Nonperturbative effect on the melting of Quarkonium states

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We have studied the quasi-free dissociation of quarkonia through a complex potential by correcting both the perturbative and nonperturbative terms of the $Q\bar{Q}$ potential at T=0 through the dielectric function in real-time formalism. The remnants of the non-perturbative confining force makes the real-part of the potential more stronger and the (magnitude) imaginary-part too becomes larger and thus contribute more to the thermal width. Thereafter we explore how does the presence of a not necessarily isotropic QCD medium modify both the real and imaginary part of potential, by calculating the leading anisotropic corrections to the propagators. We found that the presence of the anisotropy makes the real-part of the potential stronger but weakens the imaginary-part slightly, overall the anisotropy enhances the dissociation points higher, compared to isotropic medium.

Potential in hot QCD medium ($m_Q >> \Lambda_{\rm QCD}$, $T << m_Q$)

The central theme of our work is how the theoretical predictions based on high temperature methods, such as HTL perturbation theory might be modified by the remnants of the non-perturbative confining force, just above the crossover or transition temperature [1]?

The medium-modification to the $Q\bar{Q}$ potential at T=0 can be obtained by correcting its both short and long-distance part with a dielectric function, $\epsilon(p)$, through its Fourier transform, $V(p)$ [2, 3]:

$$V(r,T) \;\;=\;\; \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}}(e^{i\mathbf{p}\cdot\mathbf{r}}-1)\,\frac{V(p)}{\epsilon(p)}\;,\; V(p) = -\sqrt{(2/\pi)}\frac{\alpha}{p^2} - \frac{4\sigma}{\sqrt{2\pi}p^4}.$$

The dielectric permittivity: $\epsilon^{-1}(p) = -\lim_{\omega\to 0} p^2 D_{11}^{00}(\omega,p)$ will be obtained from the resummed propagators in HTl perturbation theory.

HTL Self-energies and Propagators

♦ Retarded self-energy [4] $\Pi^{ij}(P) = -g^2\int d^3\mathbf{k}\, v^i \frac{\partial f(\mathbf{k})}{\partial k^l}\left(\delta^{jl} + \frac{v^j p^l}{P\cdot V + i\epsilon}\right)$ which can be evaluated by the anisotropic phase-space distribution ($\xi \ll 1$) [5]: $f_{\rm aniso}(\mathbf{k}) = f_{\rm iso}\left(\sqrt{\mathbf{k}^2+\xi(\mathbf{k}.\mathbf{n})^2}\right)$.

♦ The retarded/advanced and symmetric self-energies contribute to the real-part and the imaginary-part of the self-energies, respectively. In Keldysh representation, the propagators and self-energies are:

$D_R^0 = D_{11}^0 - D_{12}^0$, $D_A^0 = D_{11}^0 - D_{21}^0$, $D_F^0 = D_{11}^0 + D_{22}^0$

$\Pi_R = \Pi_{11}+\Pi_{12},\ \Pi_A = \Pi_{11}+\Pi_{21},\ \Pi_F = \Pi_{11}+\Pi_{22}.$

Resummation Isotropic medium: $D^L_{R,A(iso)} = D^{L(0)}_{R,A} + D^{L(0)}_{R,A}\Pi^L_{R,A(iso)}D^L_{R,A(iso)}$

Anisotropic medium: $D^L_{R,A(aniso)} = D^{L(0)}_{R,A}\,\Pi^L_{R,A(aniso)}D^L_{R,A(iso)} + D^{L(0)}_{R,A}\,\Pi^L_{R,A(iso)}D^L_{R,A(aniso)}$

Retarded/Advanced self-energy

Isotropic part: $\Pi^L_{R,A(0)}(P) = m_D^2\left(\frac{p_0}{2p}\ln\frac{p_0+p\pm i\epsilon}{p_0-p\pm i\epsilon} - 1\right)$

Anisotropic part: $\Pi^L_{R,A(1)}(P) = \frac{m_D^2}{6}\left(1+\frac{3}{2}\cos(2\theta_p)\right) + \Pi^L_{R,A(0)}(P)\left[\cos(2\theta_p) - \frac{{p_0}^2}{2p^2}\times(1+3\cos(2\theta_p))\right]$

