Initial state in relativistic nuclear collisions and Color Glass Condensate

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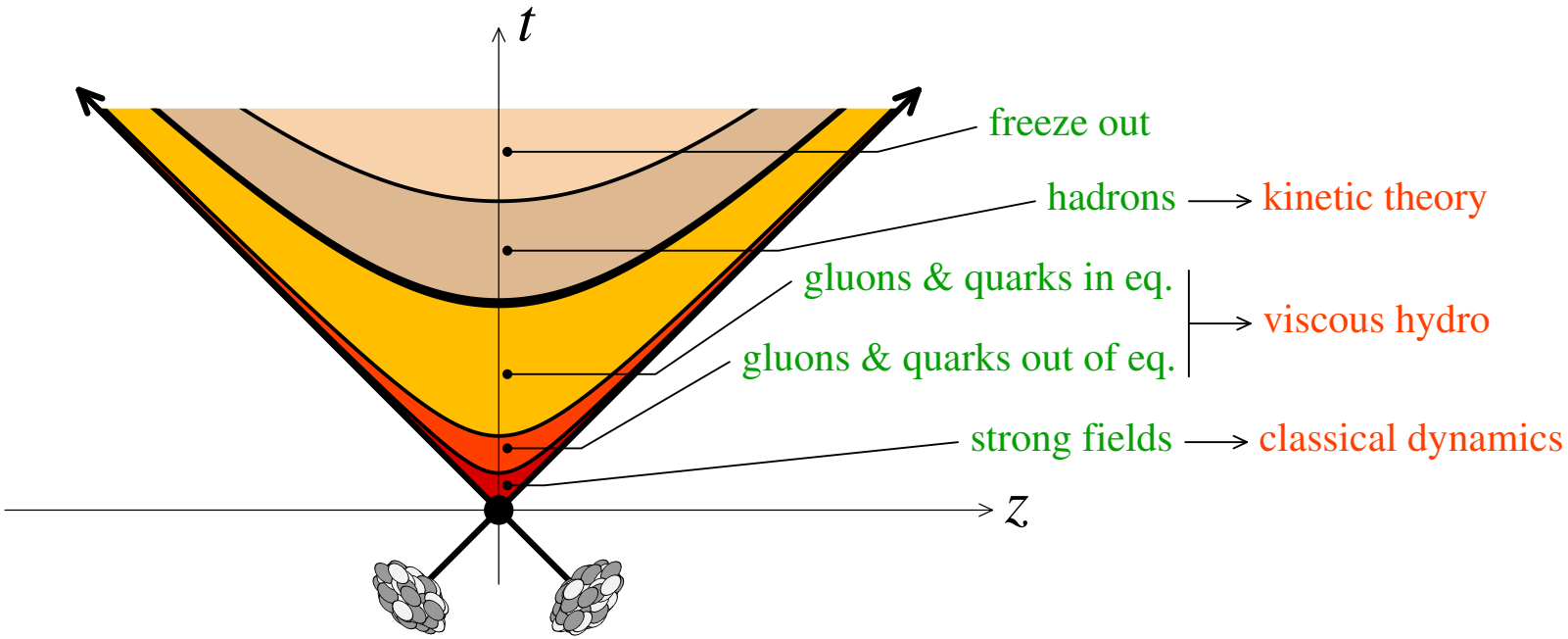
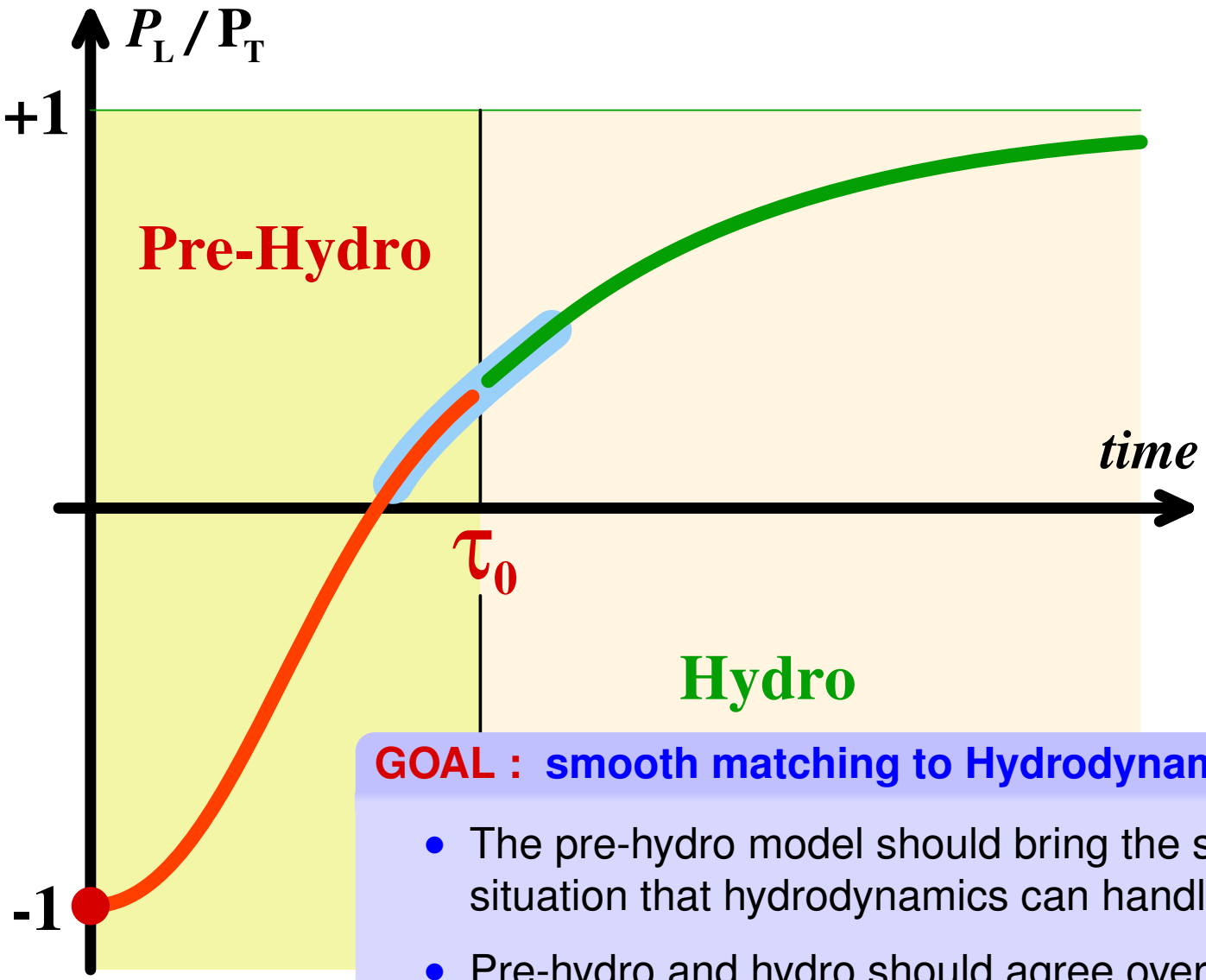
Stages of a nucleus-nucleus collision



- Hydrodynamics successful at describing the bulk evolution
- In this talk : **Initial state** $\Leftrightarrow$ **Pre-hydro evolution**
- What I will not talk about : CGC in pA and pp collisions
  [see the talks by Venugopalan, Dusling, Tribedy, Schenke]



GOAL : smooth matching to Hydrodynamics

- The pre-hydro model should bring the system to a situation that hydrodynamics can handle
- Pre-hydro and hydro should agree over some range of time $\Rightarrow$ no τ_0 dependence
- NOTE : the anisotropic hydro model of Ryblewski, Strickland et al. may help for a matching at small τ_0

Terminology

- Weakly coupled : $g \ll 1$
- Strongly coupled : $g \gg 1$

- Weakly interacting : $g\mathcal{A} \ll 1 \qquad g^2 f(\mathbf{p}) \ll 1$

  $(2 \to 2) \gg (2 \to 3), (3 \to 2), \cdots$

- Strongly interacting : $g\mathcal{A} \sim 1 \qquad g^2 f(\mathbf{p}) \sim 1$

  $(2 \to 2) \sim (2 \to 3) \sim (3 \to 2) \sim \cdots$

Terminology

- Weakly coupled : $g \ll 1$
- Strongly

Strongly coupled	$\Rightarrow$	Strongly interacting
Weakly coupled	$\not\Rightarrow$	Weakly interacting

- Weakly interacting : $g\mathcal{A} \ll 1 \qquad g^2 f(\mathbf{p}) \ll 1$

  $(2 \to 2) \gg (2 \to 3), (3 \to 2), \cdots$

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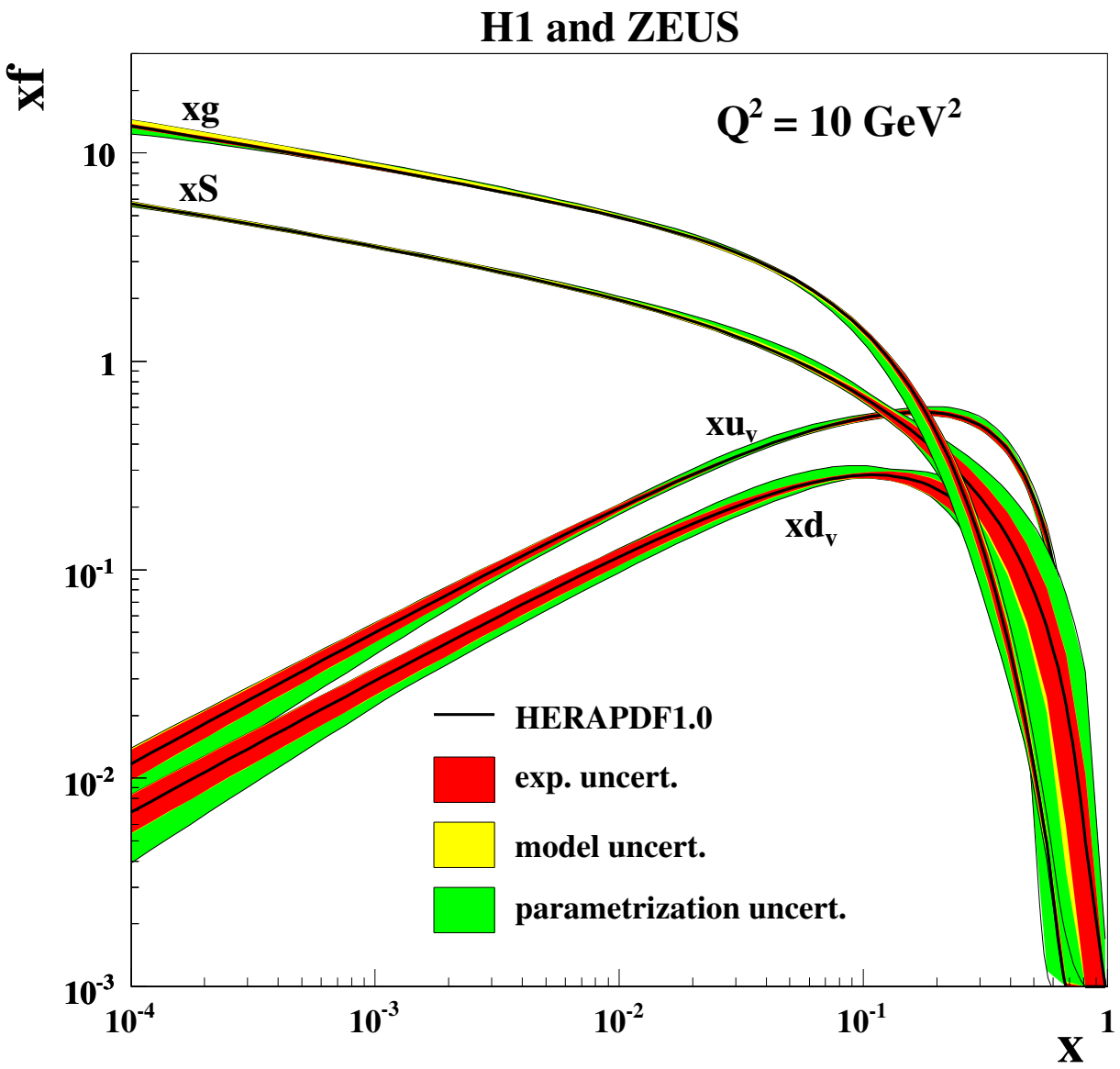
Outline

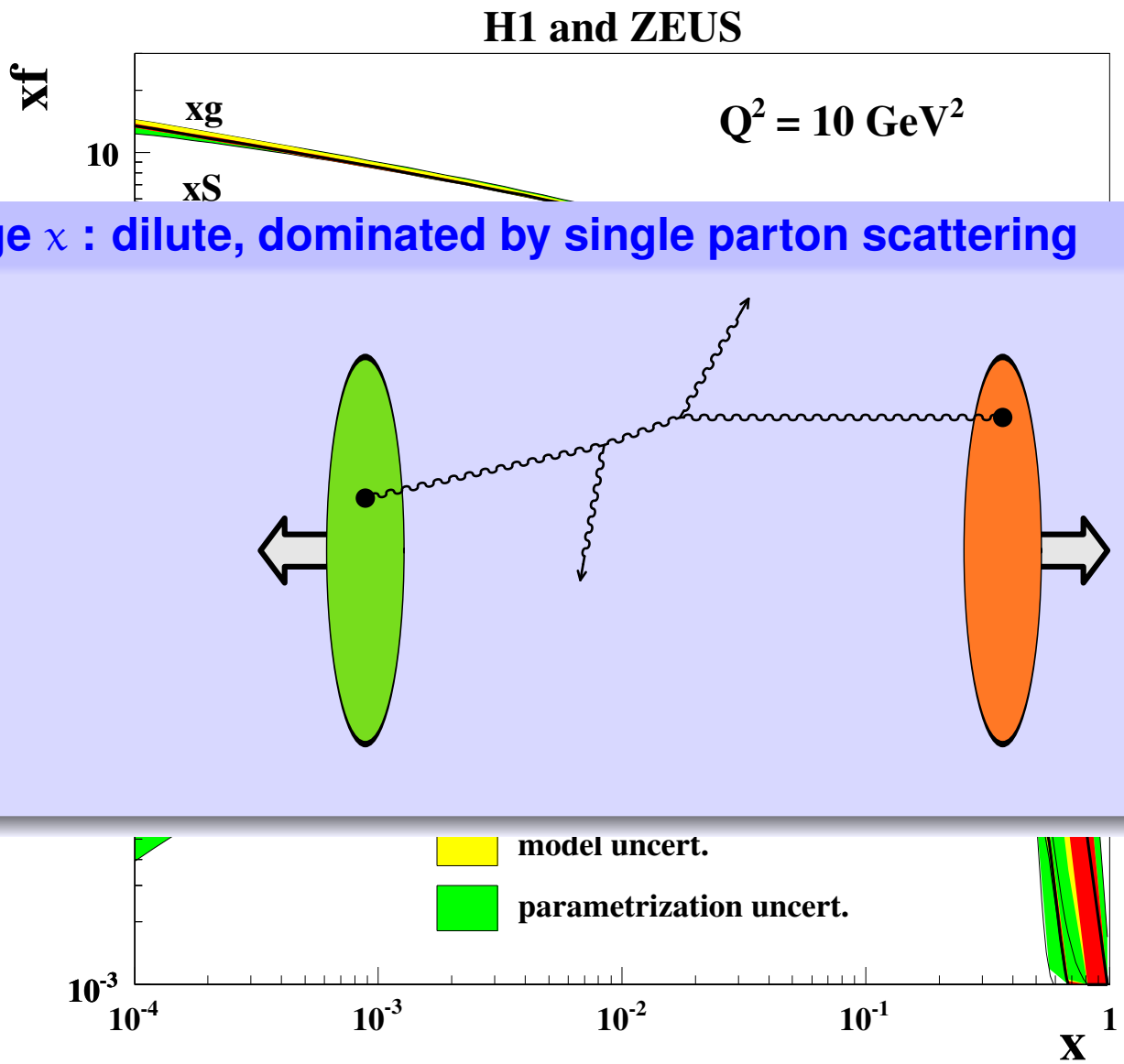
1. Initial state evolution and factorization
2. Pre-hydrodynamical evolution

