Cosmological Unification of String Theories

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based on:
hep-th/0611317, S.H. and Ian Swanson
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arXiv:0709.2166, S.H. and Ian Swanson
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and work in progress


Outline

Bosonic string solutions with nonzero tachyon

Dimension-changing solutions in the bosonic string

Transitions from type 0 to type II string theory

Lightlike tachyon condensation in Type 0

Other examples

Conclusions
Introduction

We understand THIS very well:

We also understand perturbations by weak SUSY breaking –
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- Anti-D-branes
- Fluxes
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- Fluxes

We also understand perturbations by weak SUSY breaking –

We don't yet understand strong SUSY breaking very well!

Introduction

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- Understand cosmology and strong time dependence
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- Understand connections between string theories – each of the possible $10^{500}$ universes should contain all the others!
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GOALS:
▶ Understand cosmology and strong time dependence
▶ Understand strong SUSY breaking
▶ Understand connections between string theories – each of the possible $10^{500}$ universes should contain all the others!

Each of these questions is related to the others – if we understand one, we will understand all of them.

What are the simplest concrete models of connections and transitions?
What I will describe today

- A class of $\alpha'$-exact classical solutions of string theory
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- With D not necessarily equal to 10 or 26

- D changes dynamically
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- A class of $\alpha'$-exact classical solutions of string theory
- With $D$ not necessarily equal to $10$ or $26$
- $D$ changes dynamically
- Spacetime SUSY changes dynamically

Worldsheet SUSY changes dynamically
What I will describe today

- A class of α’-exact classical solutions of string theory
- With D not necessarily equal to 10 or 26
- D changes dynamically
- Spacetime SUSY changes dynamically
- Worldsheet SUSY changes dynamically
- All involve closed string tachyon condensation, treated exactly in α’ and in the tachyon strength.

THIS TALK IS A CONJECTURE-FREE ZONE!
Analogy with open string tachyon dynamics

D-brane decay

Solitons
Parallel with Open String Tachyon Dynamics

An analogy arises for the (bosonic) closed string tachyon, representing an instability of spacetime itself.

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Tachyonic perturbations

We are interested in tachyonic perturbations of unstable string theories. In the bosonic string, for instance, the tachyon $T(X)$ couples to the worldsheet as a normal-ordered potential $:T(X):$.

We will now discuss a large class of solvable and exactly marginal perturbations of this form.
Consider a theory with stress tensor

\[
T_{++} = -\frac{1}{\alpha'} : \partial_{\sigma^+} X^\mu \partial_{\sigma^+} X_\mu : + \partial_{\sigma^+}^2 (V_\mu X^\mu)
\]

\[
T_{--} = -\frac{1}{\alpha'} : \partial_{\sigma^-} X^\mu \partial_{\sigma^-} X_\mu : + \partial_{\sigma^-}^2 (V_\mu X^\mu)
\]

where colons represent normal ordering of the 2D theory. Here, \(\sigma^\pm\) are particular light-cone combinations of the worldsheet coordinates \(\sigma^{0,1}\):

\[
\sigma^\pm = -\sigma^0 \pm \sigma^1
\]

**Physical states of the string** correspond to local operators \(\mathcal{U}\) that are Virasoro primaries of weight one. That is, their operator product expansion (OPE) with the stress tensor satisfies:

\[
T_{++}(\sigma)\mathcal{U}(\tau) \sim \frac{\mathcal{U}(\tau)}{(\sigma^+ - \tau^+)^2} + \frac{\partial_+ \mathcal{U}(\tau)}{\sigma^+ - \tau^+}
\]

and similarly for \(T_{--}\).
A profile $\mathcal{T}(X)$ for the tachyon corresponds to the vertex operator

$$\mathcal{U}_M \equiv: \mathcal{T}(X):$$

and admits the following on-shell condition:

$$\partial_\mu \partial^\mu \mathcal{T}(X) - 2 V^\mu \partial_\mu \mathcal{T}(X) + \frac{4}{\alpha'} \mathcal{T}(X) = 0$$

For tachyon profiles of the form

$$\mathcal{T}(X) = \mu^2 \exp(B_\mu X^\mu)$$

this condition is

$$B^2 - 2V \cdot B = -4/\alpha'$$
Bubble of nothing

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A general value of $B_\mu$ will lead to a nontrivial interacting theory when the strength $\mu^2$ of the perturbation is treated as non-infinitesimal.