Symmetric self-energy

Isotropic part: $\Pi^L_{F(0)}(P) = -2\pi i m_D^2\frac{T}{p}\Theta(p^2-{p_0}^2)$

Anisotropic part:$\Pi^L_{F(1)}(P) = \frac{3}{2}\pi i m_D^2\frac{T}{p}\left(\sin^2\theta_p + (3\cos^2\theta_p - 1)\frac{{p_0}^2}{p^2}\right)\times\Theta(p^2-{p_0}^2)$

Propagators

Real Part: $\Re D^{00}_{R,A}(0,p) = -\frac{1}{(p^2+m_D^2)} + \xi\frac{m_D^2}{6(p^2+m_D^2)^2}\left(3\cos 2\theta_p - 1\right),$

Imaginary part: $\Im D^{00}_F(0,p) = \frac{-2\pi T m_D^2}{p(p^2+m_D^2)^2} + \xi\left(\frac{3\pi T m_D^2}{2p(p^2+m_D^2)^2}\sin^2\theta_p - \frac{4\pi T m_D^4}{p(p^2+m_D^2)^3}\left(\sin^2\theta_p - \frac{1}{3}\right)\right).$

A. Isotropic medium

♦ The medium-corrections to the potential at T=0 are always accompanied by both real and imaginary parts, where the real-part and imaginart-part cause the Debye screening and the Landau damping[7, 9], respectively (with $\hat{r} = rm_D$):

Real part: $\Re\, V_{(0)}(r,T) = \left(\frac{2\sigma}{m_D} - \alpha m_D\right)\frac{\exp(-\hat{r})}{\hat{r}} - \frac{2\sigma}{m_D\hat{r}} + \frac{2\sigma}{m_D} - \alpha m_D$

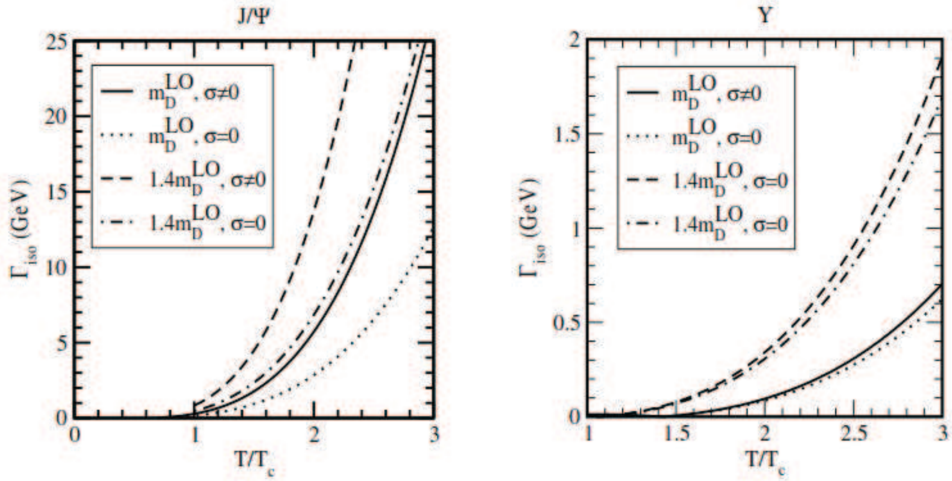
Imaginary part: originates from the static limit of symmetric self energy and plays an important role in weakening the bound state peak or transforming it to mere threshold enhancement and leads to a finite width (Γ) for the resonance peak in the spectral function, which, in turn, determines the dissociation temperature [8, 9, 10, 11, 12].

♦ Coulombic part : $\Im\, V_{1(iso)}(\mathbf{r},T) = -\alpha T\left[-\frac{\hat{r}^2}{9}\left(-4+3\gamma_E+3\log\hat{r}\right)\right]$

♦ String part: $\Im V_{2(iso)}(r,T) = \frac{2\sigma T}{m_D^2}\frac{\hat{r}^2}{6} + \frac{(-107+60\gamma_E+60\log(\hat{r}))\hat{r}^4}{3600} + O(\hat{r}^5).$

♦ **Imaginary part of the potential** $\Im\, V_{iso}(r,T) = -T\left(\alpha\frac{\hat{r}^2}{3} + + \frac{\sigma}{m_D^2}\frac{\hat{r}^4}{30}\right)\log(\frac{1}{\hat{r}})$