Color Glass Condensate in Heavy Ion Collisions

Parton distributions in a nucleon

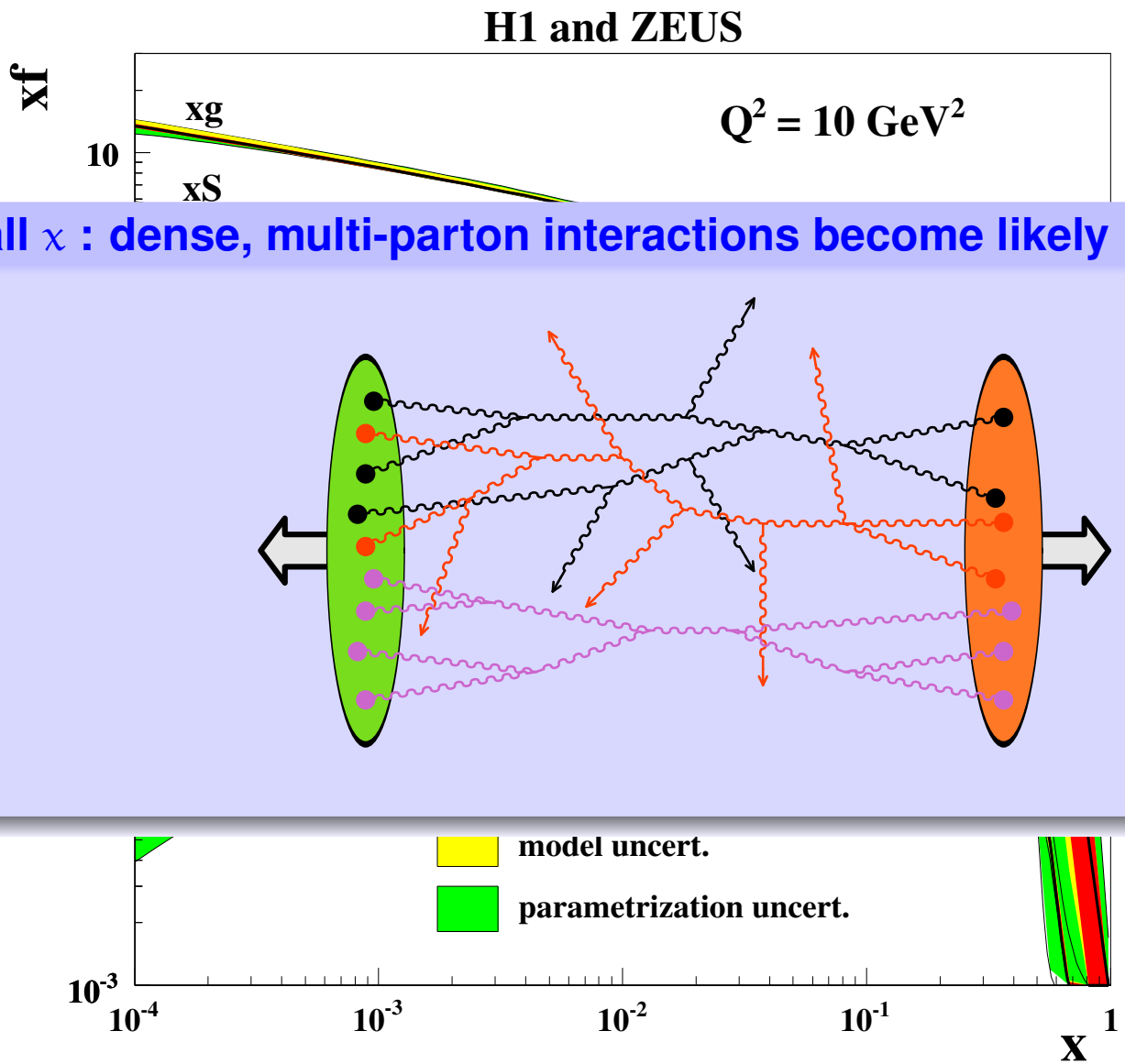


Parton distributions in a nucleon



Large x : dilute, dominated by single parton scattering

Parton distributions in a nucleon



Small x : dense, multi-parton interactions become likely

- When their occupation number becomes large, gluons can recombine :

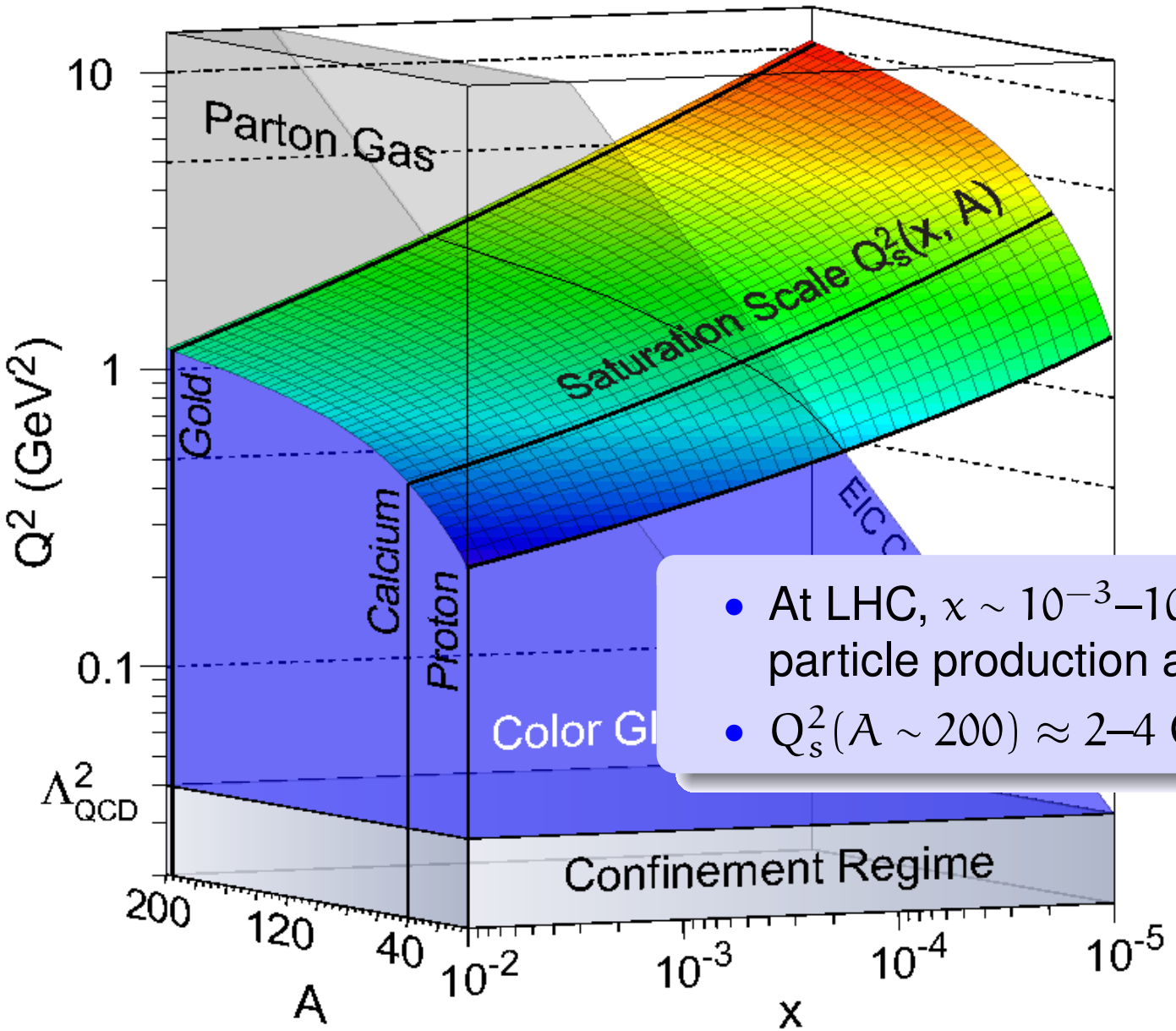
Gluon Saturation

Saturation criterion **[Gribov, Levin, Ryskin (1983)]**

$$\underbrace{\alpha_s Q^{-2}}_{\sigma_{g\,g\to g}} \times \underbrace{A^{-2/3} x G(x, Q^2)}_{\text{surface density}} \geq 1$$

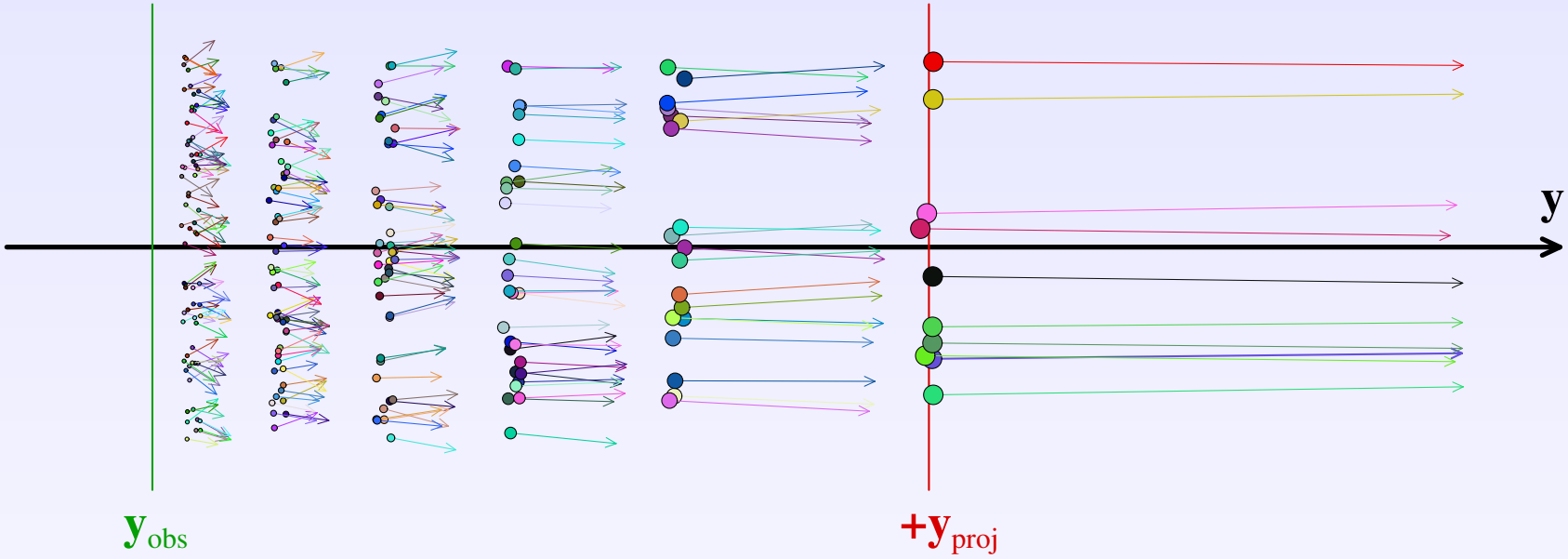
$$Q^2 \leq \underbrace{Q_s^2 \equiv \frac{\alpha_s x G(x, Q_s^2)}{A^{2/3}}}_{(\text{saturation momentum})^2} \sim A^{1/3} x^{-0.3}$$

Saturation domain



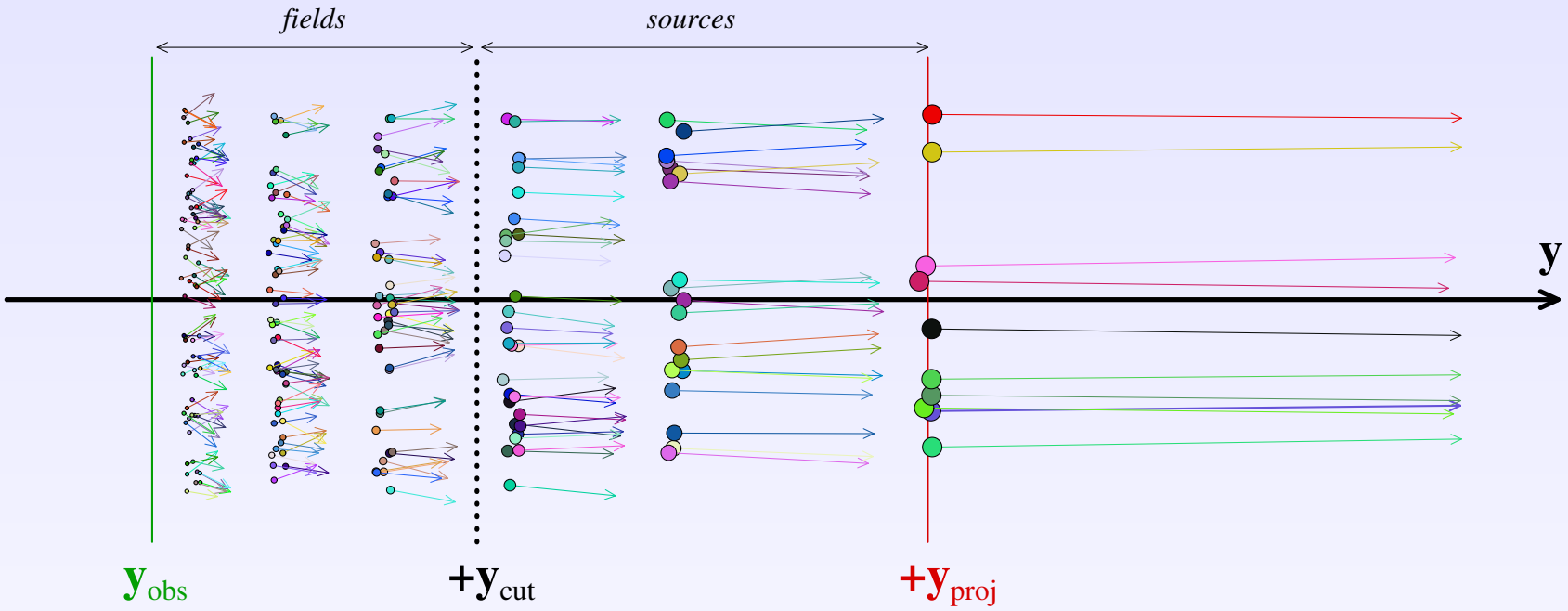
- At LHC, $x \sim 10^{-3}$–10^{-4} (for bulk particle production at mid-rapidity)
- $Q_s^2(A \sim 200) \approx 2$–$4$ GeV^2

Degrees of freedom [McLerran, Venugopalan (1994)]



- $p_\perp^2 \sim Q_s^2 \sim \Lambda_{QCD}\, e^{\lambda(y_{proj}-y)}$, $p_z \sim Q_s\, e^{y-y_{obs}}$
- Fast partons : frozen dynamics, negligible $p_\perp$ $\Rightarrow$ classical sources
- Slow partons : evolve with time $\Rightarrow$ gauge fields

Degrees of freedom [McLerran, Venugopalan (1994)]



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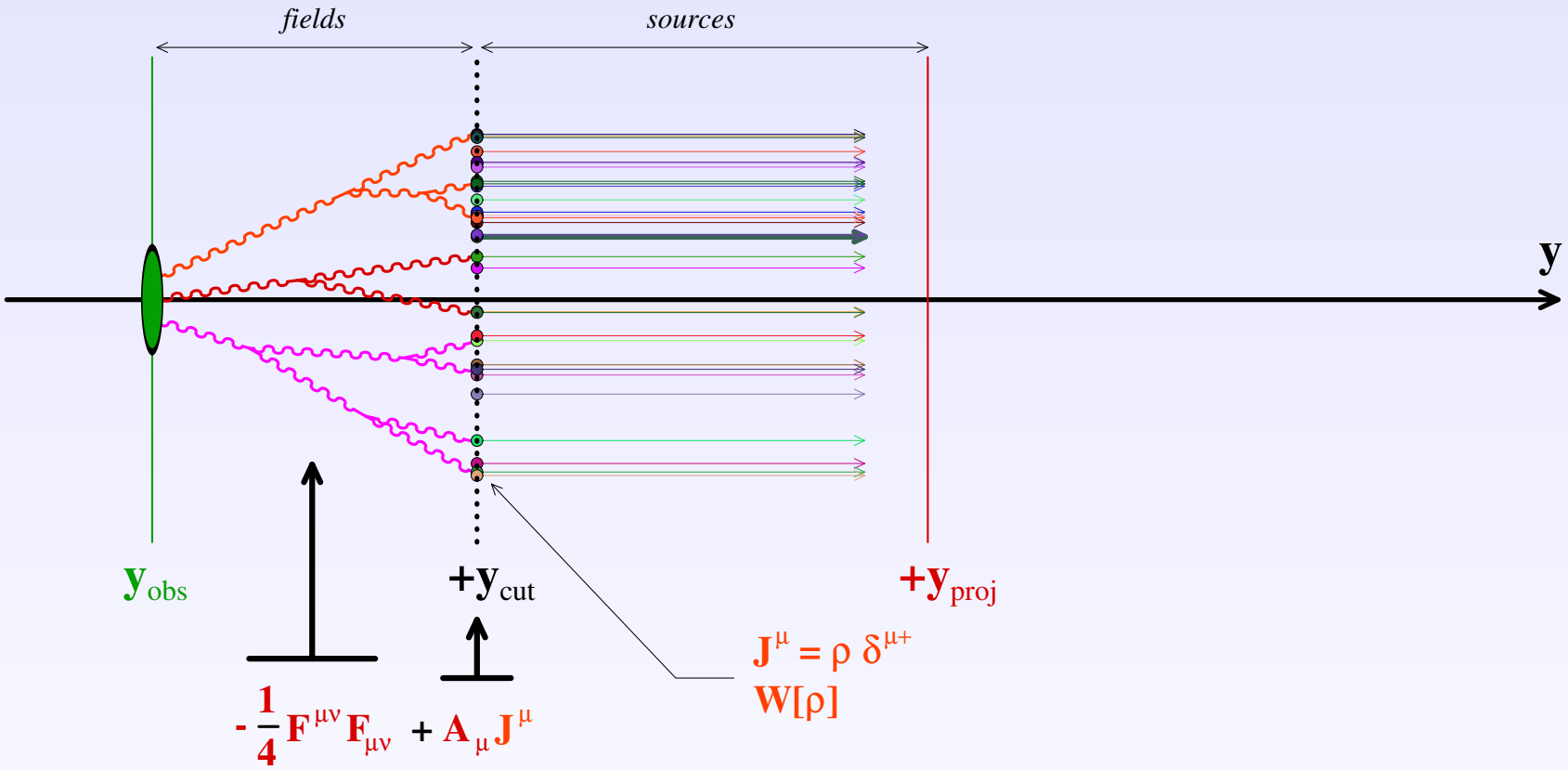