Bubble of nothing

There is a special set of choices for $B_\mu$ that renders the 2D theory well-defined and conformal to all orders in perturbation theory.
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We choose the first term in the linearized tachyon equation of motion to vanish separately.

This is tantamount to choosing the vector $B_\mu$ to be null. This renders the vertex operator $\exp (B_\mu X^\mu)$ non-singular in the vicinity of itself.
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This is tantamount to choosing the vector $B_\mu$ to be null. This renders the vertex operator $\exp(B_\mu X^\mu)$: non-singular in the vicinity of itself.

We therefore put $B_\mu$ in the form

$$B_0 = B_1 \equiv \beta/\sqrt{2}$$
$$B_i = 0, \quad i \geq 2$$

Bubble of nothing

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This gives rise to a particularly simple quantum theory. The kinetic term for $X^\pm$ appears as

$$\mathcal{L} \sim - \frac{1}{2\pi \alpha'} \left[ (\partial_{\sigma^0} X^+)(\partial_{\sigma^0} X^-) - (\partial_{\sigma^1} X^+)(\partial_{\sigma^1} X^-) \right]$$

The propagator for the $X^\pm$ fields is therefore oriented.
Bubble of nothing

- The X field has oriented propagators.
- All the interaction vertices in the theory depend only on $X^+$.
- There are no non-trivial Feynman diagrams in the theory.
- This constitutes an interacting quantum theory, \textit{without quantum corrections}.

(In conformal gauge, prior to enforcing gauge constraints, the theory is not unitary.)
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- There are no non-trivial Feynman diagrams in the theory.
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The tachyon couples to the worldsheet in the term

$$\mathcal{L} \sim - \frac{1}{2\pi} \mu^2 \exp(\beta X^+)$$

Classically, $X^+$ is harmonic, and acts as a source for $X^-$. 
Bubble of nothing

By writing the solution to the Laplace equation for $X^+$ as

$$X^+ = f_+(\sigma^+) + f_-(\sigma^-)$$

the general solution for $X^-$ can be expressed as follows:

$$X^- = g_+(\sigma^+) + g_-(\sigma^-) + \frac{\alpha' \beta \mu^2}{4} \left[ \int_{\sigma^+}^{\infty} dy^+ \exp (\beta f_+(y^+)) \right] \left[ \int_{\sigma^-}^{\infty} dy^- \exp (\beta f_-(y^-)) \right]$$

We thus see that the theory is exactly solvable.
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We thus see that the theory is exactly solvable.

All interaction vertices in the theory depend only on $X^+$, and therefore correspond to diagrams composed strictly from outgoing lines:

Physical interpretation

The solution can be thought of as a phase boundary in spacetime between the $T = 0$ phase and the $T > 0$ phase.
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The spacetime picture is therefore a phase bubble expanding out from a nucleation point:

![Space-time diagram showing a phase bubble expanding from a nucleation point.](image)

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The state collides with the bubble wall and is forced out of the region with nonzero tachyon. (The solution has $\mu^2 = 1$, $\beta = 0.1$, and the trajectory corresponds to $p^+ = 3$, $H_\perp \equiv \frac{\alpha' p_i^2}{2} = 4$.)