Decay Width: $\Gamma_{(\mathrm{iso})} = T\left(\frac{4}{\alpha m_Q^2} + \frac{12\sigma}{\alpha^2 m_Q^4}\right)\, m_D^2\,\log\frac{\alpha m_Q}{2m_D}$



$\Longrightarrow$ the remnant of confining (nonperturbative) string term enhances the thermal width of J/ψ and Υ states

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B. Potential in anisotropic medium

♦ How does the presence of a not necessarily isotropic QCD medium modify the potential (both the real and imaginary part) acting between a static quark and antiquark pair?

$$
\begin{aligned}
\bullet\text{CoulombicPart} : \Re V_1(\mathbf{r},\xi,T) &= -\alpha m_D\left[\frac{e^{-\hat{r}}}{\hat{r}}+1+\xi\left[\frac{(e^{-\hat{r}}-1)}{6}+e^{-\hat{r}}\left(\frac{1}{6}+\frac{1}{2\hat{r}}+\frac{1}{\hat{r}^2}\right)\right.\right.\\
&\left.\left.+\frac{(e^{-\hat{r}}-1)}{\hat{r}^3}\right](1-3\cos^2\theta_r)\right]\\
\text{StringPart} : \Re V_{2(aniso)}(\mathbf{r},\xi,T) &= \frac{2\sigma}{m_D}\left[\frac{(e^{-\hat{r}}-1)}{\hat{r}}+1+2\xi\left[\left(\left(\frac{(e^{-\hat{r}}-1)}{6\hat{r}}+\frac{e^{-\hat{r}}}{12}+\frac{1}{6}\right)\right.\right.\right.\\
&\left.\left.\left.+\left(e^{-\hat{r}}\left(\frac{1}{\hat{r}^2}+\frac{5}{12\hat{r}}+\frac{1}{12}\right)+\frac{1}{12\hat{r}}+\frac{(e^{-\hat{r}}-1)}{\hat{r}^3}\right)(1-3\cos^2\theta_r)\right]\right]\ .
\end{aligned}
$$

• becomes stronger with the increase of anisotropy because the (effective) Debye mass $m_D(\xi,T)$ in an anisotropic medium is always smaller than in an isotropic medium. In particular, the potential for quark pairs aligned in the direction of anisotropy are stronger than aligned in transverse direction.

Imaginary part

♦ Coulombic term: $ImV_{1(1)}(\mathbf{r},\xi,T)=(\alpha T\xi)\left[\psi_1^{(1)}(\hat{r},\theta_r)+\psi_1^{(2)}(\hat{r},\theta_r)\right]$,

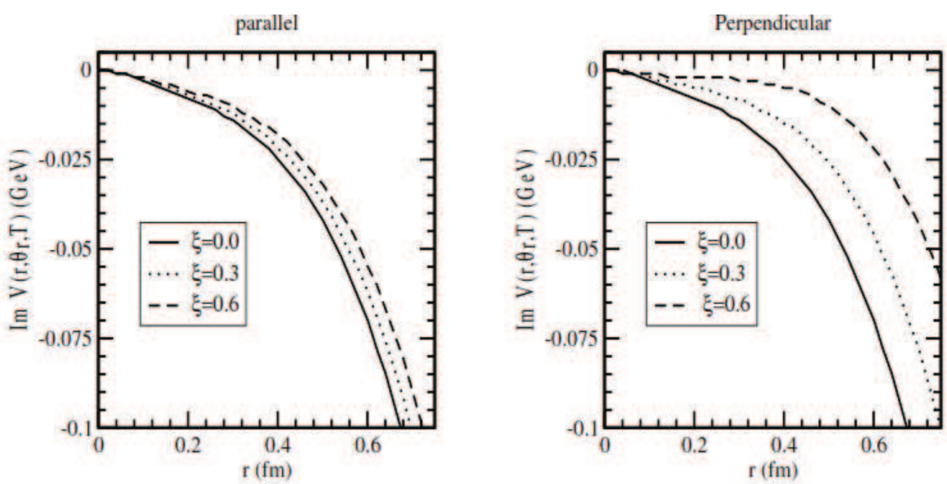
$$
\begin{aligned}
\psi_1^{(1)}(\hat{r},\theta_r) &= \frac{\hat{r}^2}{600}\left[123-90\gamma_E-90\log\hat{r}+\cos(2\theta_r)\left(-31+30\gamma_E+30\log\hat{r}\right)\right]\\
\psi_1^{(2)}(\hat{r},\theta_r) &= \frac{\hat{r}^2}{90}(-4+3\cos(2\theta_r))
\end{aligned}
$$