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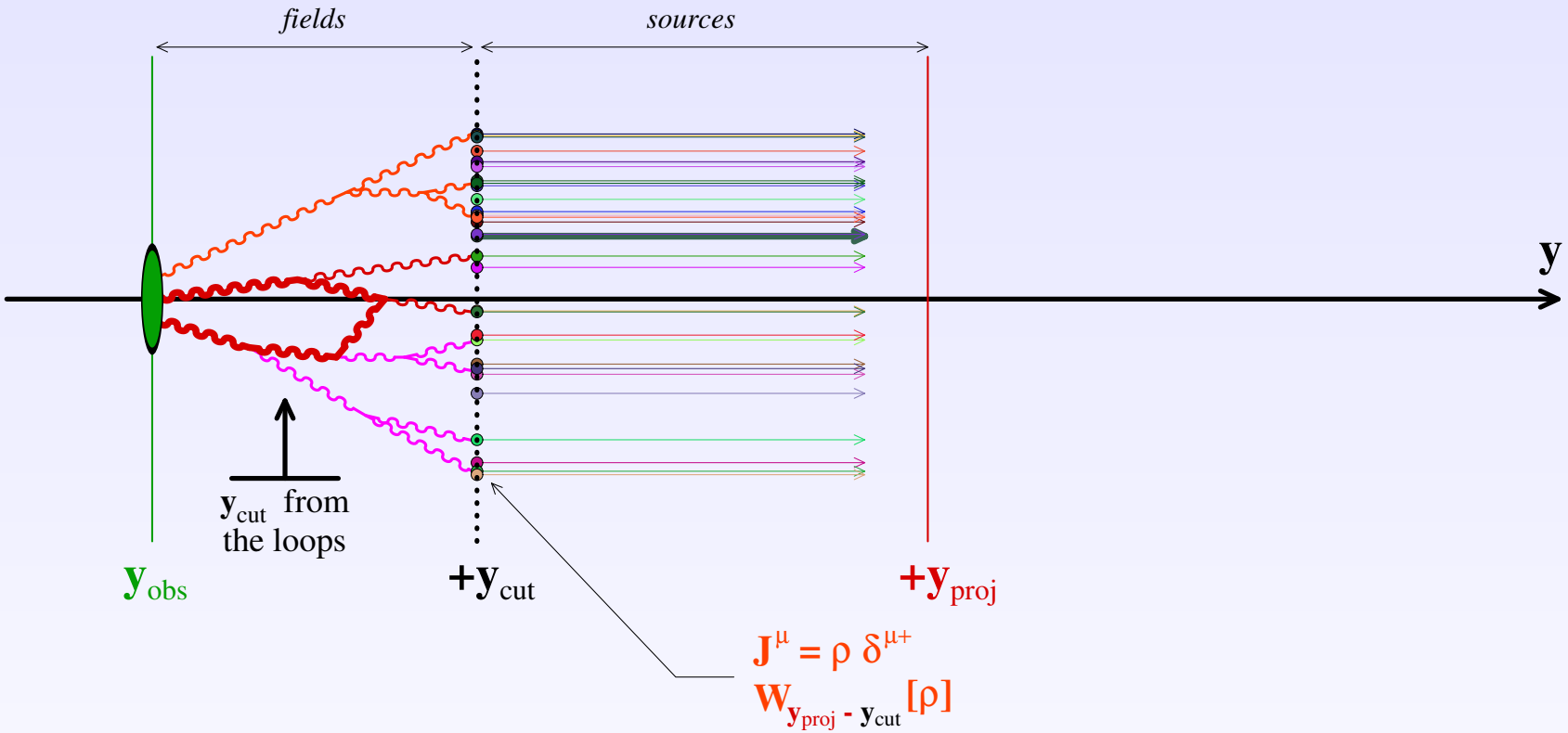
Cancellation of the cutoff dependence



- The cutoff y_{cut} is arbitrary and should not affect the result
- The probability density $W[\rho]$ changes with the cutoff
- Loop corrections cancel the cutoff dependence from $W[\rho]$

B-JIMWLK evolution equation

[Jalilian-Marian, Kovner, Leonidov, Weigert (1998)]
[Balitsky (1996)] [Iancu, Leonidov, McLerran (2001)]

B-JIMWLK equation at Leading Log

$$\frac{\partial W_Y[\rho]}{\partial Y} = \underbrace{\frac{1}{2}\int_{\vec{x}_\perp,\vec{y}_\perp} \frac{\delta}{\delta\rho_a(\vec{x}_\perp)}\chi_{ab}(\vec{x}_\perp,\vec{y}_\perp)\frac{\delta}{\delta\rho_b(\vec{y}_\perp)}}_{\mathcal{H}\quad\text{(JIMWLK Hamiltonian)}} W_Y[\rho]$$

- Mean field approx. (BK equation) : [Kovchegov (1999)]
- Langevin form of B-JIMWLK : [Blaizot, Iancu, Weigert (2003)]
- First numerical solution : [Rummukainen, Weigert (2004)]

B-JIMWLK evolution equation

[Jalilian-Marian, Kovner, Leonidov, Weigert (1998)]
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B-JIMWL

Recent developments :

Running coupling correction

[Lappi, Mäntysaari (2012)]

B-JIMWLK equation at Next to Leading Log

[Kovner, Lublinsky, Mulian (2013)]
[Caron-Huot (2013)][Balitsky, Chirilli (2013)]

- Me
- Langevin form of B-JIMWLK : **[Blaizot, Iancu, Weigert (2003)]**
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B-JIMWLK evolution equation

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B-JIMWL

Recent developments :

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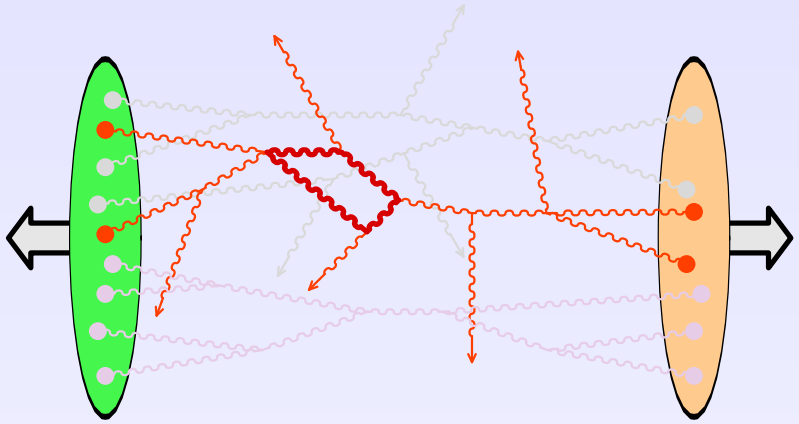
- Me
- Lange
- First numerical solution : [Rummukainen, Weigert (2004)]

Open questions for practical uses :

- Does the NLO evolution preserve the positivity of $W[\rho]$? (non trivial if the JIMWLK Hamiltonian contains higher derivatives at NLO)
- Can the NLO B-JIMWLK equation still be mapped into a Langevin equation?

Power counting in the saturated regime

$$\mathcal{S} = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{\text{slow gluons}} + \int \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}} A_\mu$$



In the saturated regime: $J^\mu \sim g^{-1}$

$$g^{-2}\ g^{\#\text{ of external gluons}}\ g^{2\times(\#\text{ of loops})}$$

- No dependence on the number of sources J^μ
  - ▷ infinite number of graphs at each order in g^2

Example : expansion of $T^{\mu\nu}$ in powers of g^2

$$T^{\mu\nu} \sim \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

Inclusive observables at Leading Order

[FG, Venugopalan (2006)]

- The Leading Order is the sum of all the tree diagrams
  Expressible in terms of classical solutions of Yang-Mills equations :
  $$\mathcal{D}_\mu \mathcal{F}^{\mu\nu} = J_1^\nu + J_2^\nu$$

- Boundary conditions : $\lim_{x^0 \to -\infty} \mathcal{A}^\mu(x) = 0$
  (WARNING : this is not true for exclusive observables!)

Components of the energy-momentum tensor at LO :

$$T^{00}_{\rm LO} = \frac{1}{2}\big[\underbrace{\mathbf{E}^2 + \mathbf{B}^2}_{\text{class. fields}}\big] \qquad T^{0i}_{\rm LO} = \big[\mathbf{E} \times \mathbf{B}\big]^i$$

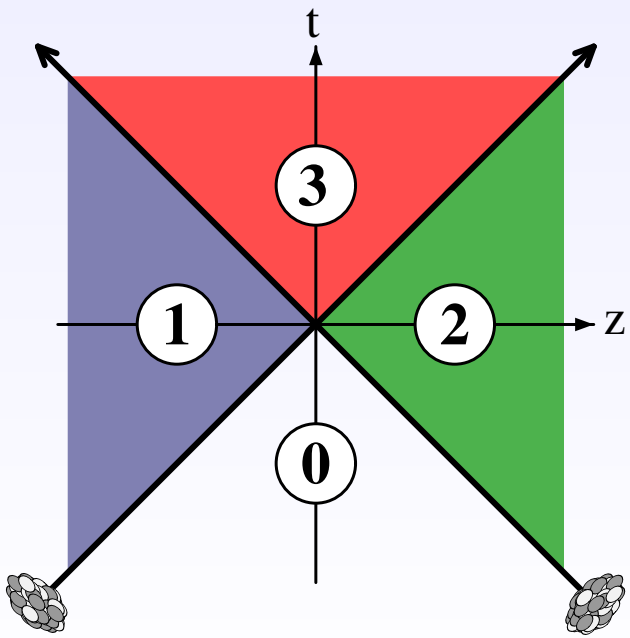
$$T^{ij}_{\rm LO} = \frac{\delta^{ij}}{2}\big[\mathbf{E}^2 + \mathbf{B}^2\big] - \big[\mathbf{E}^i\mathbf{E}^j + \mathbf{B}^i\mathbf{B}^j\big]$$

Space-time evolution of the classical field

[Kovner, McLerran, Weigert (1995)]
[Krasnitz, Venugopalan (1999)] [Lappi (2003)]

- Sources located on the light-cone :

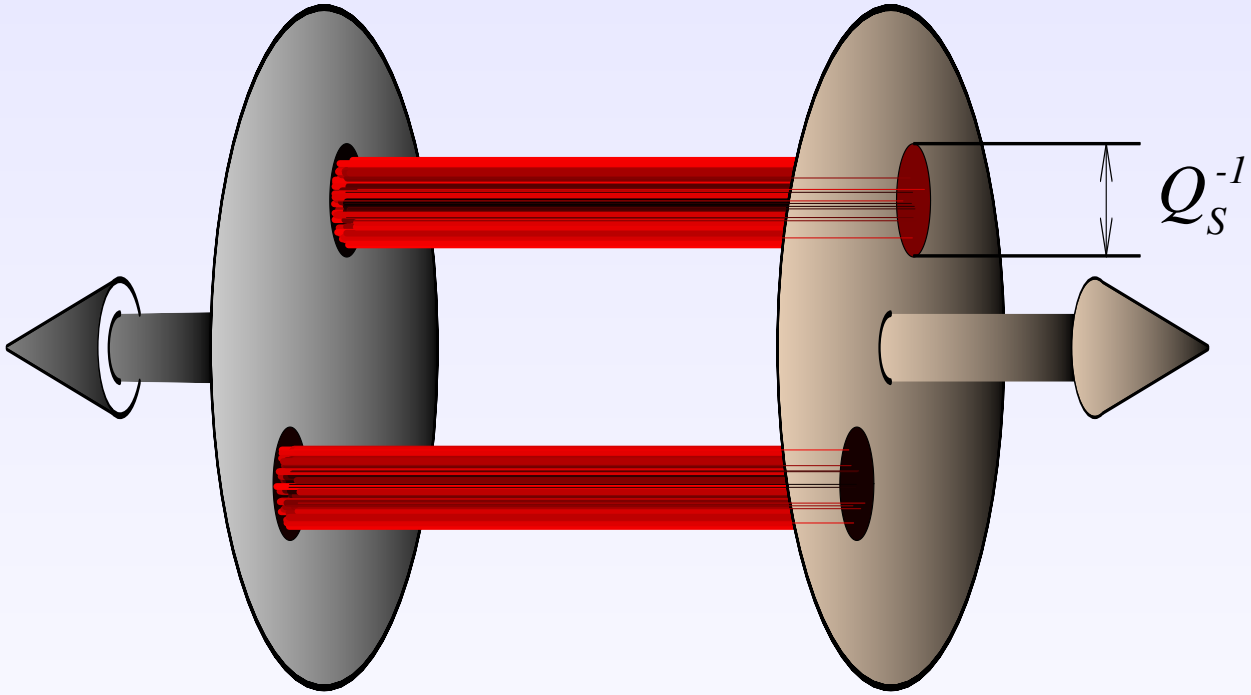
$$J^\mu = \delta^{\mu+} \underbrace{\rho_1(x^-, \mathbf{x}_\perp)}_{\sim\delta(x^-)} + \delta^{\mu-} \underbrace{\rho_2(x^+, \mathbf{x}_\perp)}_{\sim\delta(x^+)}$$



- Region 0 : $\mathcal{A}^\mu = 0$
- Regions 1,2 : $\mathcal{A}^\mu$ depends only on ρ_1 or ρ_2 (known analytically)
- Region 3 : $\mathcal{A}^\mu$ = radiated field known analytically at $\tau = 0^+$ numerical solution for $\tau > 0$

Field lines at $\tau \ll Q_s^{-1}$: Glasma flux tubes

[McLerran, Lappi (2006)]

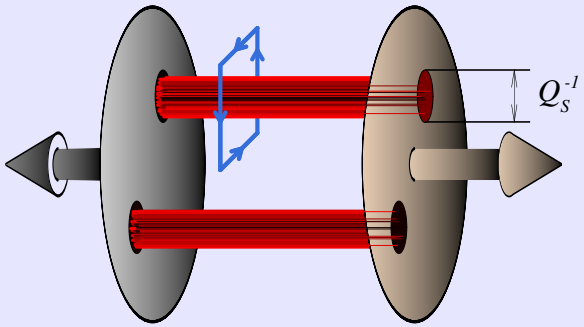


- Seed for the long range rapidity correlations (ridge)
  [Dumitru, FG, McLerran, Venugopalan (2008)]
  [Dusling, FG, Lappi, Venugopalan (2009)]

Field lines at $\tau \ll Q_s^{-1}$: Glasma flux tubes

[McLerran, Lappi (2006)]

[Dumitru, Nara, Petreska (2013)]
[Dumitru, Lappi, Nara (2014)]



$$W \equiv \left\langle P \exp ig \int_\gamma dx^i \mathcal{A}^i \right\rangle$$

$W \sim \exp(-\text{Area})$ for Area $\times\, Q_s^2 \gtrsim 1$

$\Rightarrow$ magnetic flux bundled in domains of area $\sim Q_s^{-2}$

- Seed for the long range rapidity correlations (ridge)
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Inclusive observables at Next to Leading Order

[FG, Lappi, Venugopalan (2007–2008)]

- Observables at NLO can be obtained from the LO by "fiddling" with the initial condition of the classical field :

$$\mathcal{O}_{\rm NLO} = \left[\frac{1}{2} \int_{u,v} \Gamma_2(u,v) \, \frac{\partial}{\partial \mathcal{A}_{\rm init}(u)} \frac{\partial}{\partial \mathcal{A}_{\rm init}(v)} + \int_u \alpha(u) \, \frac{\partial}{\partial \mathcal{A}_{\rm init}(u)} \right] \mathcal{O}_{\rm LO}$$

  - NLO : the time evolution remains classical;
    $\hbar$ only enters in the initial condition
  - NNLO : $\hbar$ starts appearing in the time evolution itself
  - NOT true for exclusive observables
  - This formula is the basis for proving the factorization of the $W[\rho]$ and their universality (at Leading Log)

Does it flow ?

Conditions for hydrodynamics

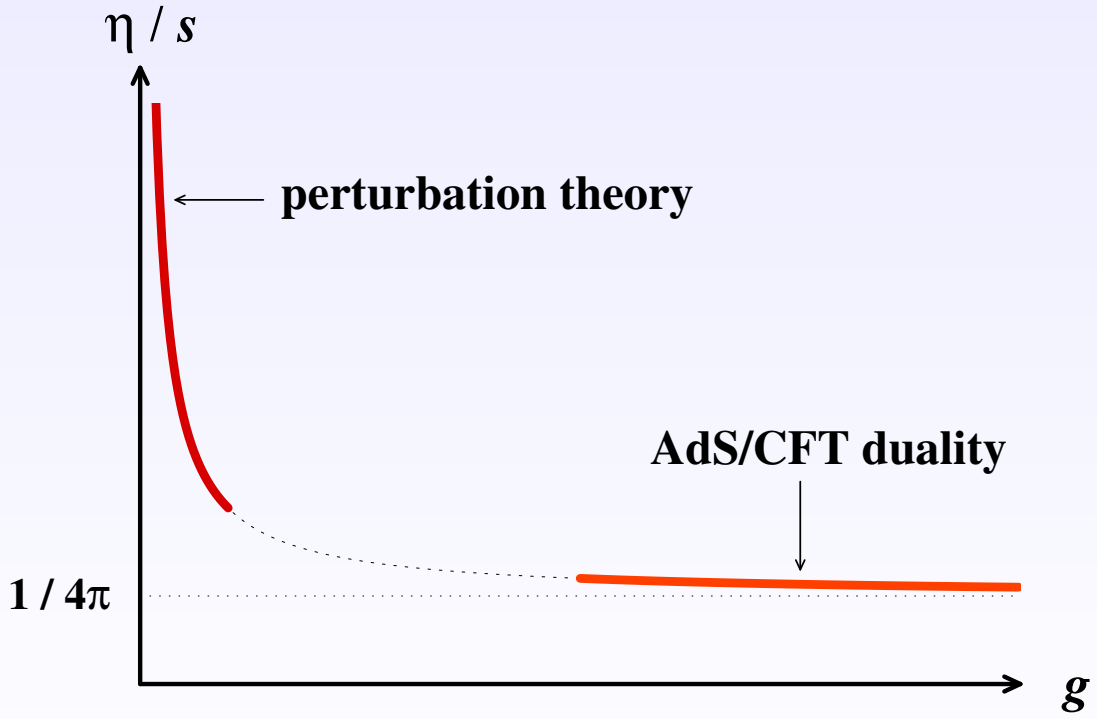
- The initial P_L/P_T should not be too small
  (for the stability of hydro codes)

- The ratio η/s should be small enough
  (for an efficient transfer from spatial to momentum anisotropy)

Shear viscosity at weak and strong coupling (in equilibrium)

Weak coupling QCD result [Arnold, Moore, Yaffe (2000)]

$$\frac{\eta}{s} \approx \frac{5.1}{g^4 \ln\left(\frac{2.4}{g}\right)}$$



Is there another possibility?

