mu\nu}\, D^\mu \boldsymbol{X}^i\, D^\nu \boldsymbol{X}^j\Big)\, \boldsymbol{X}^{ij}
\end{aligned}
$$

$$
\begin{aligned}
L_4^{(4)} &= \tfrac{1}{12}\Big[D_\mu \boldsymbol{X}^i\, D_\nu \boldsymbol{X}^j\, D^\nu \boldsymbol{X}^i\, D^\mu \boldsymbol{X}^j + D_\mu \boldsymbol{X}^i\, D_\nu \boldsymbol{X}^j\, D^\mu \boldsymbol{X}^j\, D^\nu \boldsymbol{X}^i \\
&+ D_\mu \boldsymbol{X}^i\, D_\nu \boldsymbol{X}^i\, D^\nu \boldsymbol{X}^j\, D^\mu \boldsymbol{X}^j - D_\mu \boldsymbol{X}^i\, D^\mu \boldsymbol{X}^i\, D_\nu \boldsymbol{X}^j\, D^\nu \boldsymbol{X}^j \\
&- \tfrac{1}{2} D_\mu \boldsymbol{X}^i\, D_\nu \boldsymbol{X}^j\, D^\mu \boldsymbol{X}^i\, D^\nu \boldsymbol{X}^j\Big] \\
L_5^{(4)} &= \tfrac{g_{\mathsf{YM}}^2}{12}\Big[\boldsymbol{X}^{kj}\, D_\mu \boldsymbol{X}^k\, \boldsymbol{X}^{ij}\, D^\mu \boldsymbol{X}^i + \boldsymbol{X}^{ij}\, D_\mu \boldsymbol{X}^k\, \boldsymbol{X}^{ik}\, D^\mu \boldsymbol{X}^j \\
&- 2\boldsymbol{X}^{kj}\, \boldsymbol{X}^{ik}\, D_\mu \boldsymbol{X}^j\, D^\mu \boldsymbol{X}^i - 2\boldsymbol{X}^{ki}\, \boldsymbol{X}^{jk}\, D_\mu \boldsymbol{X}^j\, D^\mu \boldsymbol{X}^i \\
&- \boldsymbol{X}^{ij}\, \boldsymbol{X}^{ij}\, D_\mu \boldsymbol{X}^k\, D^\mu \boldsymbol{X}^k - \tfrac{1}{2}\boldsymbol{X}^{ij}\, D_\mu \boldsymbol{X}^k\, \boldsymbol{X}^{ij}\, D^\mu \boldsymbol{X}^k\Big] \\
L_6^{(4)} &= \tfrac{g_{\mathsf{YM}}^4}{12}\Big[\boldsymbol{X}^{ij}\boldsymbol{X}^{kl}\boldsymbol{X}^{ik}\boldsymbol{X}^{jl} + \tfrac{1}{2}\boldsymbol{X}^{ij}\boldsymbol{X}^{jk}\boldsymbol{X}^{kl}\boldsymbol{X}^{li} \\
&- \tfrac{1}{4}\boldsymbol{X}^{ij}\boldsymbol{X}^{ij}\boldsymbol{X}^{kl}\boldsymbol{X}^{kl} - \tfrac{1}{8}\boldsymbol{X}^{ij}\boldsymbol{X}^{kl}\boldsymbol{X}^{ij}\boldsymbol{X}^{kl}\Big]
\end{aligned}
$$

- We have been able to show that this is dual, under the dNS transformation, to:

$$
\begin{aligned}
L =\ & \mathrm{tr}\Big[\tfrac{1}{2}\epsilon^{\mu\nu\rho}\boldsymbol{B}_\mu\boldsymbol{F}_{\nu\rho} - \tfrac{1}{2}\hat{D}_\mu\boldsymbol{X}^I\hat{D}^\mu\boldsymbol{X}^I \\
&+ \tfrac{1}{12}\Big(\hat{D}_\mu\boldsymbol{X}^I\,\hat{D}_\nu\boldsymbol{X}^J\,\hat{D}^\nu\boldsymbol{X}^I\,\hat{D}^\mu\boldsymbol{X}^J + \hat{D}_\mu\boldsymbol{X}^I\,\hat{D}_\nu\boldsymbol{X}^J\,\hat{D}^\mu\boldsymbol{X}^J\,\hat{D}^\nu\boldsymbol{X}^I \\
&\qquad + \hat{D}_\mu\boldsymbol{X}^I\,\hat{D}_\nu\boldsymbol{X}^I\,\hat{D}^\nu\boldsymbol{X}^J\,\hat{D}^\mu\boldsymbol{X}^J - \hat{D}_\mu\boldsymbol{X}^I\,\hat{D}^\mu\boldsymbol{X}^I\,\hat{D}_\nu\boldsymbol{X}^J\,\hat{D}^\nu\boldsymbol{X}^J \\
&\qquad - \tfrac{1}{2}\hat{D}_\mu\boldsymbol{X}^I\,\hat{D}_\nu\boldsymbol{X}^J\,\hat{D}^\mu\boldsymbol{X}^I\,\hat{D}^\nu\boldsymbol{X}^J\Big) \\
&+ \tfrac{1}{12}\Big(\tfrac{1}{2}\boldsymbol{X}^{LKJ}\hat{D}_\mu\boldsymbol{X}^K\boldsymbol{X}^{LIJ}\hat{D}^\mu\boldsymbol{X}^I + \tfrac{1}{2}\boldsymbol{X}^{LIJ}\hat{D}_\mu\boldsymbol{X}^K\boldsymbol{X}^{LIK}\hat{D}^\mu\boldsymbol{X}^J \\
&\qquad - \boldsymbol{X}^{LKJ}\boldsymbol{X}^{LIK}\hat{D}_\mu\boldsymbol{X}^J\hat{D}^\mu\boldsymbol{X}^I - \boldsymbol{X}^{LKI}\boldsymbol{X}^{LJK}\hat{D}_\mu\boldsymbol{X}^J\hat{D}^\mu\boldsymbol{X}^I \\
&\qquad - \tfrac{1}{3}\boldsymbol{X}^{LIJ}\boldsymbol{X}^{LIJ}\hat{D}_\mu\boldsymbol{X}^K\hat{D}^\mu\boldsymbol{X}^K - \tfrac{1}{6}\boldsymbol{X}^{LIJ}\,\hat{D}_\mu\boldsymbol{X}^K\boldsymbol{X}^{LIJ}\hat{D}^\mu\boldsymbol{X}^K\Big) \\
&- \tfrac{1}{6}\epsilon_{\rho\mu\nu}\hat{D}^\rho\boldsymbol{X}^I\,\hat{D}^\mu\boldsymbol{X}^J\,\hat{D}^\nu\boldsymbol{X}^K\boldsymbol{X}^{IJK} - V(\boldsymbol{X})\Big]
\end{aligned}
$$

- In the previous expression,

$$
\begin{aligned}
\hat{D}_\mu\boldsymbol{X}^I &= \partial_\mu\boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - \boldsymbol{B}_\mu X_+^I \\
\boldsymbol{X}^{IJK} &= X_+^I[\boldsymbol{X}^J, \boldsymbol{X}^K] + X_+^J[\boldsymbol{X}^K, \boldsymbol{X}^I] + X_+^K[\boldsymbol{X}^I, \boldsymbol{X}^J]
\end{aligned}
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- Here $V(X)$ is the potential:

$$V(X) = \frac{1}{12}\boldsymbol{X}^{IJK}\boldsymbol{X}^{IJK} + \frac{1}{108}\Big[\boldsymbol{X}^{NIJ}\boldsymbol{X}^{NKL}\boldsymbol{X}^{MIK}\boldsymbol{X}^{MJL} + \frac{1}{2}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MJK}\boldsymbol{X}^{NKL}\boldsymbol{X}^{MLI} - \frac{1}{4}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\boldsymbol{X}^{MKL} - \frac{1}{8}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\Big]$$

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$$\boldsymbol{X}^{IJK} = X_+^I[\boldsymbol{X}^J, \boldsymbol{X}^K] + X_+^J[\boldsymbol{X}^K, \boldsymbol{X}^I] + X_+^K[\boldsymbol{X}^I, \boldsymbol{X}^J]$$

- Here $V(X)$ is the potential:

$$V(X) = \frac{1}{12}\boldsymbol{X}^{IJK}\boldsymbol{X}^{IJK} + \frac{1}{108}\Big[\boldsymbol{X}^{NIJ}\boldsymbol{X}^{NKL}\boldsymbol{X}^{MIK}\boldsymbol{X}^{MJL} + \frac{1}{2}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MJK}\boldsymbol{X}^{NKL}\boldsymbol{X}^{MLI} - \frac{1}{4}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\boldsymbol{X}^{MKL} - \frac{1}{8}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\Big]$$

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- We see that the dual Lagrangian is $SO(8)$ invariant.
- It's worth noting that this depends crucially on the relative coefficients of various terms in the original Lagrangian.

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- We conjecture that SO(8) enhancement holds to all orders in α'.

- Unfortunately the all-orders corrections are not known for SYM, so we don't have a starting point from which to check this.

Outline

Motivation and background

The Higgs mechanism

Lorentzian 3-algebras

Higher-order corrections for Lorentzian 3-algebras

Conclusions

Summary

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- But we don't seem to be there yet.
- The existence of a large-order orbifold (deconstruction) limit provides a way (the only one so far) to relate the membrane theory to $D2$-branes. One would like to understand compactification of transverse or longitudinal directions, as we do for D-branes.
- An interesting mechanism has been identified to dualise the $D2$-brane action into a superconformal, $SO(8)$ invariant one. The result is a Lorentzian 3-algebra and this structure is preserved by α' corrections.

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...if you were as tiny as a graviton
You could enter these dimensions and go wandering on

And they'd find you...