♦ Anisotropic part in string term:

$$
\begin{aligned}
ImV_{2(1)}(r,\xi,T) &= -\frac{2\sigma T}{m_D^2}\xi\left[\psi_2^{(1)}(\hat{r},\theta)+\psi_2^{(2)}(\hat{r},\theta)\right] \\
\psi_2^{(1)}(\hat{r},\theta) &= \frac{r^2}{10}+\frac{(-739+420\gamma_E+420\log(\hat{r}))\hat{r}^4}{39200} \\
&+ \left(-\frac{r^2}{20}+\frac{(176-105\gamma_E-105\log(\hat{r}))\hat{r}^4}{14700})\right)\cos^2\theta_r \\
\psi_2^{(2)}(\hat{r},\theta) &= -\frac{4}{3}\left[\frac{7\hat{r}^2}{120}-\frac{11\hat{r}^4}{3360}+O(\hat{r}^5)\right]-4\left[-\frac{\hat{r}^2}{60}+\frac{\hat{r}^4}{840}+O(\hat{r}^5)\right]\cos^2\theta_r
\end{aligned}
$$

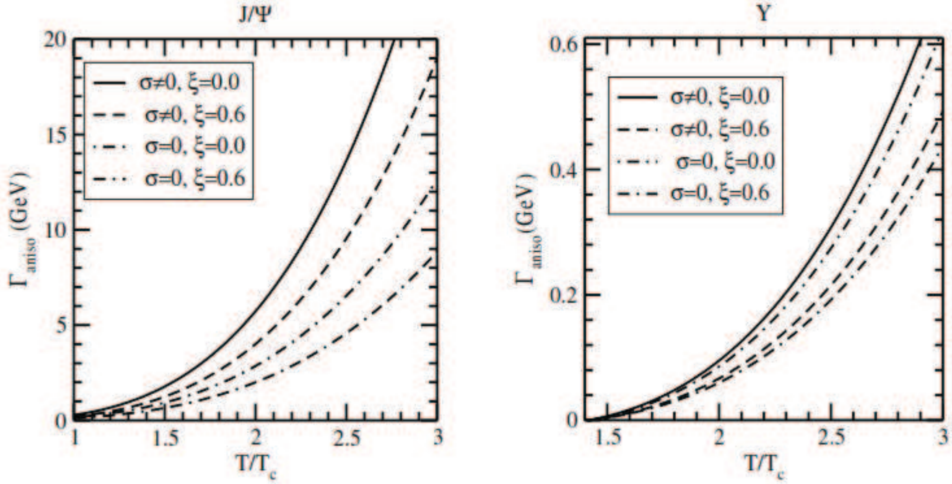
- **Imaginary potential in anisotropic medium**

$$
\begin{aligned}
ImV(r,\xi,T) = -\alpha T\hat{r}^2\log(\frac{1}{\hat{r}})\left(\frac{1}{3}-\xi\frac{3-\cos 2\theta}{20}\right) \\
-\frac{2\sigma T}{m_D^2}\hat{r}^4\log(\frac{1}{\hat{r}})\frac{1}{20}\left(\frac{1}{3}-\xi\frac{2-\cos 2\theta}{14}\right)
\end{aligned}
$$