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The solution can be thought of as a bubble of nothing.
Let’s now introduce some dependence on a third direction:

\[ T(X^+, X_2) = + \frac{\mu^2}{2 \alpha'} \exp(\beta X^+) \delta X_2^2 : + T_0(X^+) \]

\[ T_0(X^+) = \frac{\mu^2 X^+}{\alpha' q \sqrt{2}} \exp(\beta X^+) + \mu^2 \exp(\beta X^+) \]

Dimension-changing solutions in the bosonic string

States with modes of \( X_2 \) excited are pushed out along the bubble wall: the physics is essentially the same as the bubble of nothing.
Dimension-changing solutions in the bosonic string

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So these string states are pushed out to infinity and disappear from the theory in the late-time limit:

![Diagram showing bubble interior and flat space with states with $X_2$ excited]

Dimension-changing solutions in the bosonic string

There is a less generic class of states with no energy in the $X_2$ direction.
Dimension-changing solutions in the bosonic string

There is a less generic class of states with no energy in the $X_2$ direction.

These propagate \textit{through the domain wall and into the bubble region}.

The result is that the amount of matter on the worldsheet decreases dynamically as a function of time.
Dimension-changing solutions in the bosonic string

In other words, the number of dimensions in the target space decreases as a function of time.

Question: What happens to the central charge if the spacetime dimension shifts? How can the perturbation be marginal?
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Question: What happens to the central charge if the spacetime dimension shifts? How can the perturbation be marginal?

The theory is solvable, so we should be able to answer this question exactly.

In fact, quantum corrections in this theory truncate at one-loop order:
Dimension-changing solutions in the bosonic string

The one-loop diagrams can be thought of as a set of effective vertices for $X^+$, associated with integrating out the massive field $X_2$.

In fact: in the far future, all corrections coming from integrating out $X_2$ decay away, except for three contributions:

- the effective tachyon,
- the dilaton,
- the string-frame metric.
Dimension-changing solutions in the bosonic string

The remaining contributions are always nonzero, coming from the following diagrams:

\[ \Delta(\partial_+ \Phi) = \quad \\\n\Delta G_{++} = \quad \]

Write the renormalized dilaton gradient and string-frame metric as:

\[ \hat{V}_\mu \equiv V_\mu + \Delta V_\mu \]
\[ \hat{G}^{\mu \nu} \equiv G_{\mu \nu} + \Delta G_{\mu \nu} \]
In the $X^+ \to \infty$ limit, we therefore get

$$c^{\text{dilaton}} = 6\alpha' \hat{G}^{\mu\nu} \hat{V}_\mu \hat{V}_\nu = -(D - 26) + 1$$

The result is that the shift in central charge contribution from the dilaton precisely cancels the central charge shift due to the reduction in spacetime dimension.
Dimension-changing solutions in the bosonic string

This mechanism of **central charge transfer** works equally well when the tachyon has a quadratic minimum in several transverse directions:

\[
c_{\text{dilaton}} = 6\alpha' \hat{G}^{\mu\nu} \hat{V}_\mu \hat{V}_\nu = -6\alpha' q^2 + \frac{nq\beta\alpha'}{\sqrt{2}} - \frac{n\alpha'^2 q^2 \beta^2}{8} = -(D - 26) + n
\]

We can get rid of as many dimensions as we want.
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We can get rid of as many dimensions as we want.

We can even get down to \( D=2! \)

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- Lightlike tachyon condensation in Type 0
- Other examples
- Conclusions
Transitions from type 0 to type II string theory

The type 0 theory also has a rich structure of transitions – including dimension-reducing transitions of the type we have just discussed for the bosonic string. (We will not review these here.)

Let us consider some other types of transitions that the type 0 string can undergo, in a linear dilaton background.
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The type 0 theory also has a rich structure of transitions – including dimension-reducing transitions of the type we have just discussed for the bosonic string. (We will not review these here.)

Let us consider some other types of transitions that the type 0 string can undergo, in a linear dilaton background.

Instead of starting with type 0 on a smooth space, we can consider starting on a $\mathbb{Z}_2$ orbifold of flat space.