From kinetic theory :

$$\frac{\eta}{s} \sim \frac{\text{mean free path}}{\text{de Broglie wavelength}}$$

- (**de Broglie wavelength**)$^{-1} \sim Q$
- (**mean free path**)$^{-1} \sim \underbrace{g^4 Q^{-2}}_{\text{cross section}} \times \underbrace{\int_k f_k}_{\text{density}} \underbrace{(1 + f_k)}_{\text{Bose enhancement}}$

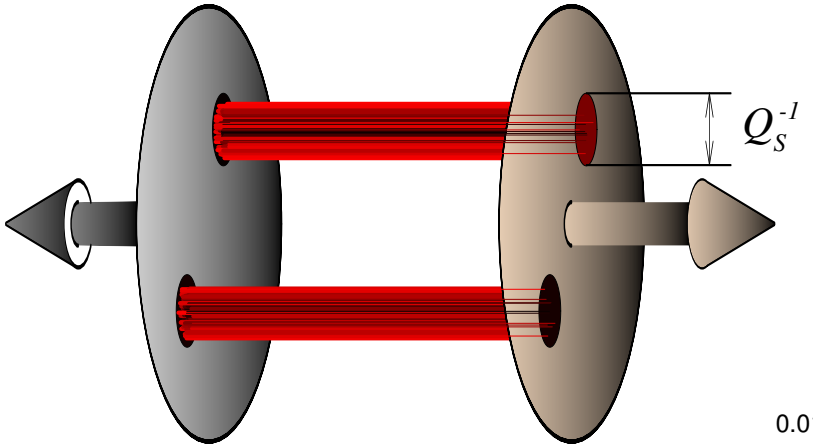
If $g \ll 1$ but $f_k \sim g^{-2}$ (weakly coupled, but strongly interacting)

$$\frac{\eta}{s} \sim g^0$$

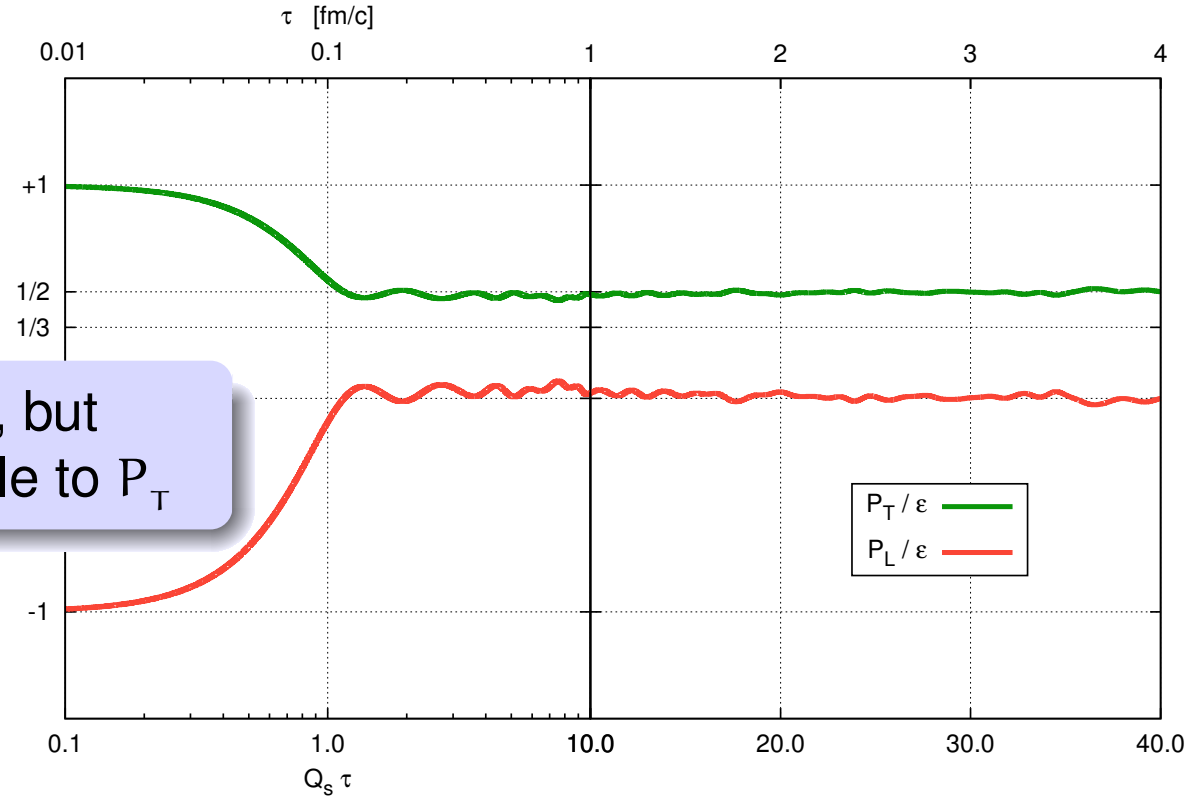
(even w/o quasiparticles, $B \sim \frac{Q^2}{g}$ has the same effect)

[Asakawa, Bass, Mueller (2006)]

CGC at LO : strong pressure anisotropy at all times

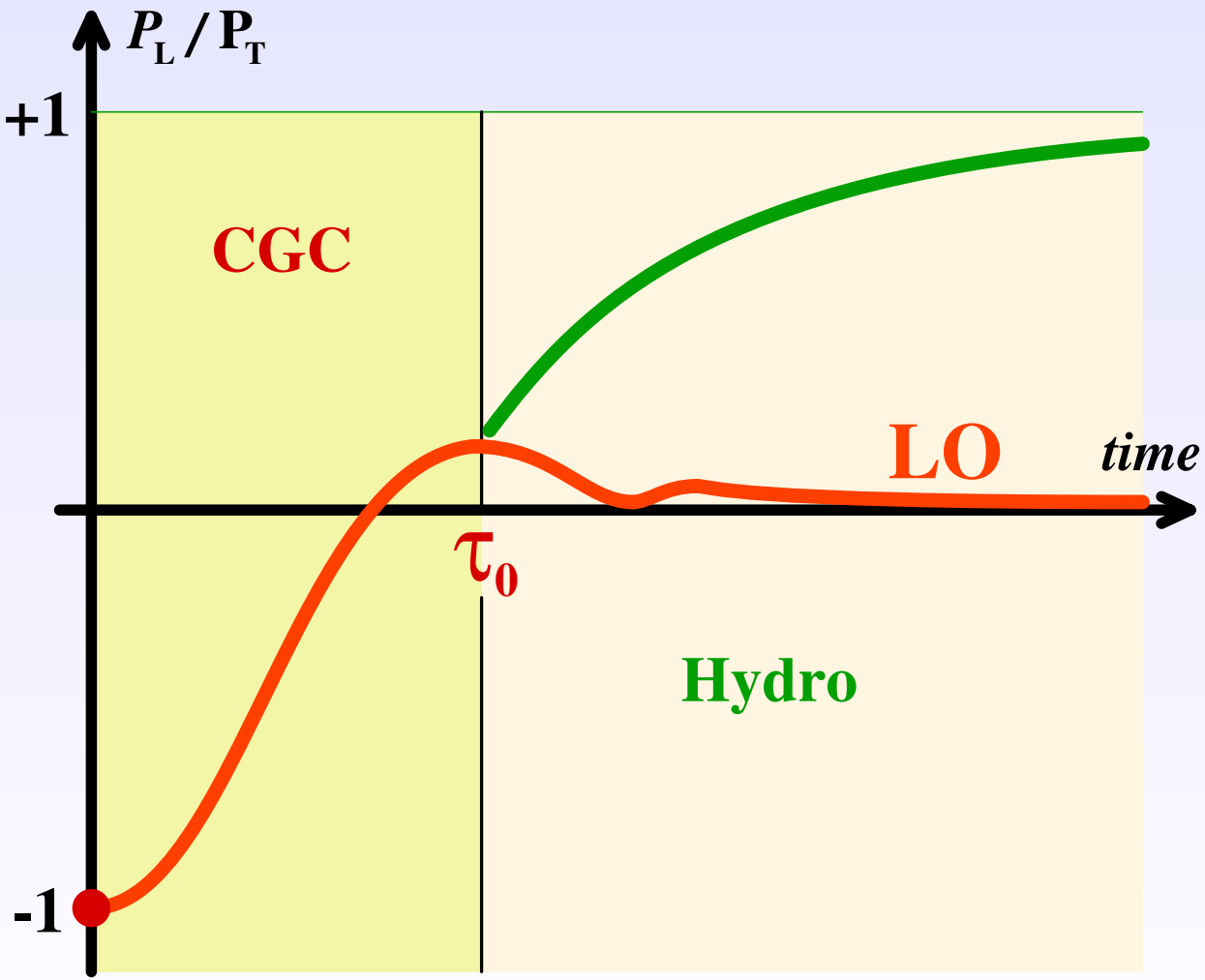


When $\mathbf{E} \parallel \mathbf{B}$:
$P_T = \epsilon, P_L = -\epsilon$



P_L rises to positive values, but never becomes comparable to P_T

CGC at LO : unsatisfactory matching to hydrodynamics



CGC at LO : unsatisfactory matching to hydrodynamics

P_L / P_T

Matching to hydro :

- Compute $T^{\mu\nu}$ from CGC
- Find time-like eigenvector : $u_\mu T^{\mu\nu} = \epsilon\, u^\nu$
- Get pressure from some equation of state $P = f(\epsilon)$
- Get viscous stress as difference between full and ideal $T^{\mu\nu}$

"CGC initial conditions" very often means :

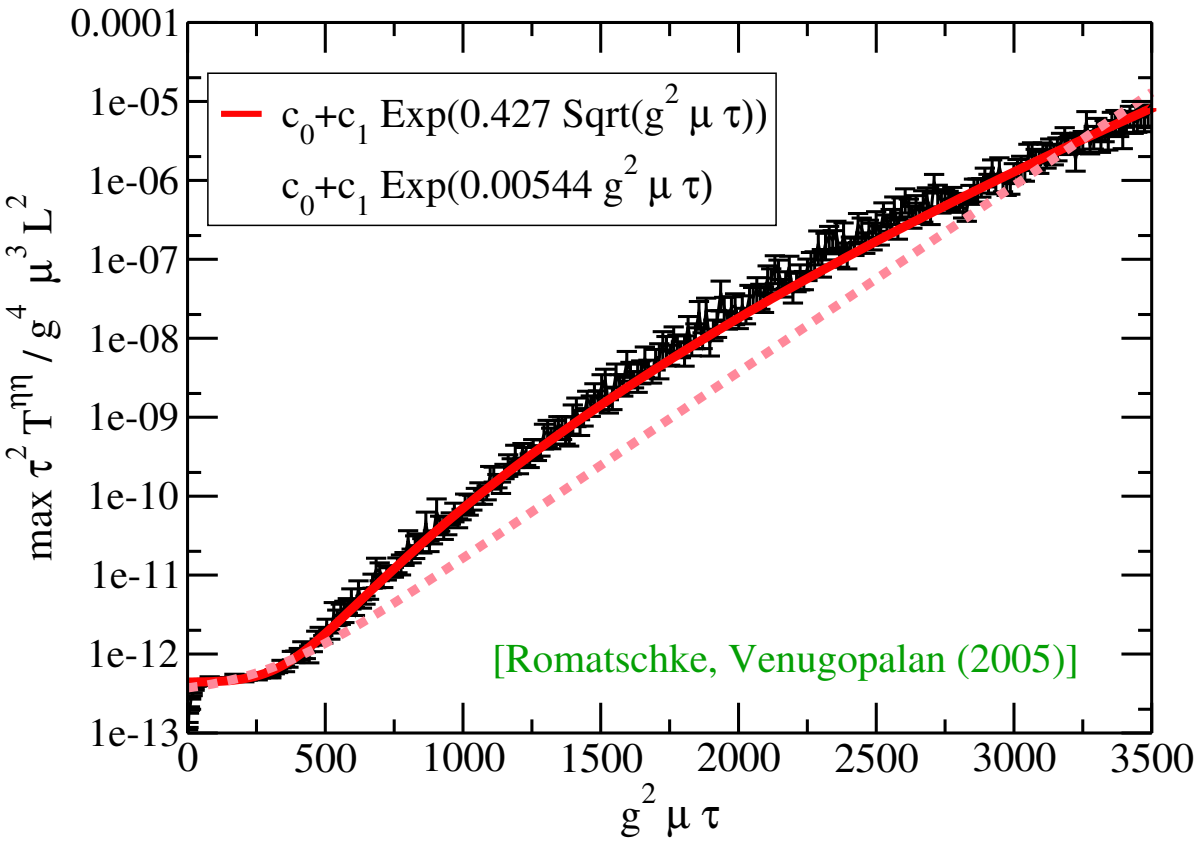
- $\epsilon = T^{00}$ from CGC (or a CGC-inspired model)
- Initial flow neglected, Viscous stress $= 0$

NOTE : glasma fields start to flow at $\tau \sim Q_s^{-1}$:

[Krasnitz, Nara, Venugopalan (2002)] [Chen, Fries (2013)]

CGC at NLO : instabilities

[Mrowczynski (1988), Romatschke, Strickland (2003), Arnold, Lenaghan, Moore (2003), Rebhan, Romatschke, Strickland (2005), Arnold, Lenaghan, Moore, Yaffe (2005), Romatschke, Rebhan (2006), Bodeker, Rummukainen (2007), Fujii, Itakura (2008),...,**Attems, Rebhan, Strickland (2012)**, **Fukushima (2013)**]



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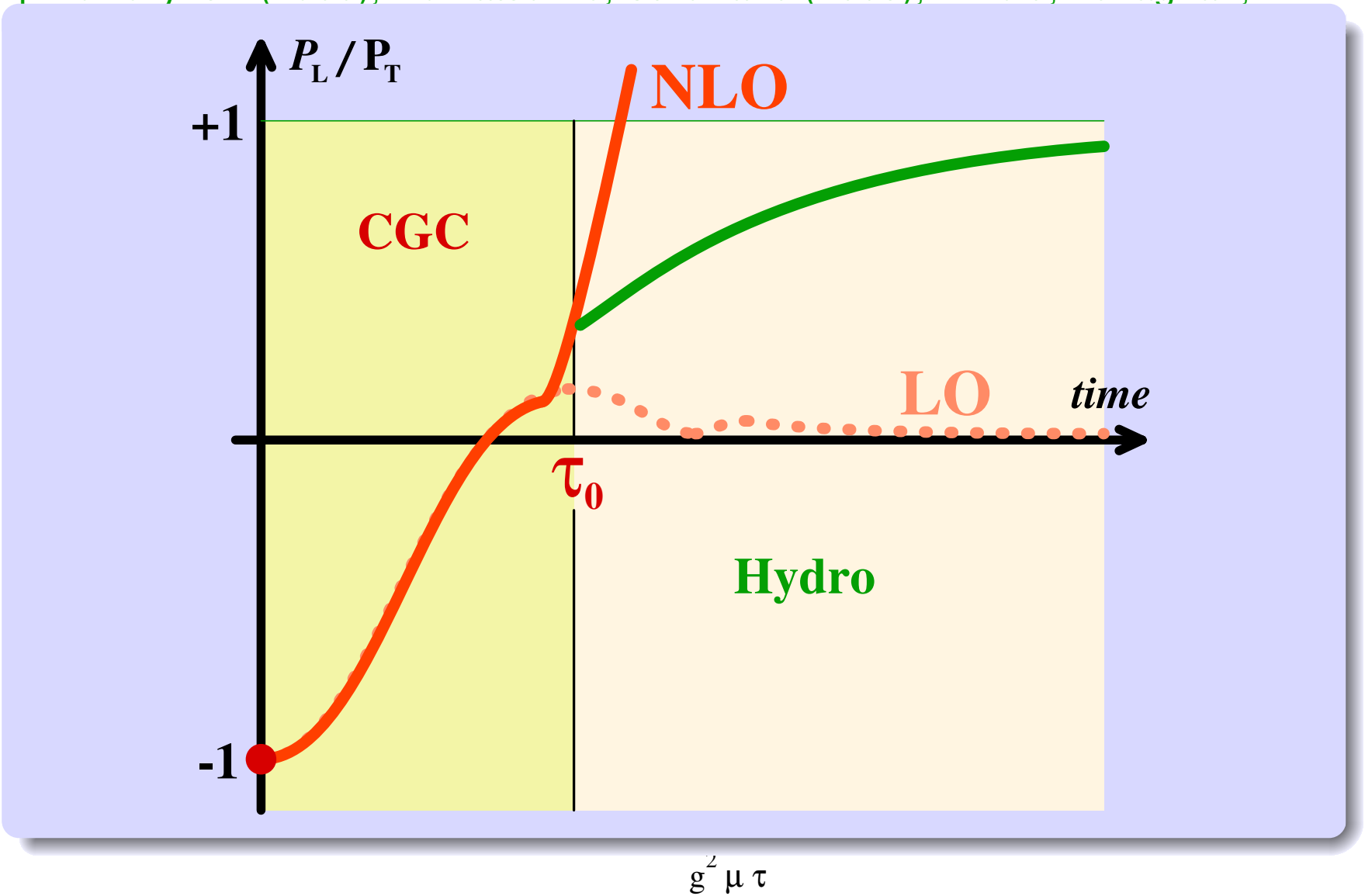