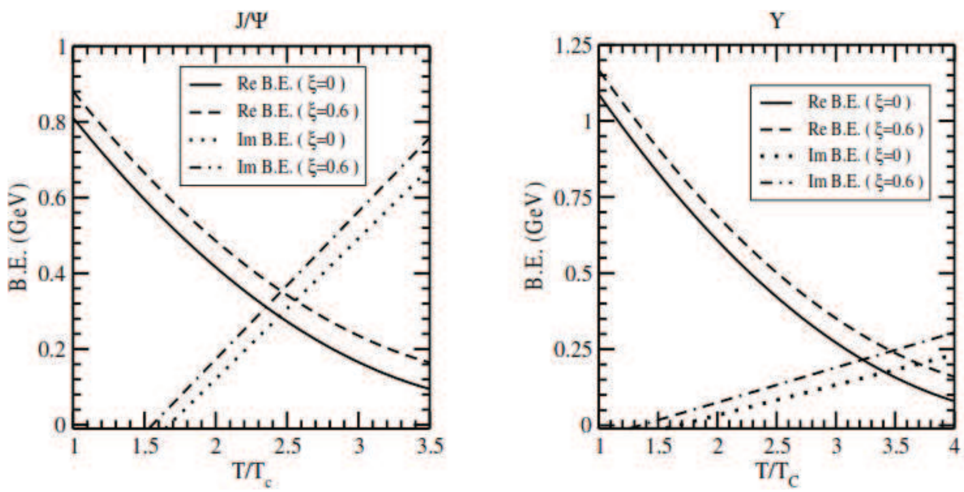
♦ **Decay width**[13]

$$
\begin{aligned}
\Gamma_{\rm aniso} &= \int d^3\mathbf{r}|\Psi(r)|^2\left[\alpha T\hat{r}^2\log(\frac{1}{\hat{r}})\left(\frac{1}{3}-\xi\frac{3-\cos 2\theta}{20}\right)+\frac{2\sigma T}{m_D^2}\hat{r}^4\log(\frac{1}{\hat{r}})\frac{1}{20}\left(\frac{1}{3}-\xi\frac{2-\cos 2\theta}{14}\right)\right] \\
&= \left(\frac{4Tm_D^2}{\alpha m_Q^2}+\frac{12\sigma T m_D^2}{\alpha^2 m_Q^4}\right)\left(1-\frac{\xi}{2}\right)\log\frac{\alpha m_Q}{2m_D}
\end{aligned}
$$



• becomes smaller than in isotropic medium and gets narrower with the increase of anisotropy because Γ is proportional to the (square) Debye mass, which decreases in the anisotropic medium due to the effective reduction of the local parton density around a test (heavy) quark compared to isotropic medium.

Real and Imaginary Binding Energies:



• the inclusion of nonperturbative string term makes the quarkonium states more bound in the anisotropic medium too, so the binding of $Q\bar{Q}$ pairs becomes stronger with respect to their isotropic counterpart and increases with the increase of anisotropy because the (real part) potential becomes deeper due to the weaker screening. In contrast to the real part, the imaginary part of the binding energy increases with the temperature but decreases with the anisotropy.

Dissociation temperatures have been calculated both from the intersections of the real and imaginary binding energies [14, 15], and from the criterion on the width of the resonance: $\Gamma \geq 2\mathrm{Re\ B.E.}$ [16] (lattice Debye mass (left) and leading-order (right)).

Method	State	$\xi = 0.0$	$\xi = 0.3$	$\xi = 0.6$
Re B.E.=Im B.E.	J/ψ	1.33	1.34	1.35
	Υ	1.91	1.93	1.94
Γ=2B.E.	J/ψ	1.02	1.06	1.12
	Υ	1.88	1.92	2.02

Method	State	$\xi = 0.0$	$\xi = 0.3$	$\xi = 0.6$
Re B.E.=Im B.E.	J/ψ	2.45	2.46	2.47
	Υ	3.40	3.45	3.46
Γ=2B.E.	J/ψ	1.40	1.46	1.54
	Υ	3.10	3.17	3.26

Conclusion

Isotropic medium: inclusion of the linear/string term, in addition to the Coulomb term, makes the real part of the potential more attractive. So, as a consequence the quarkonium states become more bound compared to the medium modification to the Coulomb term alone. Moreover the string term affects the imaginary part too where its magnitude is increased by the string contribution. As a result, the (thermal) width of the states are broadened due to the presence of string term and makes the competition between the screening and the broadening due to damping interesting and plays an important role in the dissociation mechanism. We have found that the quarkonium states are dissociated at higher temperature compared to the medium-consideration of the Coulomb term only.

Anisotropic Medium The anisotropy behaves as an additional handle to decipher the properties of quarkonium states, namely, the binding of $Q\bar{Q}$ pairs gets stronger with respect to their isotropic counterpart because both the real and imaginary part of the complex potential becomes deeper with the increase of anisotropy because the (effective) Debye mass becomes smaller than in isotropic medium and as a result the screening of the Coulomb and string contributions is less accentuated.

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