As an example, start with type 0 string theory in 12 dimensions, with one dimension orbifolded by a reflection:

$$ X^{11} \rightarrow - X^{11} $$
Transitions from type 0 to type II string theory

(For modular invariance, we can stipulate that this acts simultaneously as a chiral R parity, \((-1)^{F_{LW}}\).)

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That is, the orbifold symmetry acts on the worldsheet fields as:

\[
\begin{align*}
X^{0-10} : & \quad + \\
X^{11} : & \quad - \\
\tilde{G} : & \quad - \\
G : & \quad +
\end{align*}
\]

where \(G\) and \(\tilde{G}\) are the right- and left-moving worldsheet supercurrents.

Transitions from type 0 to type II string theory

The orbifold singularity has real codimension 1, with massless spacetime fermions propagating on the 10 + 1 dimensional fixed locus \(\{X^M = X^{0,\ldots,10}\}\).
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The boundary conditions at the orbifold force the tachyon \( T \) to vanish at \( X_{11} = 0 \):

\[
T(X^M, X^{11}) = -T(X^M, -X^{11})
\]
Transitions from type 0 to type II string theory

Starting with type 0 in this 12-dimensional geometry, we can consider a local tachyon profile which describes the behavior of a generic tachyon vev near the orbifold fixed locus:

\[ T = \mu \exp(\beta X^+) X_{10} X_{11}, \quad \beta q = \frac{\sqrt{2}}{\alpha'} \]

The tachyon couples to the worldsheet as a \((1, 1)\) superpotential:

\[ \mathcal{L}_{\text{int}} = \frac{i}{2\pi} \int d\theta_+ d\theta_- T(X) \]
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This gives rise to a potential and Yukawa term:

\[ \mathcal{L}_{\text{int}} = -\frac{\alpha' \mu^2}{8\pi} \exp (2\beta X^+) \cdot \left[ \left( X_{10}^2 + X_{11}^2 \right) + \frac{i\alpha' \mu}{4\pi} \left( \bar{\psi}^{10} \psi^{11} + \bar{\psi}^{11} \psi^{10} \right) \right] \]

Transitions from type 0 to type II string theory

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The orbifold symmetry $(-1)^{F_{lw}}$ acts on the remaining worldsheet fields as:

- $X^{0-10}$
- $\tilde{G}$
- $G$

We also have the generator $(-1)^{F_{w}}$ of the type 0 GSO projection. The product $(-1)^{F_{rw}} \equiv (-1)^{F_{w}} \cdot (-1)^{F_{lw}}$ acts as:

- $X^{0-9}$
- $\tilde{G}$
- $G$
Transitions from type 0 to type II string theory

We thus have the usual GSO projection of critical type II string theory. The worldsheet theory $X^+ \to \infty$ is therefore identical to the worldsheet theory of the type II superstring.

The background values of all fields are trivial, save for the dilaton, which has a lightlike gradient, rolling to weak coupling in the future.
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A type II background with flat string-frame metric and lightlike linear dilaton actually preserves sixteen Killing spinors.

Our final state is therefore a half-BPS vacuum of type II string theory.
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This exact solution establishes conclusively that the type 0 theory in supercritical dimensions can relax by tachyon condensation to a supersymmetric ground state in $D=10$!

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In all examples so far, the basic kind of string theory is unchanged between the initial and final configurations.

We now turn to a related model of lightlike tachyon condensation in type 0 string theory, where the tachyon depends only on $X^+$, and is independent of the $D - 2$ dimensions transverse to $X^\pm$. 
Lightlike tachyon condensation in type 0

In all examples so far, the basic kind of string theory is unchanged between the initial and final configurations.

We now turn to a related model of lightlike tachyon condensation in type 0 string theory, where the tachyon depends only on $X^+$, and is independent of the $D - 2$ dimensions transverse to $X^\pm$.

