REMODELING THE TOPOLOGICAL STRING - PERTURBATIVELY AND NONPERTURBATIVELY

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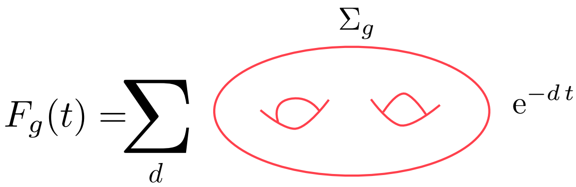
- Perturbative: [Bouchard-Klemm-M.M.-Pasquetti, 0709.1453 & 0807.0597] [Bouchard-M.M, 0709.1458]
- Nonperturbative: [M.M.-Schiappa-Weiss, 0711.1954] [M.M. 0805.3033]
- Previous work: [M.M. hep-th/0612127]

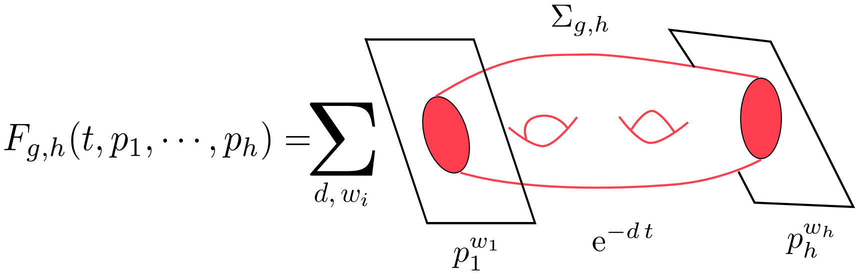
Topological strings on CYs

In terms of *complexity*:

Noncritical strings < Topological strings < Superstrings

Still, highly nontrivial! For the A model on a CY, TS amplitudes encode an *enormous* amount of geometric information:





Goals

"Compute it all at once and algebraically" [Kontsevich]

Example: noncritical strings solved in terms of *matrix models/ integrable systems*

For TS on generic CY threefolds we use instead *mirror symmetry* and the type B model. This computes "algebraically" and all at once in degree -but not in genus! Very effective for genus zero and disk invariants, where the algebraic solution is encoded by Picard-Fuchs equations

Mirror symmetry at higher genus

The generalization of mirror symmetry/B model to higher genus involves the famous holomorphic anomaly equations. These tell us that the TS amplitudes are not holomorphic in the moduli:

$$\partial_{\bar{t}} F_{g,h}(t,\bar{t}) = \text{functional of } F_{g'<g,h'\leq h}$$

[BCOV (closed), Walcher (open global), Eynard-MM-Orantin (open local), Bonelli-Tanzini, ...]

The anomaly equations give a powerful computational tool but in general they are not conclusive due to lack of boundary conditions (holomorphic ambiguity). These are provided in some cases: gap conditions of [Huang-Klemm] (cf. talk by Klemm at Strings07).

Other strategies?

Matrix models and Dijkgraaf-Vafa

In 2002, D&V found a description of type B TS on certain CY backgrounds in terms of large N matrix models

$$Z = \int \mathrm{d}M \, \mathrm{e}^{-\frac{1}{g_s} V(M)}$$

master field

type B TS on

$$u^2 + v^2 + y^2 - (V'(x))^2 + f(x) = 0$$

$$y^2 = (V'(x))^2 - f(x) \quad \text{spectral curve}$$

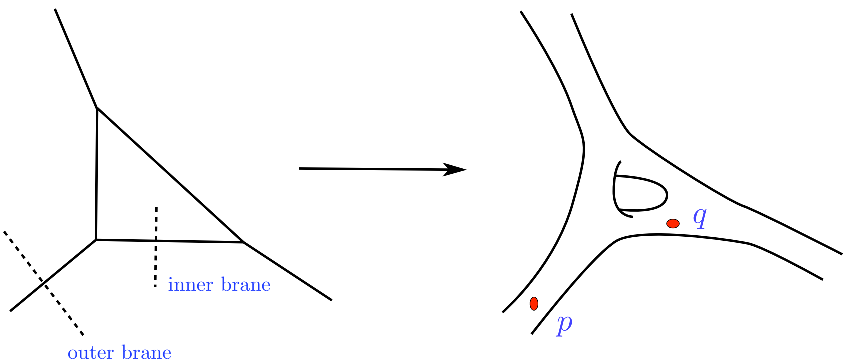
$$\rho(\lambda) = \frac{1}{2\pi t} \mathrm{Im}\, y(\lambda)$$