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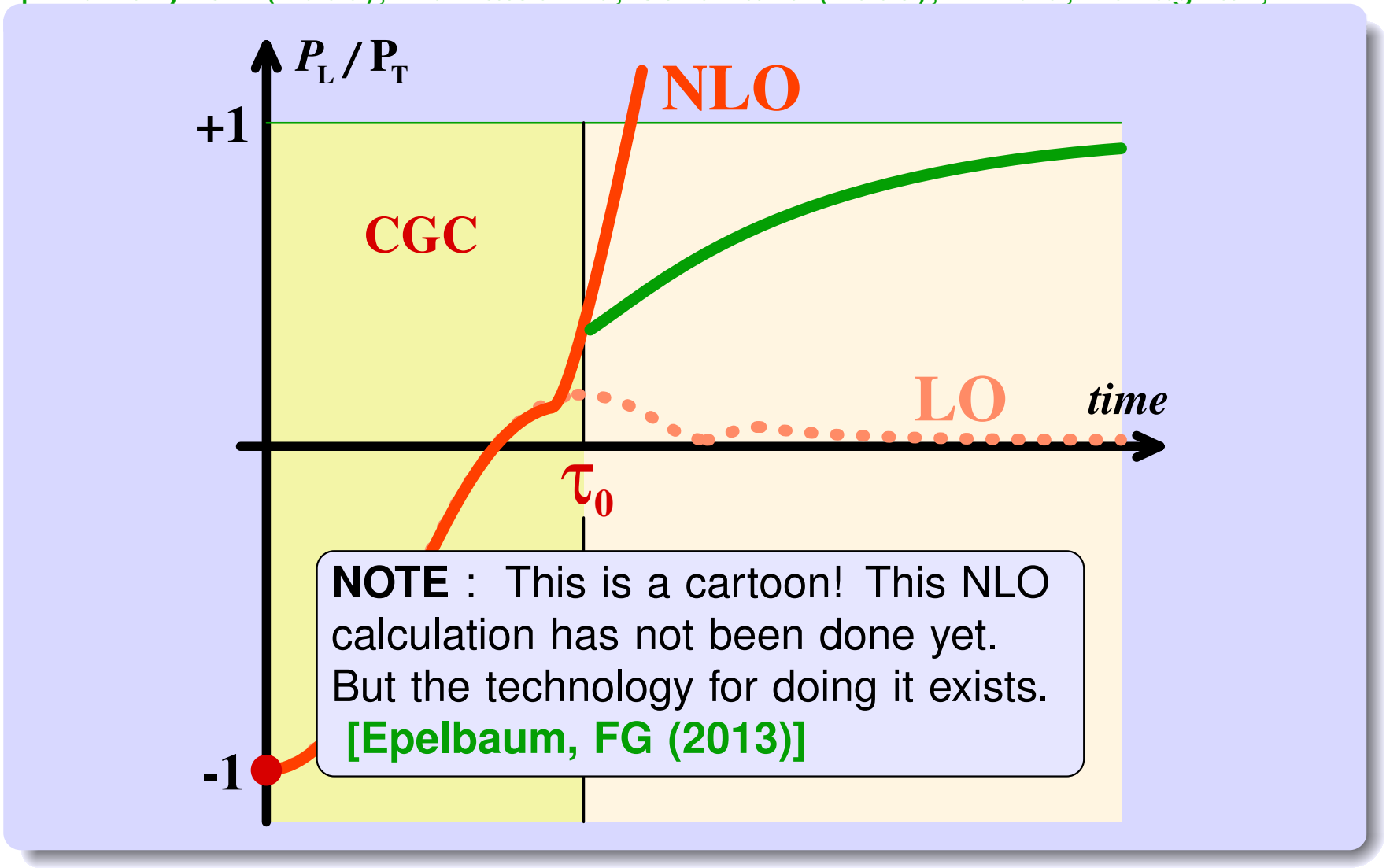
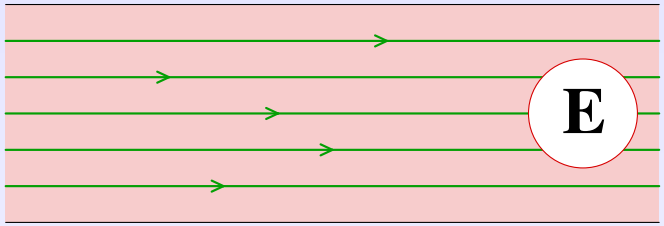
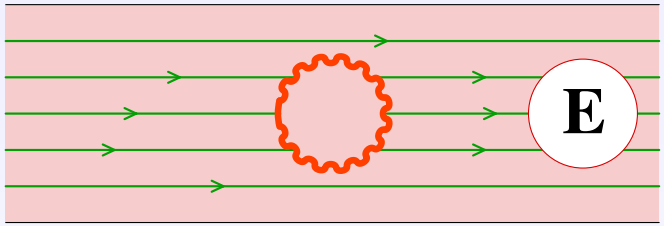


NOTE : This is a cartoon! This NLO calculation has not been done yet. But the technology for doing it exists. **[Epelbaum, FG (2013)]**

- LO = longitudinal chromo-$\mathbf{E}$ and chromo-$\mathbf{B}$ fields



- NLO = gluon loop embedded in this field

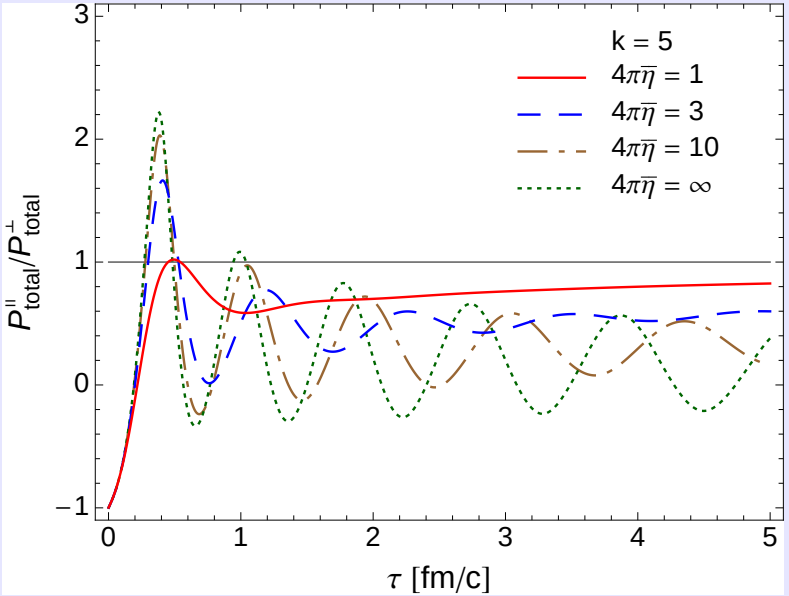


  - the loop can have an imaginary part $\Rightarrow$ gluon pair production
  - BUT : no feedback of the produced gluons on the LO field!

Color flux tube model : [Ryblewski, Florkowski (2013)]

$$(p^\mu \partial_\mu + \underbrace{g F^{\mu\nu} p_\nu}_{\text{Lorentz force}} \partial^\mu_p)\, G = \underbrace{\frac{dN}{d\Gamma}}_{\text{Schwinger}} + \underbrace{C_p[G]}_{\text{collisions}}$$

$$\partial_\mu F^{\mu\nu} = j^\nu \qquad \text{(feedback)}$$



- Field converted into particles by instability
- Nearly constant P_L/P_T
- Ratio depends on $\bar{\eta} \equiv \eta/s$

Classical statistical approximation

Three points of view / derivations

- Path integral : fields A_+, A_- on the two branches of the Schwinger-Keldysh contour. In the regime of large fields, $(A_1 \equiv A_+ - A_-) \ll (A_2 \equiv A_+ + A_-)$. Expand the action in A_1 and keep only the lowest order term.
- Diagrammatic expansion : Schwinger-Keldysh formalism in the retarded/advanced basis. Drop the 2111 vertex.
- CGC : exponentiate the operator that relates $\mathcal{O}_{\text{LO}}$ and $\mathcal{O}_{\text{NLO}}$

Practical implementation

- Classical time evolution
- Fluctuations in the initial classical field

- Dynamics fully non-linear $\Rightarrow$ no unbounded growth
- Individual classical trajectories may be chaotic $\Rightarrow$ a small initial ensemble can span a large phase space volume

Classical statistical approximation

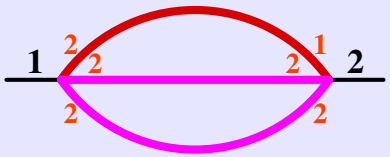
Three points of view / derivations

- CSA $\neq$ underlying theory at 2-loops and beyond
- Sources of fluctuations of the initial fields :

$$G_{22}(p) \sim \left(f_0(\mathbf{p}) + \frac{1}{2}\right)\delta(p^2)$$

quasiparticles $\hookleftarrow$ $\hookrightarrow$ vacuum fluctuations

- **Vacuum fluctuations** make the CSA **non-renormalizable**. Example of problematic graph :

Im 

$= -\dfrac{g^4}{1024\pi^3}\left(\Lambda_{\rm UV}^2 - \dfrac{2}{3}p^2\right)$

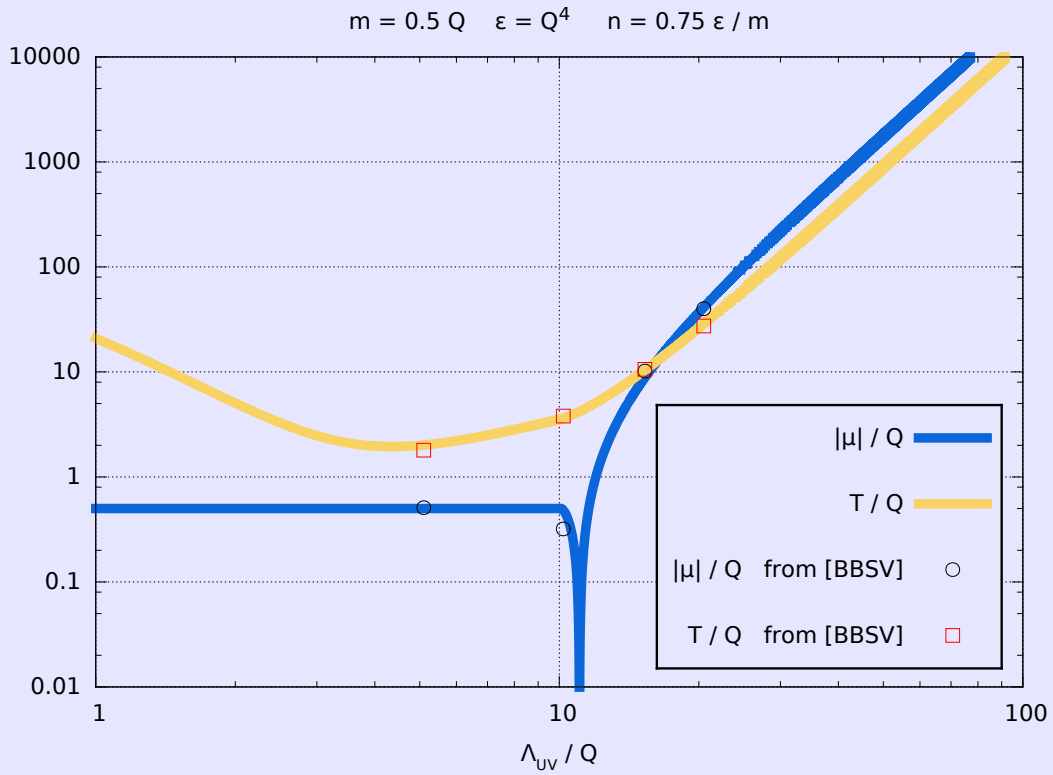
- With only **quasiparticle**-induced fluctuations :
  - Finite if $f_0(p)$ falls faster than p^{-1}
  - Super-renormalizable if $f_0(p) \sim p^{-1}$ [Aarts, Smit (1997)]

Pra

ensemble can span a large phase space volume

Classical statistical approximation

Cutoff dependence at late time :



- Sweet range : $\Lambda_{\rm UV} \sim (3-6) \times Q$
- But no continuum limit

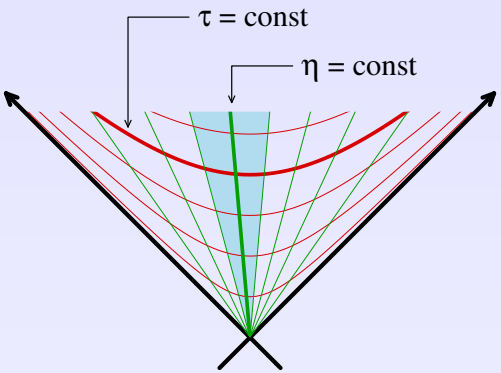
Two recent works along these lines

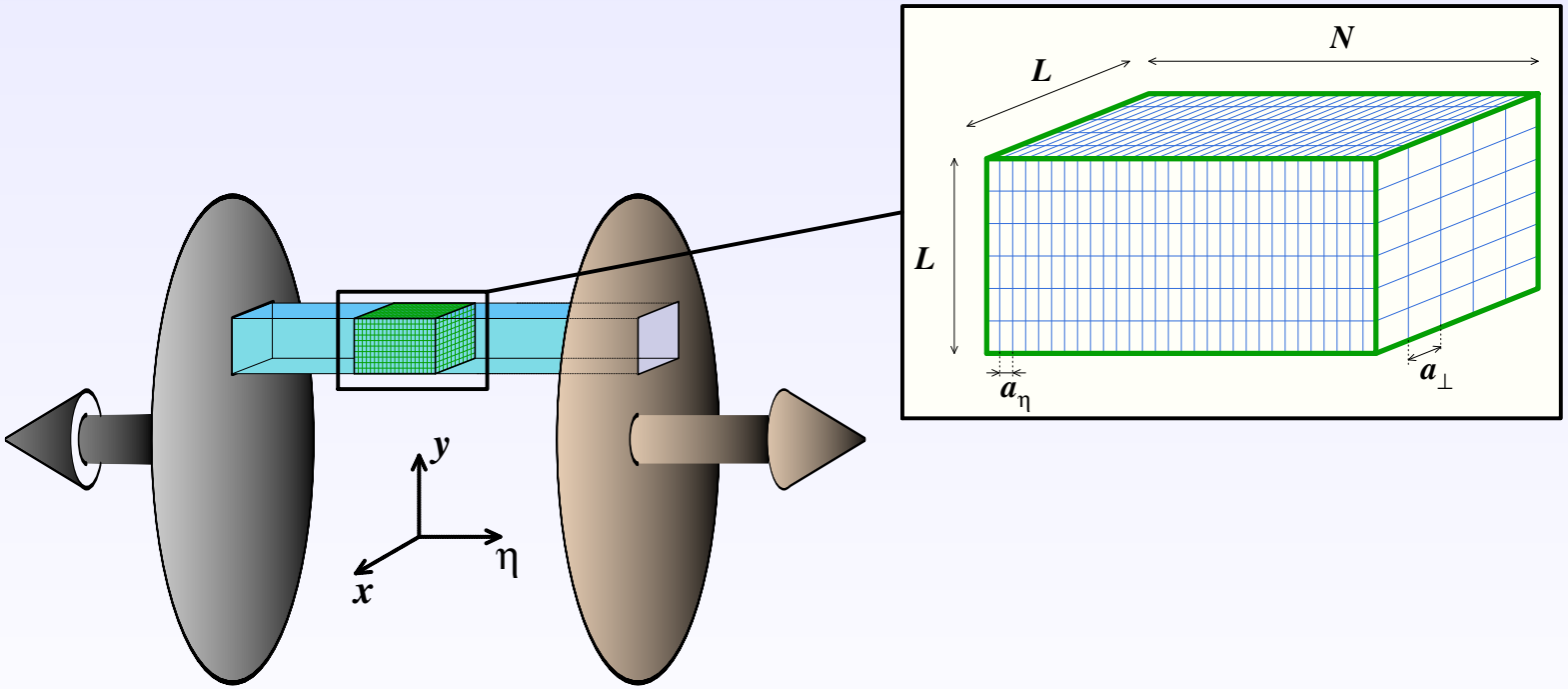
- **[Berges, Boguslavski, Schlichting, Venugopalan (2013)]**
  - Start at $Q_s\tau \gg 1$
  - Ensemble of fields that represents a highly occupied gas of gluons
  - Free parameters : initial gluon distribution
  - Fluctuations = particle-like fluctuations
  - No α_s dependence

- **[Epelbaum, FG (2013)]**
  - Start at $Q_s\tau \ll 1$
  - Ensemble of fields calculated analytically to reproduce the NLO
  - No free parameter
  - Fluctuations = vacuum fluctuations
  - Fluctuations / background field $\sim \alpha_s$

Discretization of the expanding volume

- Comoving coordinates : $\tau, \eta, \mathbf{x}_\perp$
- Simulation of a sub-volume
  + periodic boundary conditions
- $L^2 \times N$ lattice