We start with the Lagrangian for a timelike linear dilaton theory on a flat worldsheet, describing $D$ free, massless fields and their superpartners:

$$L_{\text{kin}} = \frac{1}{2\pi} G_{MN} \left[ \frac{2}{\alpha'} (\partial_+ X^M)(\partial_- X^N) - i\psi^M (\partial_- \psi^N) - i\tilde{\psi}^M (\partial_+ \tilde{\psi}^N) \right]$$

Lightlike tachyon condensation in type 0

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Lightlike tachyon condensation in type 0

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We again take the simple form  

$$  \mathcal{T} \equiv \tilde{\mu} \exp(\beta X^+) $$

Lightlike tachyon condensation in type 0

Remember that the tachyon couples to the worldsheet as a $(1, 1)$ superpotential, giving rise to a worldsheet potential and Yukawa term:

$$  \mathcal{L}_{\text{int}} = -\frac{\alpha'}{8\pi} G^{MN} \partial_M T \partial_N T + \frac{i\alpha'}{4\pi} \partial_M \partial_N T \bar{\psi}^M \psi^N $$
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\]

We also get a modified supersymmetry transformation for the fermions:

\[
\{Q_-, \psi^M\} = -\{Q_+, \tilde{\psi}^M\} = F^M
\]

\[
F^M \equiv -\sqrt{\frac{\alpha'}{8}} G^{MN} \partial_N T
\]

Lightlike tachyon condensation in type 0

Since the gradient of the tachyon is null, the worldsheet potential

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is \text{ZERO}. 
Lightlike tachyon condensation in type 0

Since the gradient of the tachyon is null, the worldsheet potential
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is ZERO.

But there is a nonvanishing $F$-term and Yukawa coupling between the lightlike fermions:
\[
F^- = +\frac{q\sqrt{\alpha'}}{2} \exp (\beta X^+)
\]
\[ \mathcal{L}_{\text{Yukawa}} = \frac{i \mu}{4\pi} \exp (\beta X^+) \tilde{\psi}^+ \psi^+ \]
where $\mu \equiv \beta^2 \alpha' \tilde{\mu}$.

The 2D interaction terms become large as $X^+ \to +\infty$:
Lightlike tachyon condensation in type 0

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The 2D \textit{interaction terms} become \textit{large} as \( X^+ \to +\infty \):

We’ll have to \textit{deal with that}!

---

Lightlike tachyon condensation in type 0

There is no worldsheet potential, so all string states pass through the domain wall of the tachyon condensate.
Lightlike tachyon condensation in type 0

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Perform a canonical change of variables so that the new variables have weak interactions in the limit $X^+ \to \infty$:

$$\{\psi^+, \psi^-, \tilde{\psi}^+, \tilde{\psi}^-, X^\mu\} \Rightarrow \{b_1, c_1, \tilde{b}_1, \tilde{c}_1, X'^\mu\}$$
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where $b_1$ and $c_1$ are a new set of ghost variables, with weights $3/2$ and $-1/2$, and $c = -11$.

These have nothing to do with the Fadeev-Popov ghosts.
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where $b_1$ and $c_1$ are a new set of ghost variables, with weights $3/2$ and $-1/2$, and $c = -11$.

These have nothing to do with the Fadeev-Popov ghosts.

Also, twelve units of central charge are transferred from the light cone fermions $\psi^\pm$ to the dilaton gradient.

Deep inside the tachyon condensate
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The new Lagrangian is free, except for the interaction term

\[ \mathcal{L}_{\text{int}} = \mu^{-1} \exp \left( -\beta X^+ \right) \tilde{b}_1 b_1 \]

and becomes free in the limit \( X^+ \to \infty \)

What is this new theory?
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In the late time limit, the theory deep inside the tachyon condensate is formally type 0 string theory, but in actuality, it is precisely equal to bosonic string theory.

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In the late time limit, the theory deep inside the tachyon condensate is formally type 0 string theory, but in actuality, it is precisely equal to bosonic string theory.