Moral: the "algebraic" approach of B model/mirror symmetry can be reformulated and generalized to higher genus in terms of matrix models. Enough (and great!) for engineering SUSY theories, but these backgrounds are very special (for example, they have no mirrors)

Toric backgrounds

What about other backgrounds? Specially interesting are *toric* CY manifolds:

- Rich *enumerative* content
- Nontrivial moduli spaces (*orbifold* points)
- Large N gauge theory duals (Chern-Simons)
- Engineering of N=2 gauge theories
- Mirror manifolds encoded by algebraic curves

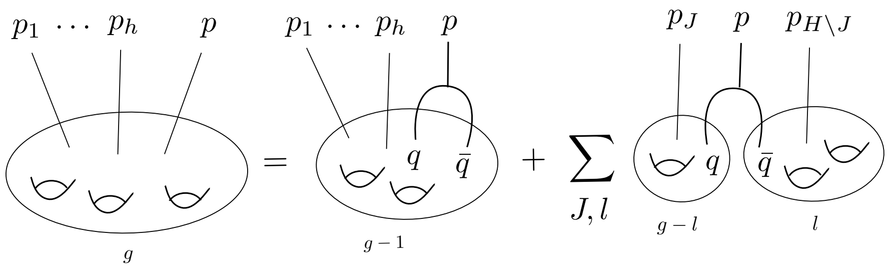


$\mathcal{O}(-3) \to \mathbb{P}^2$ $\qquad$ $y(y+x+1) + z_t\, x^3 = 0$

Remodeling the B model I

Given any algebraic curve one can define, through explicit residue formulae, amplitudes $F_{g,h}(t, p_1, \cdots, p_h), \; F_g(t)$

[Ambjorn et al, Akemann, Eynard et al]



If the curve is the spectral curve of a MM, these amplitudes solve the loop equations and give the *full, explicit* 1/N expansion

$$\log Z = \sum_{g\geq 0} F_g(t) g_s^{2g-2} \qquad \left\langle \mathrm{tr}\frac{1}{p_1 - M} \cdots \mathrm{tr}\frac{1}{p_h - M} \right\rangle^{(c)} = \sum_{g \geq 0} F_{g,h}(t, p_1, \cdots, p_h) g_s^{2g-2+h}$$

Example: $F_{0,2}(p_1, p_2) = B(p_1, p_2)$ Bergmann kernel

Remodeling the B model II

Consider now the B model on the mirror of a toric CY threefold + toric D-branes

Conjecture [M.M., BKMP]: the TS amplitudes of the B model are given, for *all g,h*, by the matrix model-like, residue amplitudes on the mirror curve

Note 1: This is of course true for the CY backgrounds considered by D&V

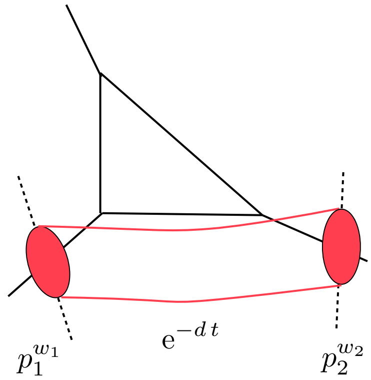
Note 2: We *don't* need an explicit matrix model realization: knowledge of the algebraic (mirror) curve is enough to compute all perturbative amplitudes.

Motivation: the residue amplitudes can also be obtained from a chiral boson on the curve [ADKMV, M.M.,D&V], which is the Kodaira-Spencer theory (=string field theory of the B model) in the noncompact CY case.