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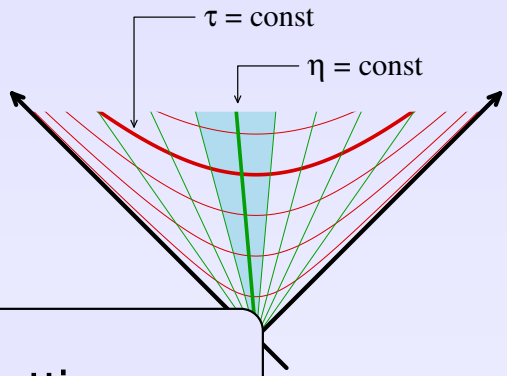
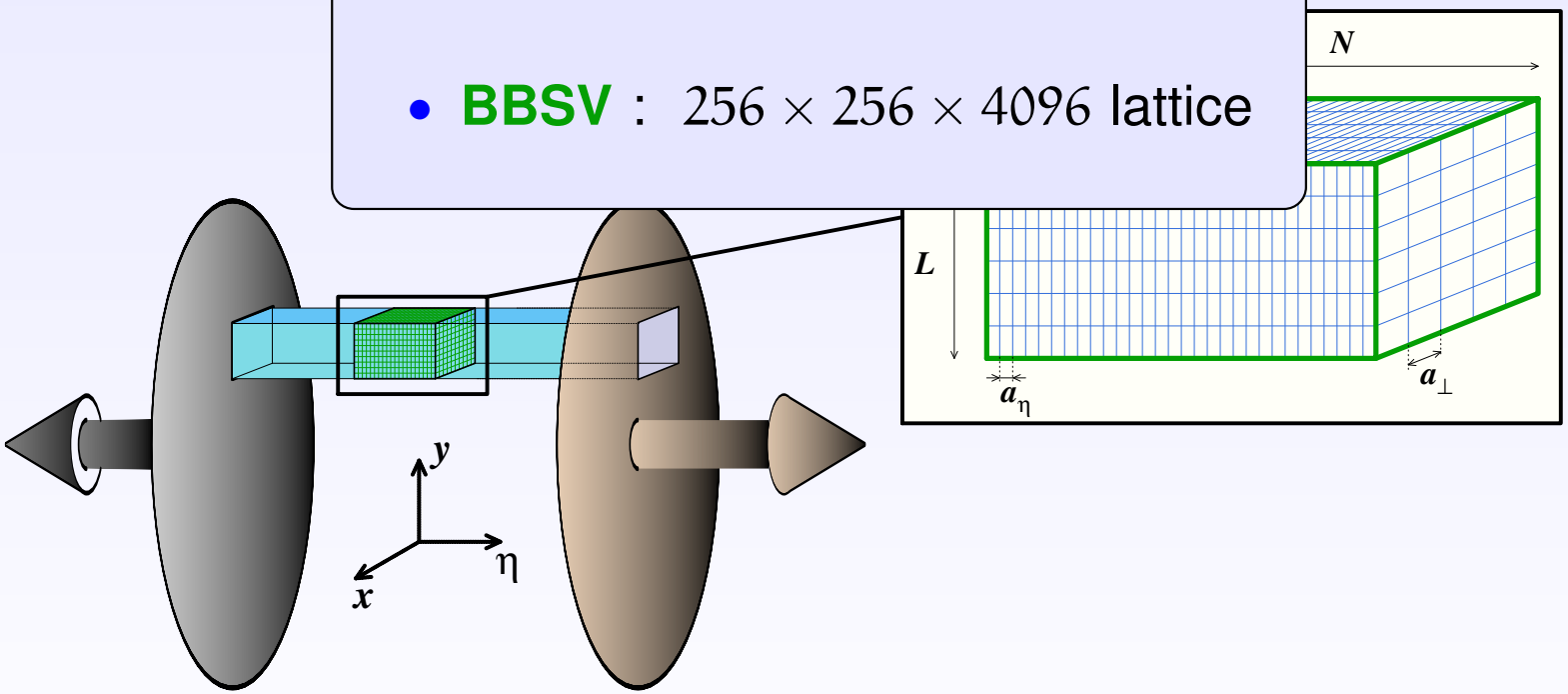


- **EG** : $64 \times 64 \times 128$ lattice
- **BBSV** : $256 \times 256 \times 4096$ lattice



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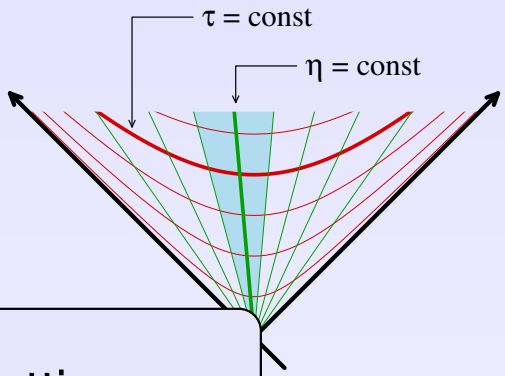

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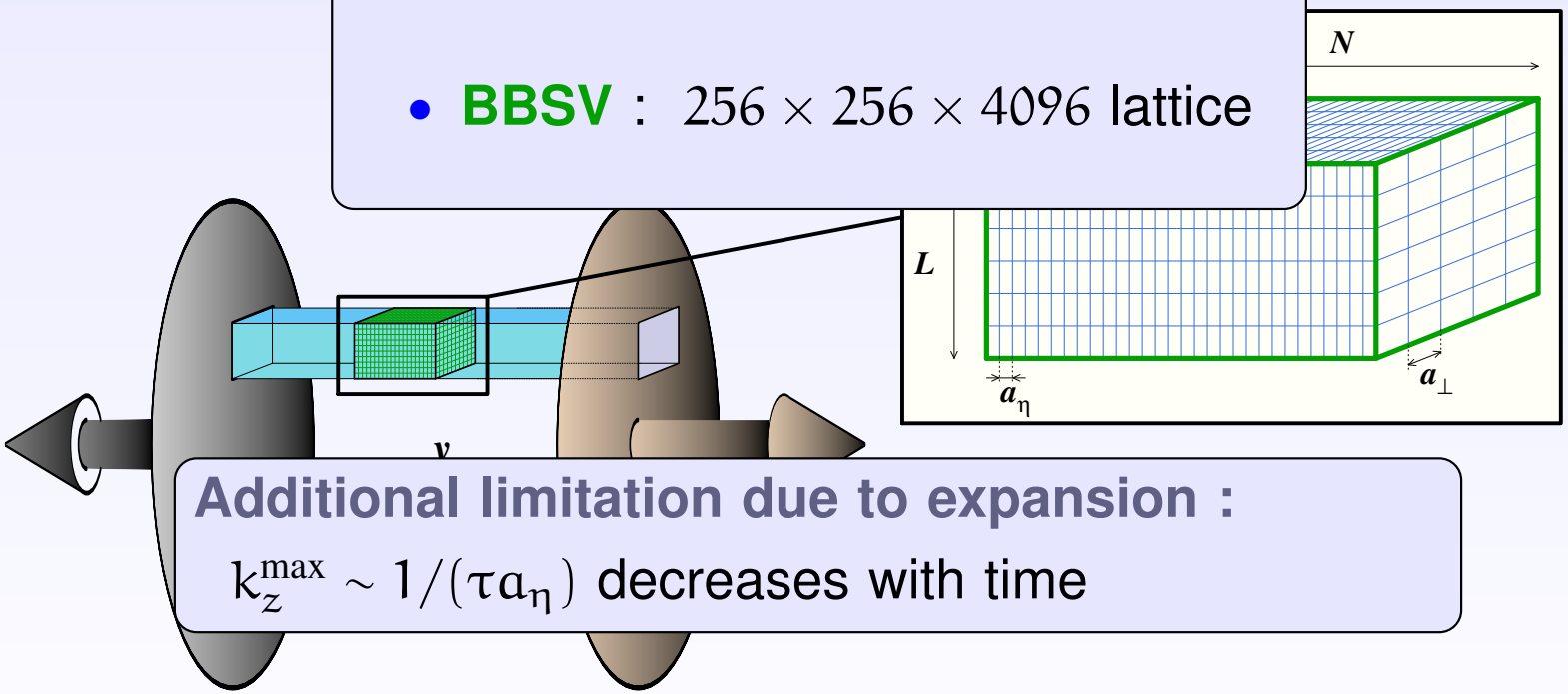


Additional limitation due to expansion :

$k_z^{max} \sim 1/(\tau a_\eta)$ decreases with time

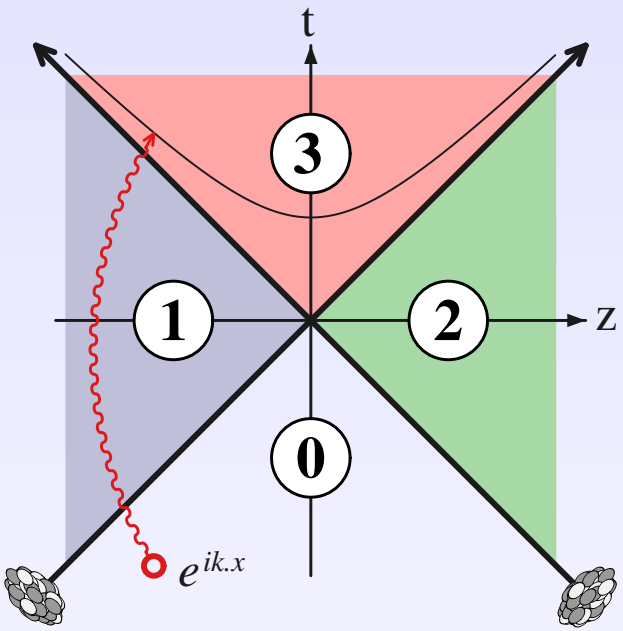
Gaussian spectra of fluctuations

EG : CGC at $\tau \ll Q_s^{-1}$ (1-loop accurate)

$$\langle \mathcal{A}^\mu \rangle = \mathcal{A}^\mu_{\rm LO} \qquad \text{Var.} = \int_{\text{modes } \mathbf{k}} \frac{1}{2}\, a_{\mathbf{k}}(\mathbf{u})\, a^*_{\mathbf{k}}(\mathbf{v})$$

$$\left[\mathcal{D}_\rho \mathcal{D}^\rho \delta^\nu_\mu - \mathcal{D}_\mu \mathcal{D}^\nu + ig\, \mathcal{F}_\mu{}^\nu\right] a^\mu_{\mathbf{k}} = 0$$

$$\lim_{x^0 \to -\infty} a_{\mathbf{k}}(x) = e^{i\mathbf{k}\cdot x}$$



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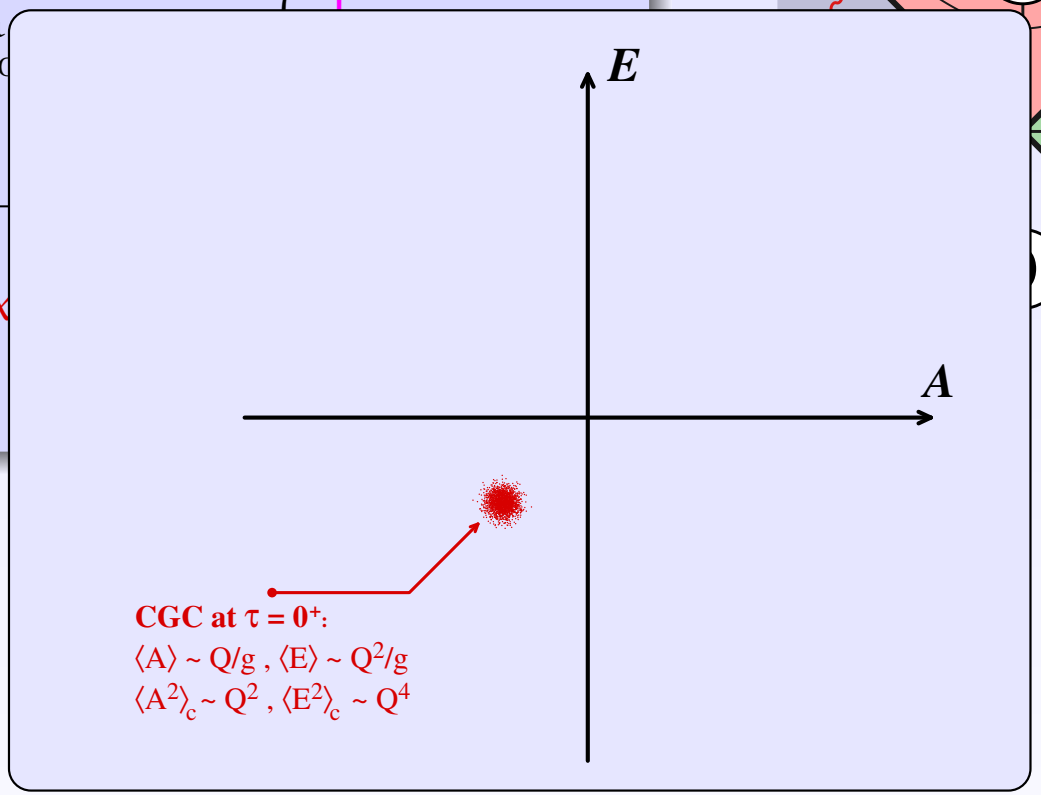

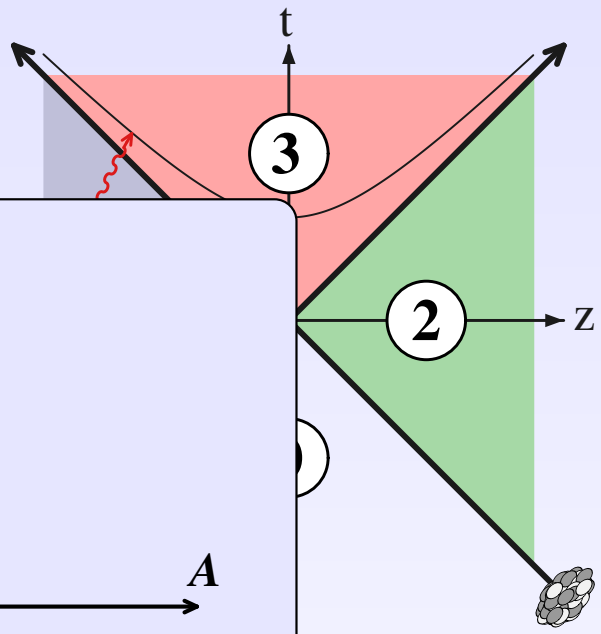

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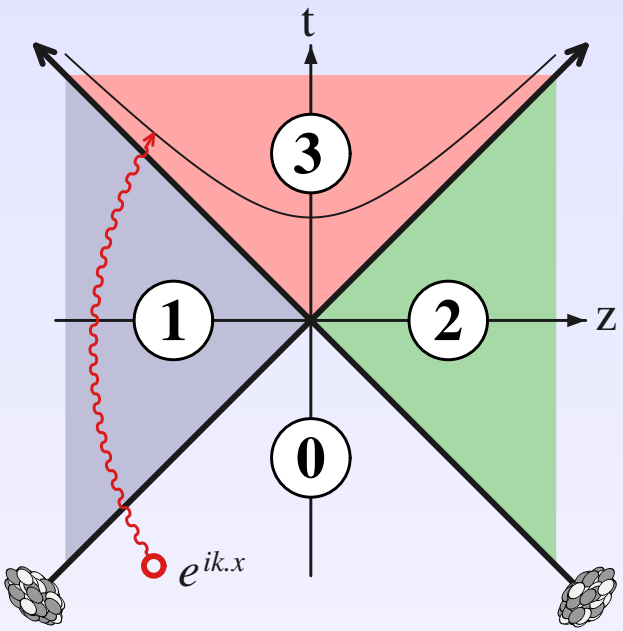

BBSV : Dense gas of free gluons at $Q_s\tau \gg 1$

$$\langle \mathcal{A}^\mu \rangle = 0 \qquad \text{Var.} = \int_{\text{modes } \mathbf{k}} f_0(\mathbf{k})\, a_{\mathbf{k}}(\mathbf{u}) a^*_{\mathbf{k}}(\mathbf{v}) \qquad a_{\mathbf{k}}(x) \equiv e^{i\mathbf{k}\cdot x}$$

$$f_0(\mathbf{k}) \sim g^{-2} \times \theta(Q_s - k)$$

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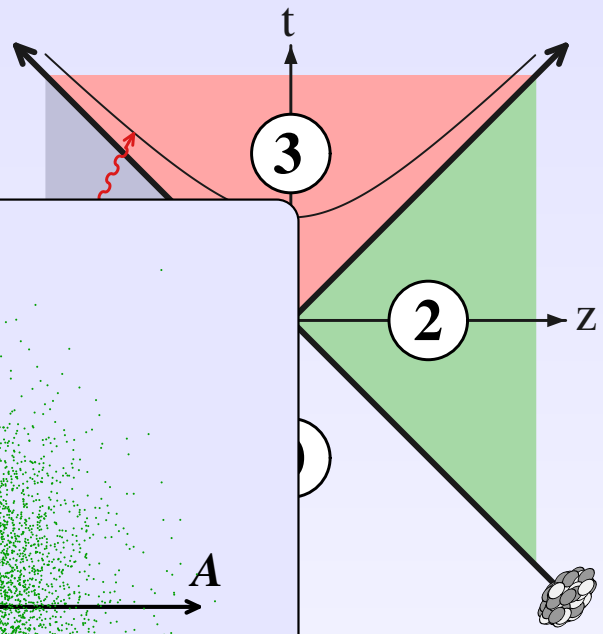