In the natural variables of the late-time limit, the theory precisely realizes a well-known mechanism, originally found by Berkovits and Vafa.
The IR limit

The total final supercurrent $G \equiv G^{LC} + G^\perp$ in IR variables is

$$G = b_1 + ic'_1 b_1 c_1 - c_1 T^{\text{mat}} + c''_1 \left( -\frac{1}{6} c^\perp - \frac{1}{2} + \alpha' q^2 \right)$$

$$+ c_1 c'_1 c''_1 \left( -\frac{i}{4} \alpha' q^2 - \frac{i}{2} + \frac{i}{24} c^\perp \right)$$
The IR limit

The total transformed stress tensor is

$$T = T^\text{mat} + T^{b_1 c_1}$$

with

$$T^{b_1 c_1} = -\frac{3i}{2} \partial_+ c_1 b_1 - \frac{i}{2} c_1 \partial_+ b_1 + \frac{i}{2} \partial_+ (c_1 \partial_+^2 c_1)$$

Plugging in $q = \sqrt{\frac{D-10}{4\alpha'}}$ and $c^\perp = \frac{3}{2} (D - 2)$:

$$G = b_1 + ic_1 b_1 c_1 - c_1 T^\text{mat} - \frac{5}{2} c_1^\prime\prime$$
The IR limit

The $X^+ \to \infty$ limit of our solution is described by a free worldsheet theory, $D$ free scalars $X'^M$ and $D - 2$ free fermions $\psi'^i$.

The total central charge of the $X^M$, $\psi^i$ system is 26, and the contribution of $-11$ from the $b_1c_1$ system brings the total central charge to 15.
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The theory has critical central charge for a SCFT interpreted as the worldsheet theory of a RNS superstring in conformal gauge.

Berkovits-Vafa construction

This type of superconformal field theory is an embedding of the bosonic string in the solution space of the superstring. [hep-th/9310170]
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This type of superconformal field theory is an embedding of the bosonic string in the solution space of the superstring. [hep-th/9310170]

For a conformal field theory $T^{\text{mat}}$ with a central charge of 26, it is possible to construct a corresponding superconformal field theory defined by $G, T$ with central charge 15.

Upon treating the superconformal theory as a superstring theory, the resulting physical states and scattering amplitudes are identical to those of the theory defined by $T^{\text{mat}}$ when treated as a bosonic string theory.
Transition to bosonic string theory

To summarize, the transition follows an instability in an initial $D$-dimensional type 0 theory.

The dynamics then spontaneously break worldsheet supersymmetry, giving rise to a bosonic string theory in the same number of dimensions deep inside the tachyonic phase.
Outline

Bosonic string solutions with nonzero tachyon

Dimension-changing solutions in the bosonic string

Transitions from type 0 to type II string theory

Lightlike tachyon condensation in Type 0

Other examples

Conclusions

$E_8$ heterotic string theory

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Consider unstable 10D heterotic string theory with a single $E_8$ gauge group, realized as a current algebra at level two.

This theory has a single real tachyon $T$.

The endpoint of tachyon condensation in this theory has been a subject of much speculation.

[Hořava + Fabinger, 2000]

We consider the theory in the background of a lightlike linear dilaton

$$\phi = -\frac{q}{\sqrt{2}} X^-$$
$E_8$ heterotic string theory

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$$\Phi = -\frac{q}{\sqrt{2}} X^-$$

This theory admits several exact solutions describing dynamical tachyon condensation to different types of endpoints.

$E_8$ heterotic string theory

One exact solution (studied by Hořava and Keeler) describes a bubble of nothing similar to the one we described in the bosonic string. The form of the solution is

$$T = \mu \exp (\beta X^+)$$

with $q\beta = \frac{\sqrt{2}}{\alpha'}$.

[Hořava and Keeler, 2007]
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with $q\beta = \frac{\sqrt{2}}{\alpha'}$.

[Hořava and Keeler, 2007]

We found another interesting exact solution, of the form

$$T = \mu X_9 \exp (\beta X^+)$$

The endpoint of this solution can be analyzed exactly. The solution does not destroy the universe, but it does reduce the dimension of the spacetime.
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In this case, the endpoint is a previously unknown string theory that is interesting in its own right.