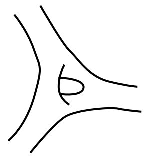
Application/test I

We get a general mirror "theorem" for *all* toric CYs and *all* open and closed topological string amplitudes: we count worldsheet instantons by expanding at large radius

$$F_{0,2}(t,p_1,p_2) = \sum_{d,w_1,w_2}$$



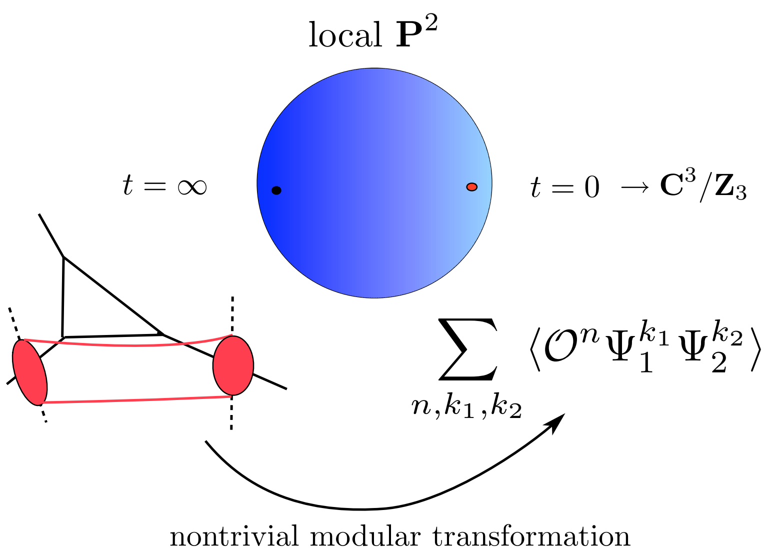
$= B(p_1,p_2)$ on



In particular, we deduce that these amplitudes satisfy *recursion relations modeled on loop equations*. This can be even applied to very classical enumerative objects, like Hurwitz numbers (which can be obtained from special limits of toric backgrounds [Bouchard-M.M.])

Application/test II

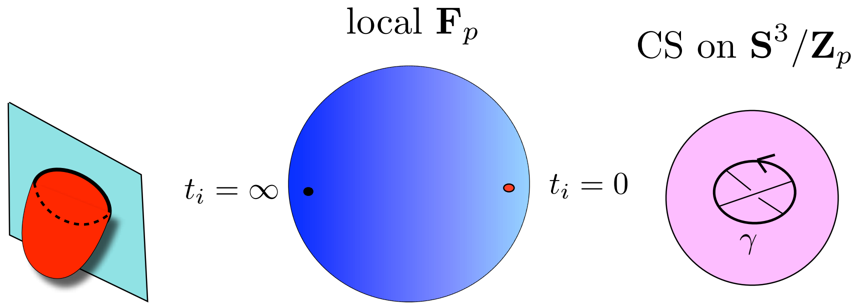
Open TS amplitudes *exact* in α' and the open moduli (previously unavailable). We can then probe *small distances* in toric CYs with background D-branes, for example compute correlators at *orbifold points* involving twist fields+boundary preserving operators [BKMP 08]



This generalizes [ABK] and can be tested against explicit calculations of open orbifold GW invariants! [Cavalieri]

Application/test III

CY background	large N CS dual
orbifold point	perturbative gauge theory
open TS amplitudes	Wilson loop vevs



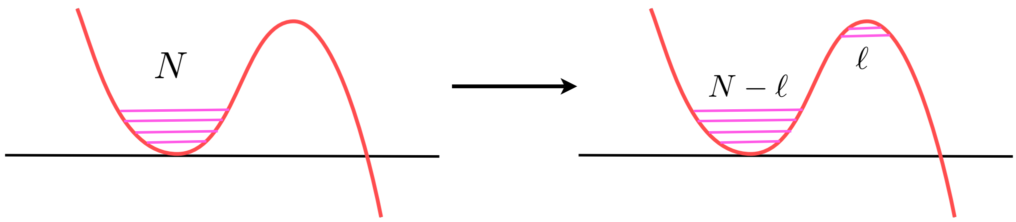
$$\langle W_R \rangle = \sum_{d_i} a_{d_i,R}\, \mathrm{e}^{-\sum_i d_i t_i} \qquad \langle W_R \rangle = \langle \mathrm{Tr}_R U_\gamma \rangle_{\mathrm{CS}}$$

nontrivial modular transformation

Explicit strong/weak 't Hooft coupling interpolation with nontrivial modular properties

Nonperturbative effects in matrix models

The partition function of a MM has nonperturbative corrections: eigenvalue tunneling / sum over all filling fractions



$$Z(t,g_s) = Z^{\mathrm{p}}(t,g_s) + \sum_{\ell \geq 1} \mathrm{e}^{-\ell A_{\mathrm{st}}(t)/g_s} C^\ell Z^{(\ell)}(t,g_s)$$