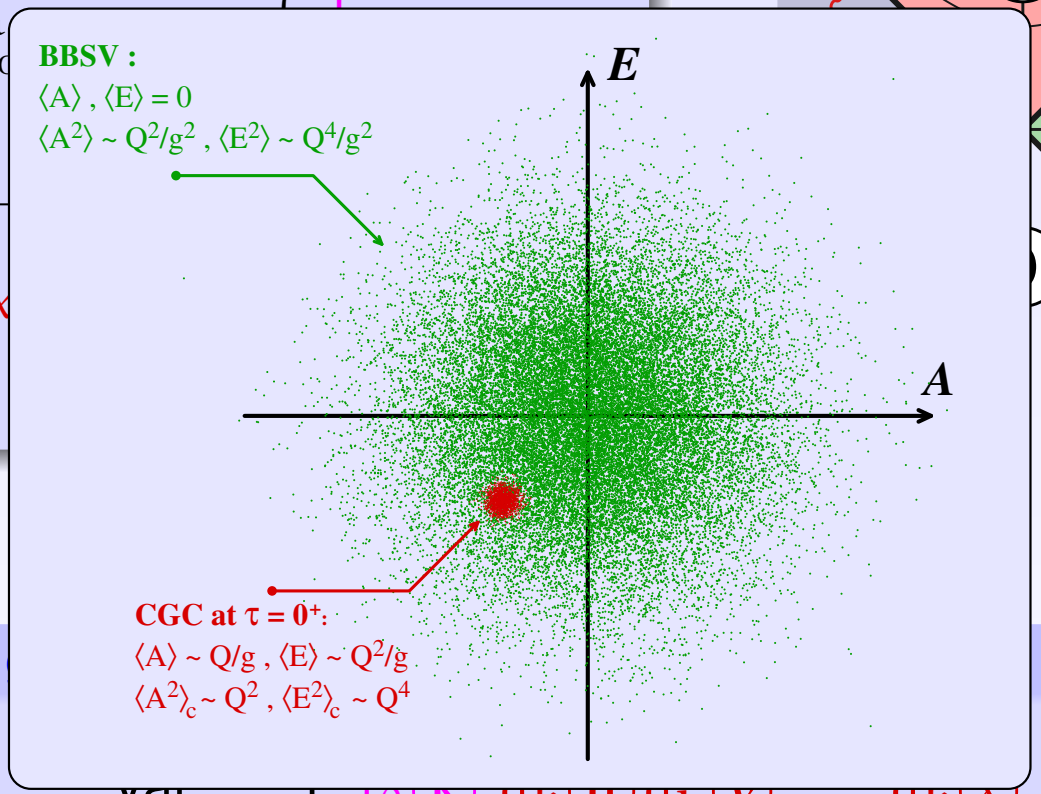

BBSV : Dense

$$\langle \mathcal{A}^\mu \rangle = 0 \qquad \mathrm{var.} = \int_{\text{modes } \mathbf{k}} f_0(\mathbf{k})\, a_{\mathbf{k}}(u) a_{\mathbf{k}}(v) \qquad a_{\mathbf{k}}(x) \equiv e^{ik\cdot x}$$

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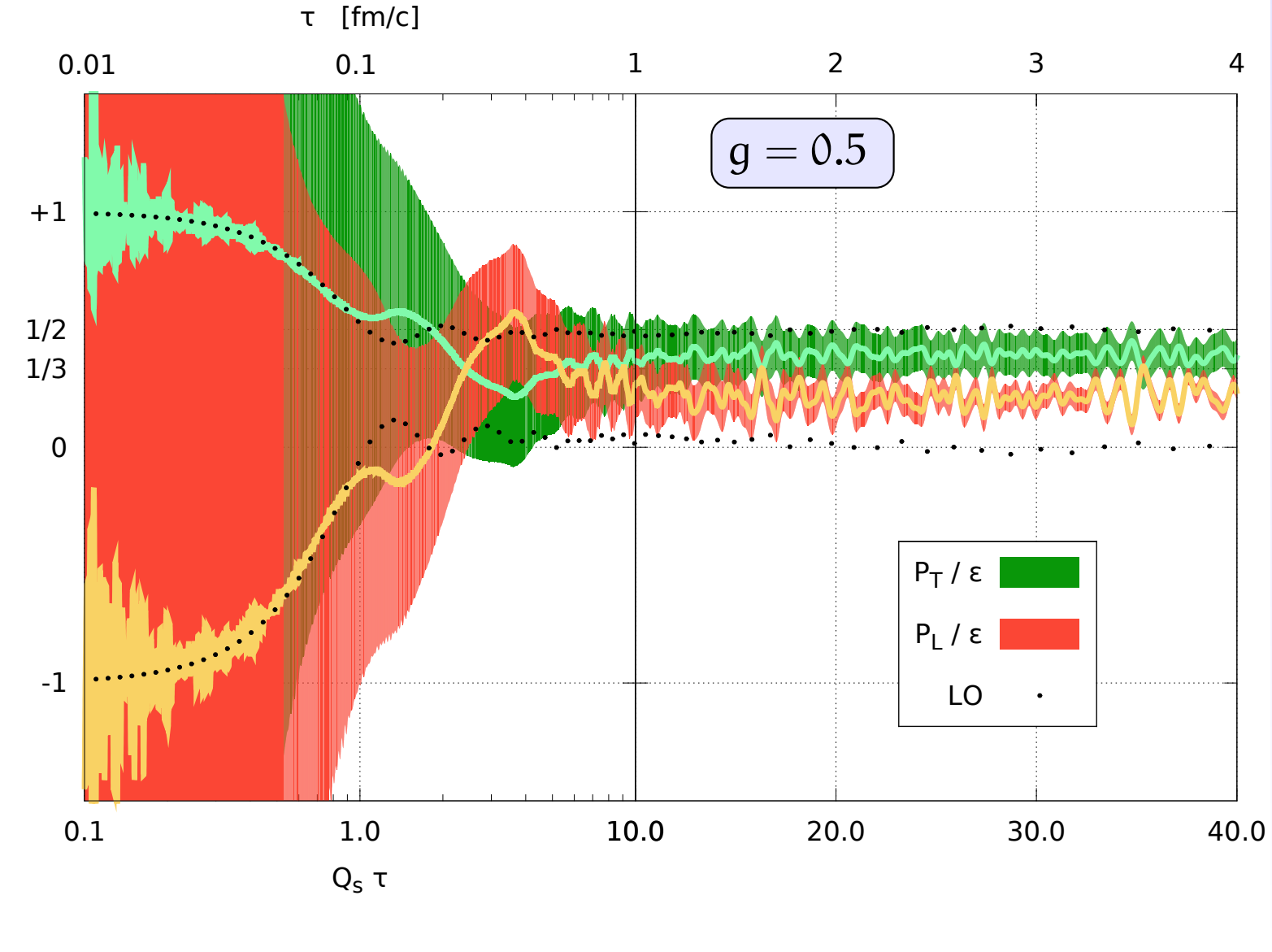
EG : IC $\equiv$ CGC at $\tau \ll Q_s^{-1}$



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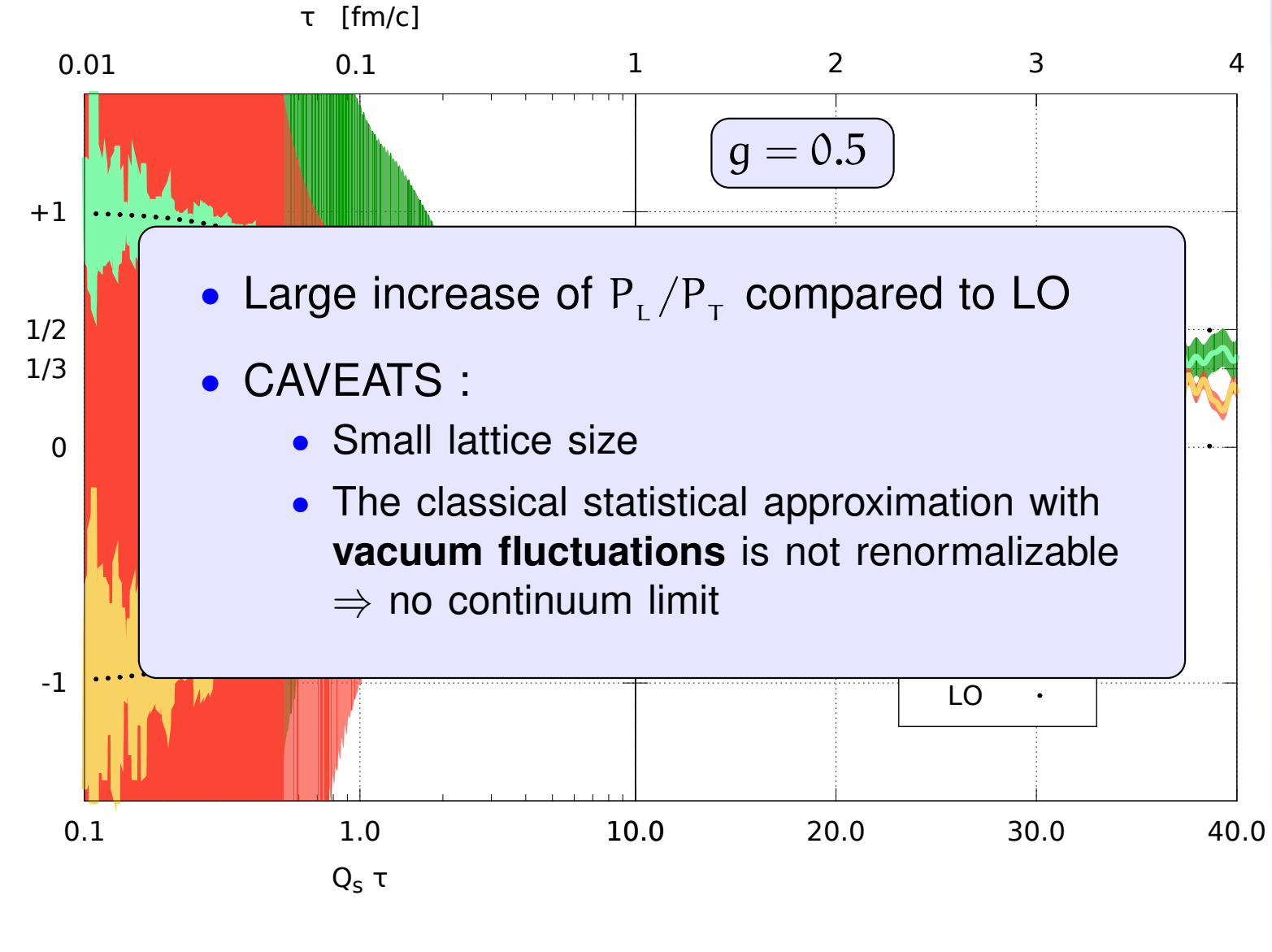
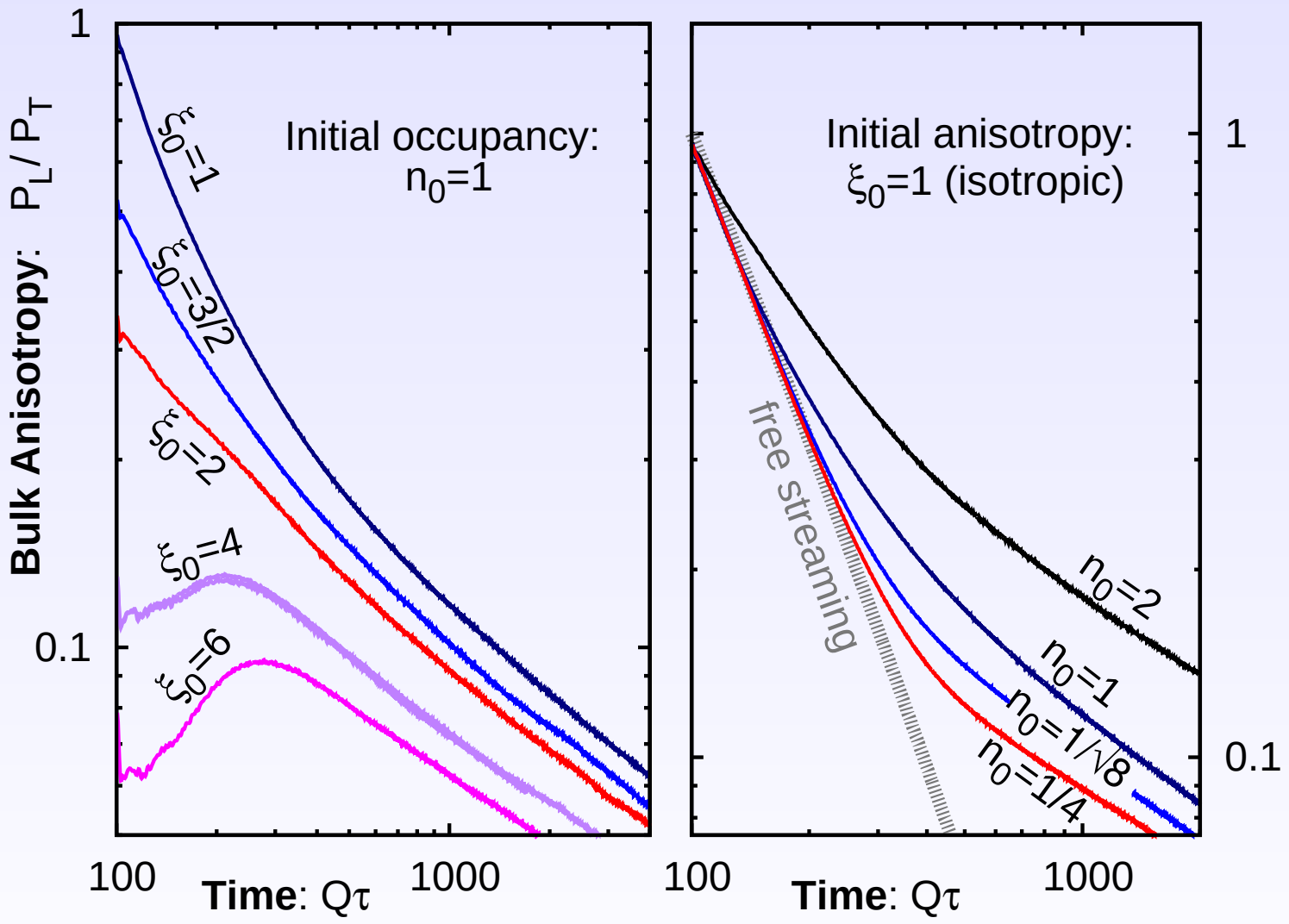


- Large increase of P_L/P_T compared to LO
- CAVEATS :
  - Small lattice size
  - The classical statistical approximation with **vacuum fluctuations** is not renormalizable $\Rightarrow$ no continuum limit

BBSV : IC $\equiv$ dense gas of free gluons at $Q_s\tau \gg 1$



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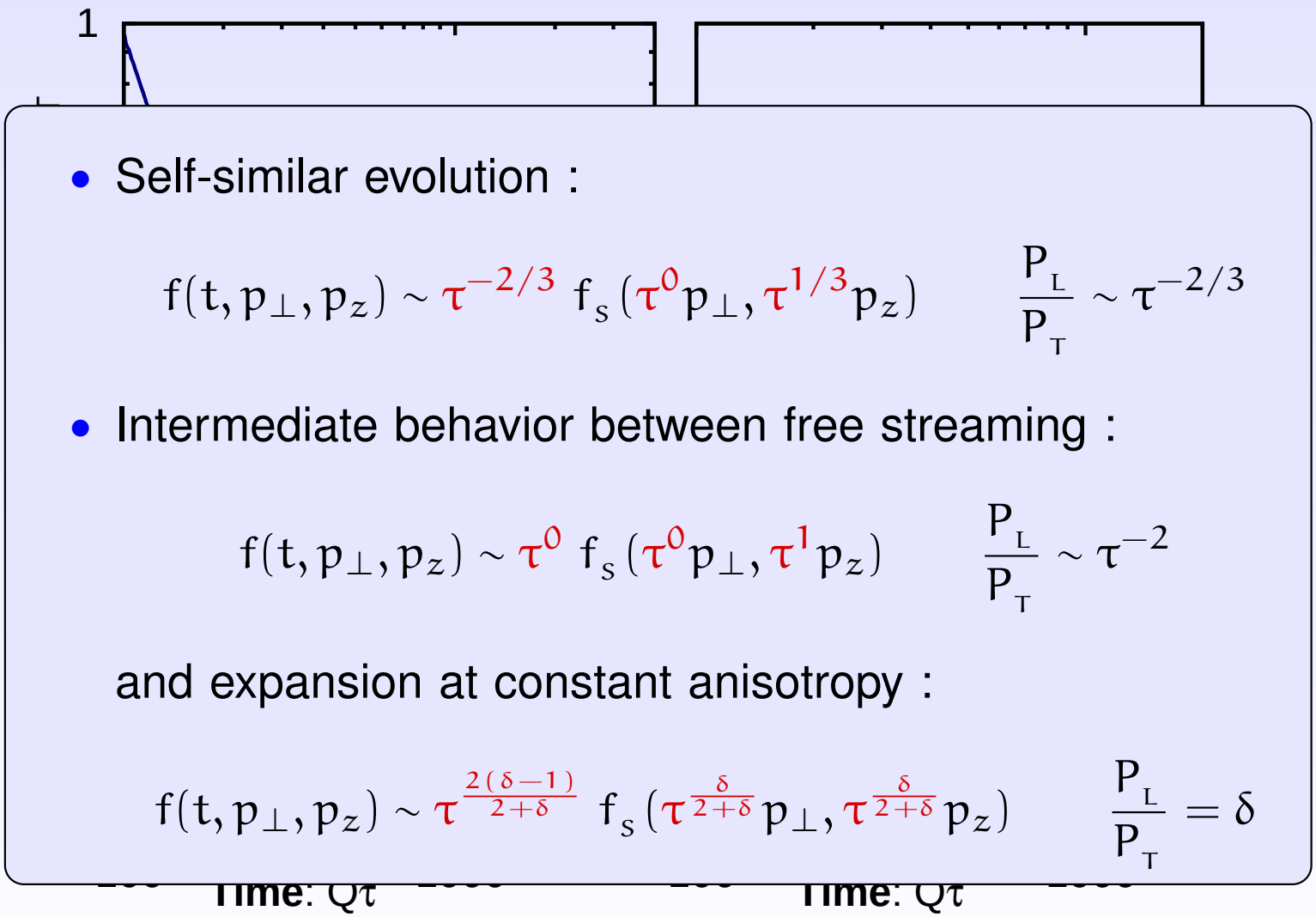


- Self-similar evolution :