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- stable (tachyon-free)
- with spacelike linear dilaton
$E_8$ heterotic string theory

In this case, the endpoint is a previously unknown string theory that is interesting in its own right.

The final state lives in nine dimensions with the following properties:

- stable (tachyon-free)
- with spacelike linear dilaton
- with no moduli or other massless fields
- $E_8$ gauge symmetry left unbroken
The tachyonic $E_8$ string

The spectrum of the final theory is

$$Z_{\text{mass}}^\text{NS}(\tau) = (q\bar{q})^{+\frac{1}{16}} \left[ 1,785 + 108,500(q\bar{q})^{+\frac{1}{2}} + O \left( q\bar{q} \right) \right]$$

$$Z_{\text{mass}}^\text{R}(\tau) = 1,984 + 4,058,880(q\bar{q})^1 + O \left( (q\bar{q})^2 \right)$$

<table>
<thead>
<tr>
<th>theory</th>
<th>sector</th>
<th>mass</th>
<th>field content</th>
<th>mult.</th>
</tr>
</thead>
<tbody>
<tr>
<td>UHE</td>
<td>NS</td>
<td>$m^2 = -2/\alpha'$</td>
<td>$\mathcal{T}$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>NS</td>
<td>$m^2 = 0$</td>
<td>$\Phi(1) + G(35) + B(28) + A(1984)$</td>
<td>2048</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>$m^2 = 0$</td>
<td>$\Lambda_+ (1984) + \Lambda_- (1984)$</td>
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</tr>
<tr>
<td>HE9</td>
<td>NS</td>
<td>$m^2 = +1/(4\alpha')$</td>
<td>$\Phi(1) + \hat{G}(27) + \hat{B}(21) + \hat{A}(1736)$</td>
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There is tachyon-free, with no supersymmetry.

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\[ \mathcal{N} = 2 \text{ string theory} \]

Consider \( \mathcal{N} = 2 \) string theory.

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The simplest null holomorphic tachyon gives rise to a transition to bosonic string theory. At the endpoint, the theory is an analogue of the Berkovits-Vafa system discussed above.

Outline

- Bosonic string solutions with nonzero tachyon
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- Conclusions
A partial catalog of exact transitions

<table>
<thead>
<tr>
<th>start</th>
<th>$D_{init}$</th>
<th>$\exp(-\beta X^+T)$</th>
<th>end</th>
<th>$D_{fin}$</th>
<th>comments</th>
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<tbody>
<tr>
<td>bos</td>
<td>D</td>
<td>$\mu^2 X_2^2 + \mathcal{I}_0$</td>
<td>bos</td>
<td>D-1</td>
<td>tuned</td>
</tr>
<tr>
<td>0</td>
<td>D</td>
<td>$\mu X_2 X_3$</td>
<td>0</td>
<td>D-2</td>
<td>natural</td>
</tr>
<tr>
<td>0 (orb)</td>
<td>D</td>
<td>$\mu X_{i+1} Y_i$</td>
<td>II</td>
<td>10</td>
<td>stable</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>$\mu$</td>
<td>bos</td>
<td>D</td>
<td>tuned</td>
</tr>
<tr>
<td></td>
<td>+ 1/2 (D-2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UHE</td>
<td>10</td>
<td>$\mu X_2$</td>
<td>HE9</td>
<td>9</td>
<td>stable</td>
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<tr>
<td>HO($^{(+1)}$)</td>
<td>11</td>
<td>$\mu X_2$</td>
<td>HO</td>
<td>10</td>
<td>stable</td>
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The Big Picture – Part I
Conclusions

- Supercritical string theory has some surprising and interesting properties.