C: non-perturbative ambiguity; corresponds to a choice of contour in the matrix integral and plays the role of a theta angle [David]

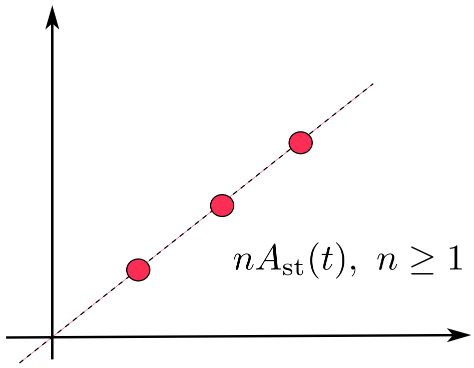
These effects have been much studied in the double-scaling limit, where they become *spacetime instantons of noncritical strings* due to ZZ branes. We studied them in the *full* matrix model in [M.M.-Schiappa-Weiss, M.M]. In particular they can be computed using the spectral curve only.

Nonperturbative effects and topological strings I

These effects are *testable* through the connection instantons/large order behavior:

$$F_g(t) \sim (2g)!(A_{\rm st}(t))^{-2g}$$

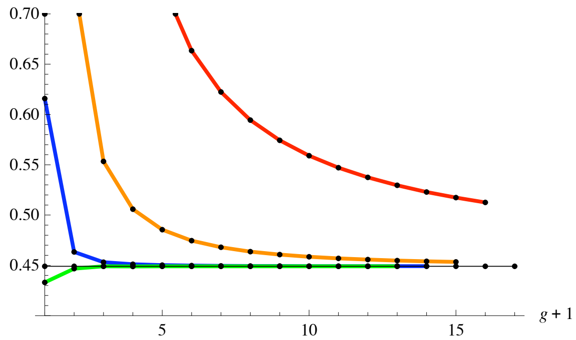
$\longrightarrow$ nontrivial information on the analytic structure of $F(t, g_s)$ as a function of g_s i.e. singularities in the Borel plane



Since we have a MM-like description of topological strings in terms of a spectral (mirror) curve, we can explicitly compute nonperturbative effects of this type in some toric backgrounds. They should correspond to *generalizations of ZZ branes to toric CYs*

Nonperturbative effects and topological strings II

We have computed the perturbative amplitudes and the nonperturbative effects for topological strings on $\mathcal{O}(-p) + \mathcal{O}(p-2) \to \mathbf{P}^1$ and its limit $p \to \infty$ (closely related to chiral 2d YM). One can do a precision test of the instanton/large order connection



$2g(F_g(t)/F_{g+1}(t))^{\frac{1}{2}}$ and its Richardson transforms for a fixed value of t, as compared to the instanton action (horizontal line)

From the point of view of type II string, these instanton effects correspond to *domain walls* [as already pointed out in D&V]

Large N duals and nonperturbative effects

In general, the TS partition function corresponds to a *fixed*, generic filling fraction of the matrix model (a fixed instanton background). In models with large N CS duals, this corresponds to expanding around a gauge theory instanton.

fix a background

$$Z_{\rm MM}(N_1, \cdots, N_p) = \underbrace{Z_{\rm TS}(t_1, \cdots, t_p)}_{\text{TS partition function}}$$

Therefore, the *total* partition function of the gauge theory -which is the natural nonperturbative object- equals the sum over *all* instanton sectors of the MM and *forces us* to include these sectors.

sum all backgrounds

$$Z_{\rm CS}(\mathbf{S}^3/\mathbf{Z}_p; N, g_s) = \sum_{N_1+\cdots+N_p=N} C_1^{N_1} \cdots C_p^{N_p} Z_{\rm MM}(N_1, \cdots, N_p)$$

Is this sum over all backgrounds the natural way to get background independent TS partition functions? [cf. Eynard 08]

Conclusions and open problems

- Toric case: there is a *generalization of special geometry* to higher genus by using residue calculus on the mirror manifold. Is it possible to extend it to the compact case?

- We obtained a precise understanding of *nonperturbative effects* in some toric backgrounds. But we need to develop instanton calculus in more general MM/TS theories. Worldsheet interpretation of these effects: a "special" D-brane? Type II interpretation?

- Large N duals force us to sum over all instanton sectors of MM. Can we use this sum to define background independent TS partition functions?