- We see that the supercritical string can be connected to the duality web of critical string theory.
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- We have found solutions that interpolate between superstring theory and purely bosonic string theory.
- The surprising feature of these connections is the crucial role of time dependence.
Conclusions

▶ Supercritical string theory has some surprising and interesting properties.

▶ We see that the supercritical string can be connected to the duality web of critical string theory.

▶ We have found solutions that interpolate between superstring theory and purely bosonic string theory.

▶ The surprising feature of these connections is the crucial role of time dependence.

▶ There may be other interesting links between theories that we have yet to discover.

▶ Thank you!
Implications for Phenomenology

The *phenomenological* implications of our models remain to be worked out –

*preliminary* results are *promising*!
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Details of the transformation

Rescale the $b_4$ field so that the new $b$ fermion appears in the supercurrent with unit normalization. To preserve all canonical commutators, however, we will rescale the $c_4$ field oppositely:

$$b_4 = \frac{2}{q\sqrt{\alpha'}} b_3 = \beta \sqrt{2\alpha'} b_3$$

$$c_4 = \frac{q\sqrt{\alpha'}}{2} c_3 = \frac{1}{\beta \sqrt{2\alpha'}} c_3$$

The IR limit

The invariance properties of the system under spatial reflection are still unclear.
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The stress tensor is invariant under the discrete symmetry reflecting the spacelike vector orthogonal to $\hat{V}_\mu$.

The supercurrent is not, however, since $V_\mu$ and $\Delta V_\mu$ appear independently in $G^{\text{LC}}$. 
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We would like to find field variables that render this discrete symmetry more manifest, such that only the vector $\hat{V}_\mu$ enters $G^\text{LC}$.

We therefore define new variables $b_2$, $c_2$, $Z^\mu$ by:

\[
Y^\pm = Z^\pm \pm \frac{i}{2\beta} c_2 \partial_+ c_2 \\
Y^\pm = Z^\pm \pm \frac{i}{2\beta} c_2 \partial_+ c_2 \\
b_3 = b_2 - \frac{2}{\beta\alpha'} (\partial_+ c_2) \left( \partial_+ Z^+ - \partial_+ Z^- \right) - \frac{1}{\beta\alpha'} c_2 \left( \partial_+^2 Z^+ - \partial_+^2 Z^- \right) \\
+ \frac{i}{2\beta^2\alpha'} c_2 (\partial_+ c_2) (\partial_+^2 c_2) \\
c_3 = c_2
\]
The IR limit

The worldsheet supersymmetry is now realized \textit{nonlinearly}.

The bosons $Z^\mu$ transform into their own derivatives, times a goldstone fermion:

$$[Q, Z^\mu] = ic_2 \partial_+ Z^\mu \quad \{Q, c_2\} = 1 + ic_2 \partial_+ c_2$$

where

$$Q \equiv \frac{1}{2\pi} \int d\sigma_1 \, G(\sigma)$$
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In the sector involving the transverse fields $X_i, \psi^i$, supersymmetry is realized in the usual linear fashion:

$$[Q, X_i] = i \sqrt{\frac{\alpha'}{2}} \psi^i$$

$$\{Q, \psi^i\} = \sqrt{\frac{2}{\alpha'}} \partial_+ X_i$$

The IR limit
At first sight, our realization of supersymmetry in the full theory is unfamiliar, with worldsheet supersymmetry realized linearly in one sector and nonlinearly in another.
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At first sight, our realization of supersymmetry in the full theory is unfamiliar, with worldsheet supersymmetry realized linearly in one sector and nonlinearly in another.

However, it turns out that this realization is equivalent to one for which worldsheet supersymmetry is realized completely nonlinearly in all sectors.

We now perform a final transformation on the system. Defining the Hermitian infinitesimal generator

\[ g \equiv -\frac{i}{2\pi} \int d\sigma_1 c_2(\sigma) G^\perp(\sigma) \]

we transform all operators in the theory according to

\[ \mathcal{O} \rightarrow U \mathcal{O} U^{-1} \]

with

\[ U \equiv \exp(ig) \]