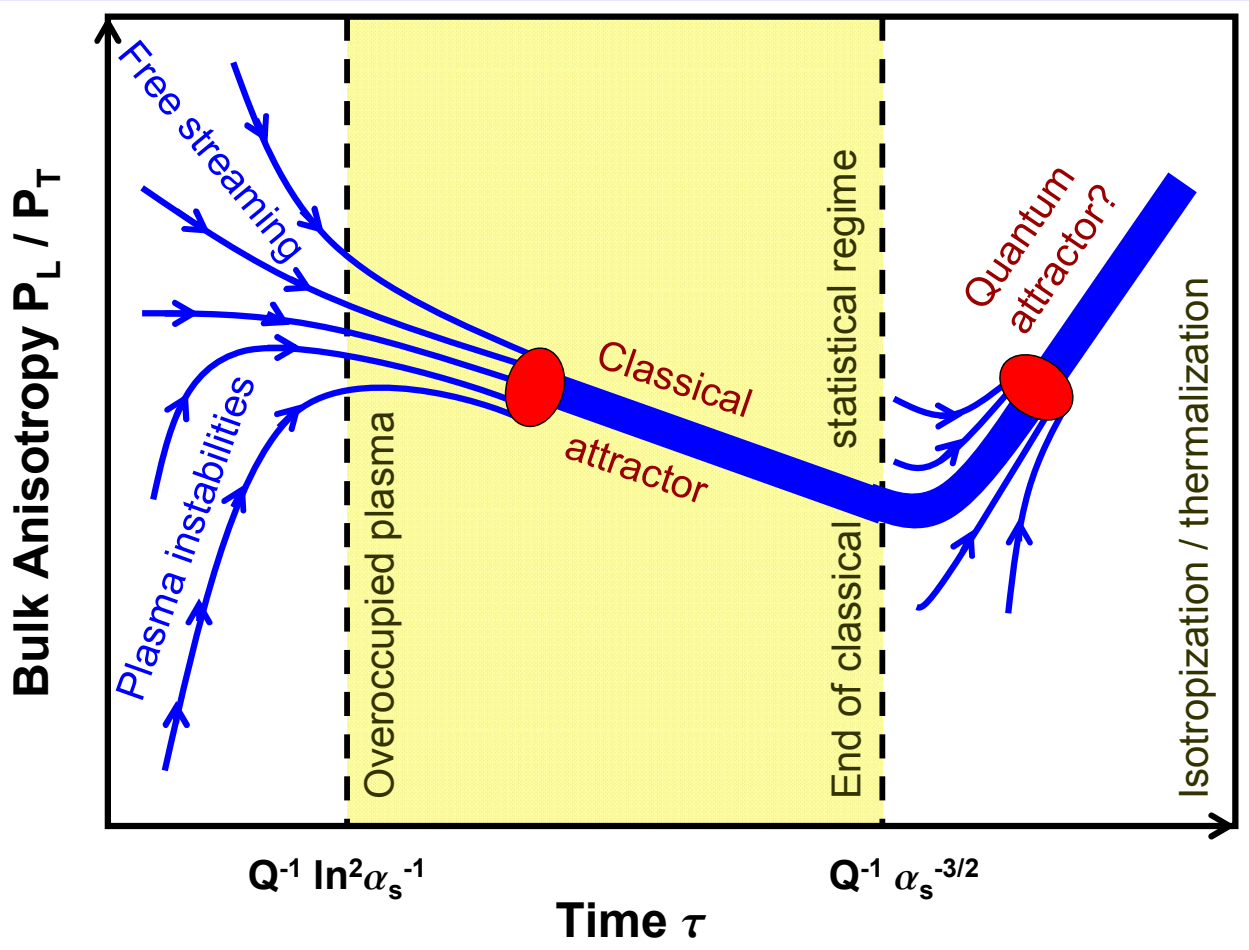
$$f(t, p_\perp, p_z) \sim \tau^{-2/3}\, f_s\left(\tau^0 p_\perp, \tau^{1/3} p_z\right) \qquad \frac{P_L}{P_T} \sim \tau^{-2/3}$$

- Intermediate behavior between free streaming :

$$f(t, p_\perp, p_z) \sim \tau^{0}\, f_s\left(\tau^0 p_\perp, \tau^{1} p_z\right) \qquad \frac{P_L}{P_T} \sim \tau^{-2}$$

and expansion at constant anisotropy :

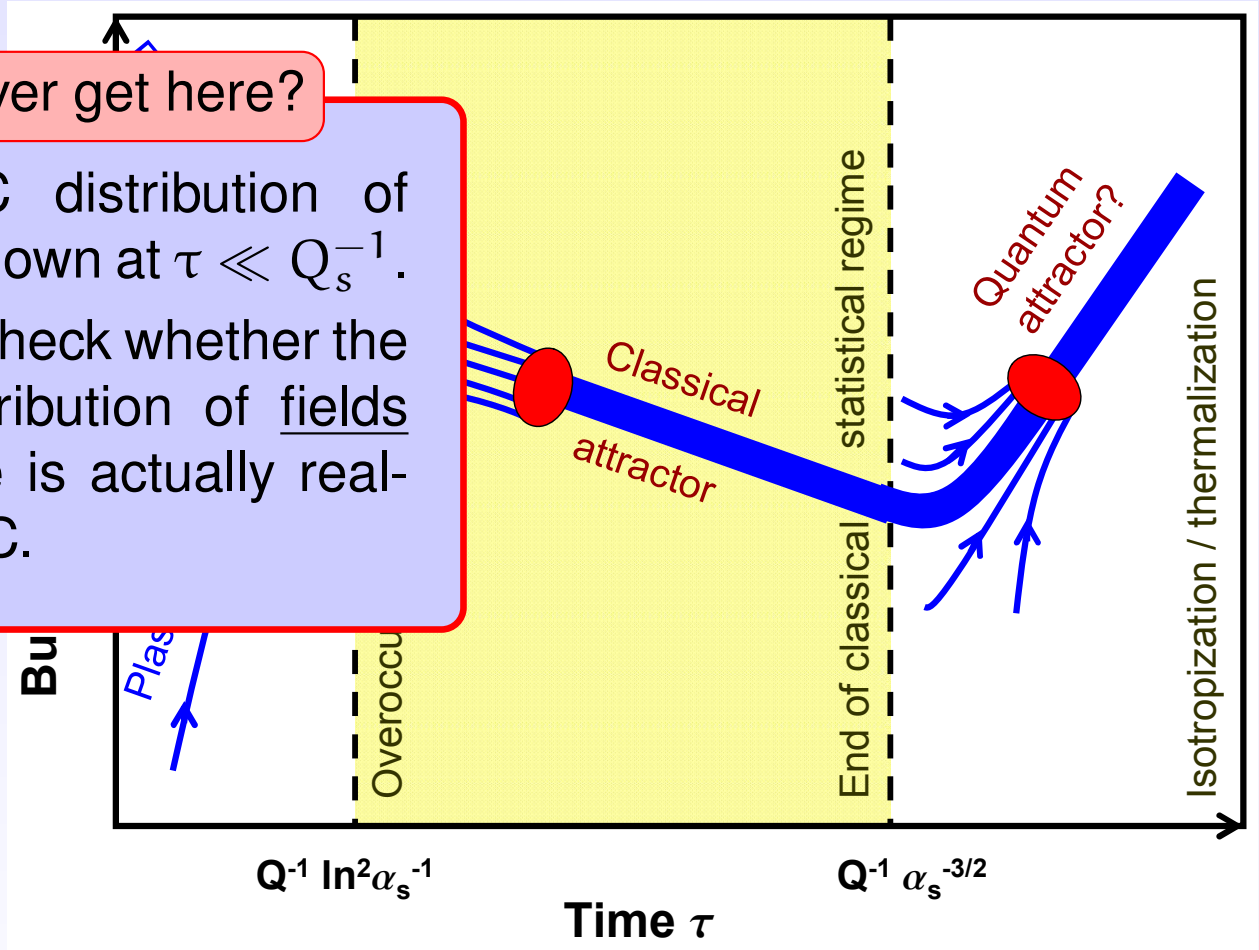
$$f(t, p_\perp, p_z) \sim \tau^{\frac{2(\delta-1)}{2+\delta}}\, f_s\left(\tau^{\frac{\delta}{2+\delta}} p_\perp, \tau^{\frac{\delta}{2+\delta}} p_z\right) \qquad \frac{P_L}{P_T} = \delta$$


Bulk Anisotropy P_L / P_T
Free streaming
Plasma instabilities
Overoccupied plasma
Classical
attractor
statistical regime
End of classical
Quantum
attractor?
Isotropization / thermalization
Q^{-1} $\ln^2\alpha_s^{-1}$
Q^{-1} $\alpha_s^{-3/2}$
Time τ

Does it ever get here?

The CGC distribution of fields is known at $\tau \ll Q_s^{-1}$.

One can check whether the initial distribution of <u>fields</u> used here is actually realized in HIC.



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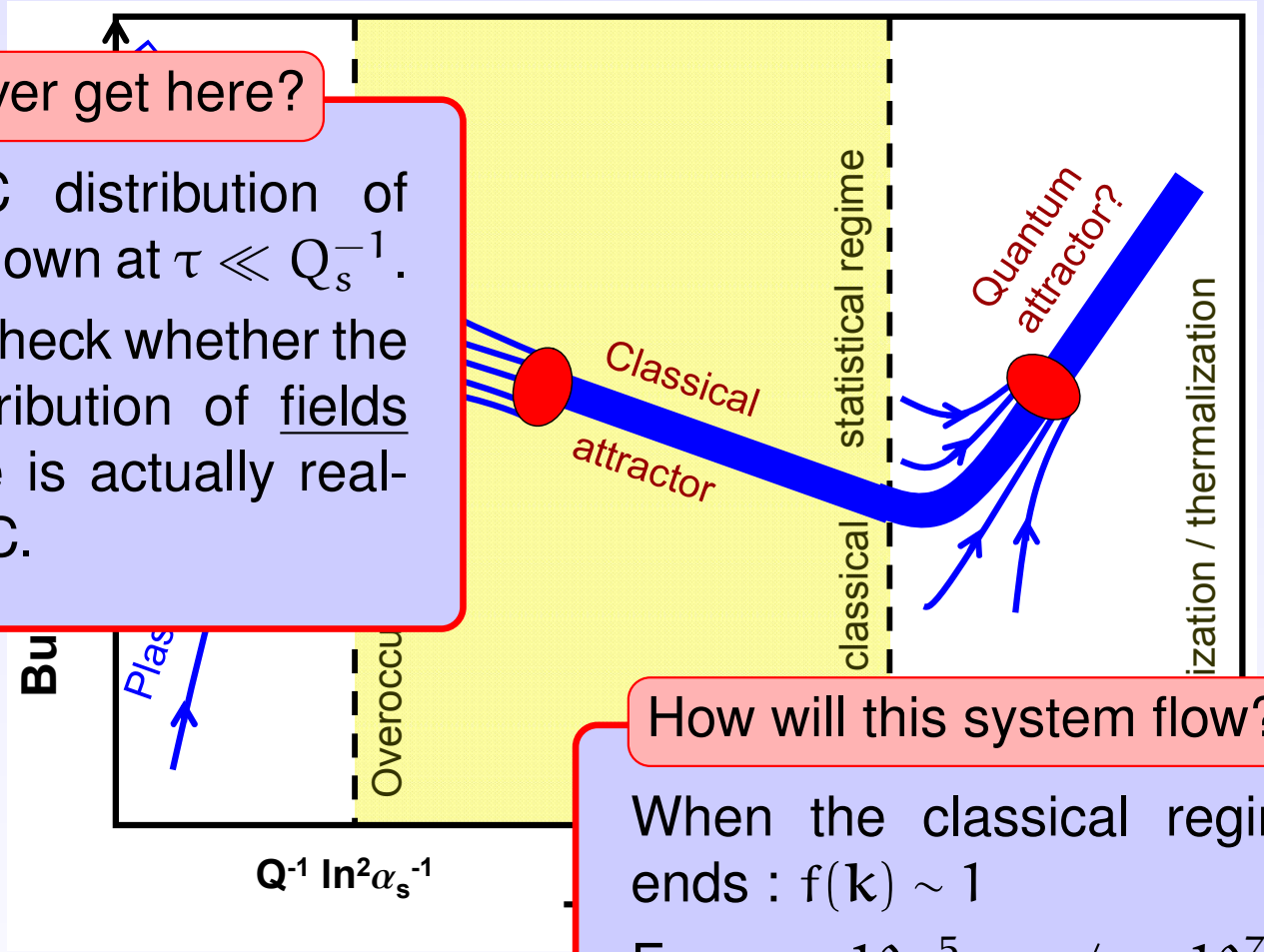
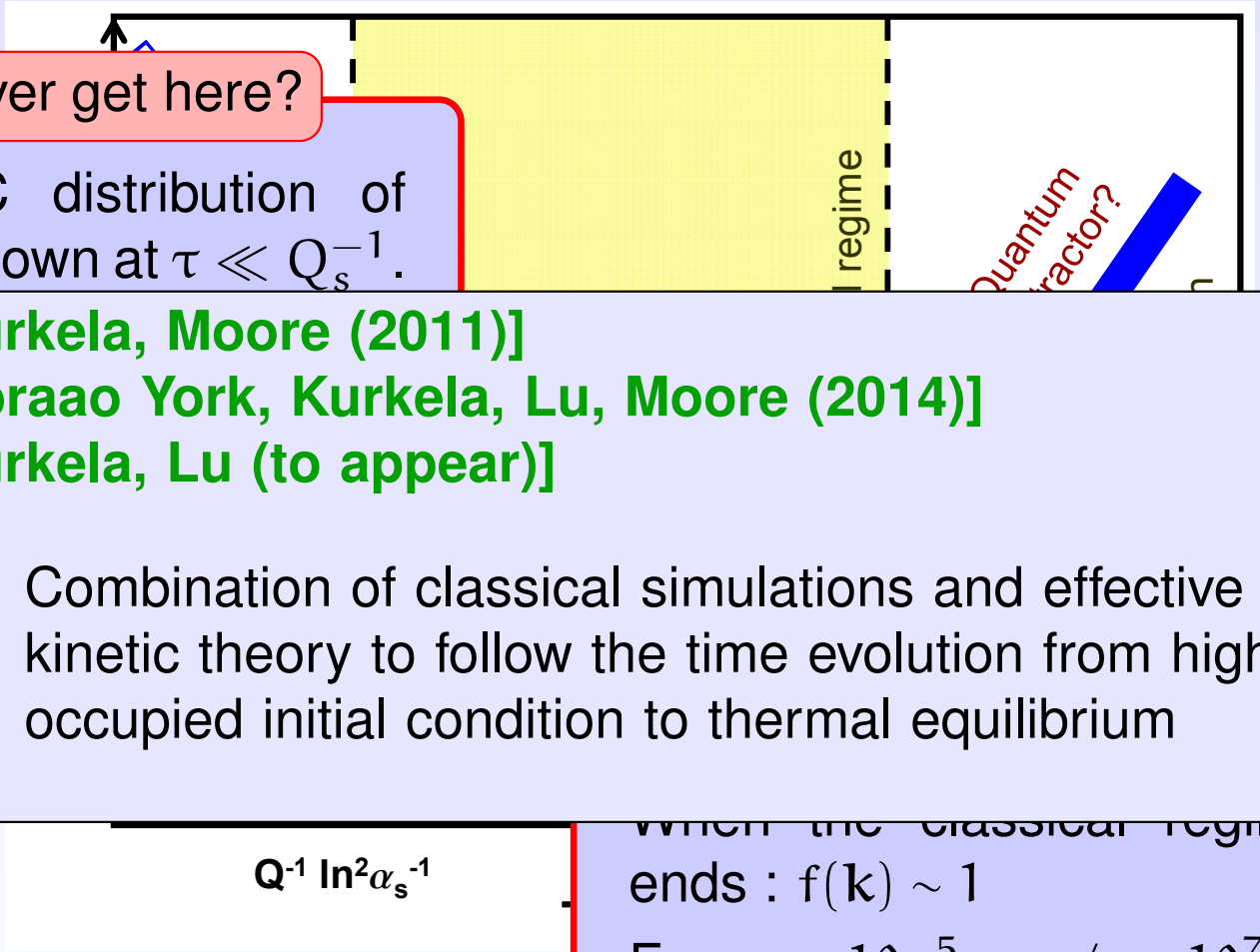


How will this system flow?

When the classical regime ends : $f(k) \sim 1$

For $\alpha_s \sim 10^{-5}$: $\eta/s \sim 10^7$



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One c
initial
used
ized in

[Kurkela, Moore (2011)]
[Abraao York, Kurkela, Lu, Moore (2014)]
[Kurkela, Lu (to appear)]

- Combination of classical simulations and effective kinetic theory to follow the time evolution from highly occupied initial condition to thermal equilibrium

When the classical regime ends : $f(k) \sim 1$

For $\alpha_s \sim 10^{-5}$: $\eta/s \sim 10^7$

Bose-Einstein condensation?

[Blaizot, FG, Liao, McLerran, Venugopalan (2012)]

In thermal equilibrium :

$$\epsilon \sim T^4 \quad n \sim T^3 \qquad n\,\epsilon^{-3/4} \sim 1$$

In the CGC at $\tau \lesssim Q_s^{-1}$:

$$\epsilon \sim \frac{Q_s^4}{g^2} \quad n \sim \frac{Q_s^3}{g^2} \qquad n\,\epsilon^{-3/4} \sim g^{-1/2}$$

- At weak coupling, and if the evolution is number conserving, there is a mismatch : the particles in excess condense in the zero mode
- Observed in classical statistical simulations (for scalar fields) **[Epelbaum, FG (2011)] [Berges, Sexty (2012)]**
  and in kinetic theory with $2 \to 2$ scattering (in QCD) **[Blaizot, Liao, McLerran (2013)]**

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In thermal equilibrium :

$$\epsilon \sim T^4 \quad n \sim T^3 \qquad n\,\epsilon^{-3/4} \sim 1$$

In the C

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  zer

- With number changing processes : transient BEC
- BEC also appears in expanding systems
- If this happened also in heavy ion collisions, phenomenological implications unclear at this point (easier isotropization,...?)

  [Floerchinger, Wetterich (2013)]

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  [Epelbaum, FG (2011)] [Berges, Sexty (2012)]
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  [Blaizot, Liao, McLerran (2013)]

Summary

Color Glass Condensate

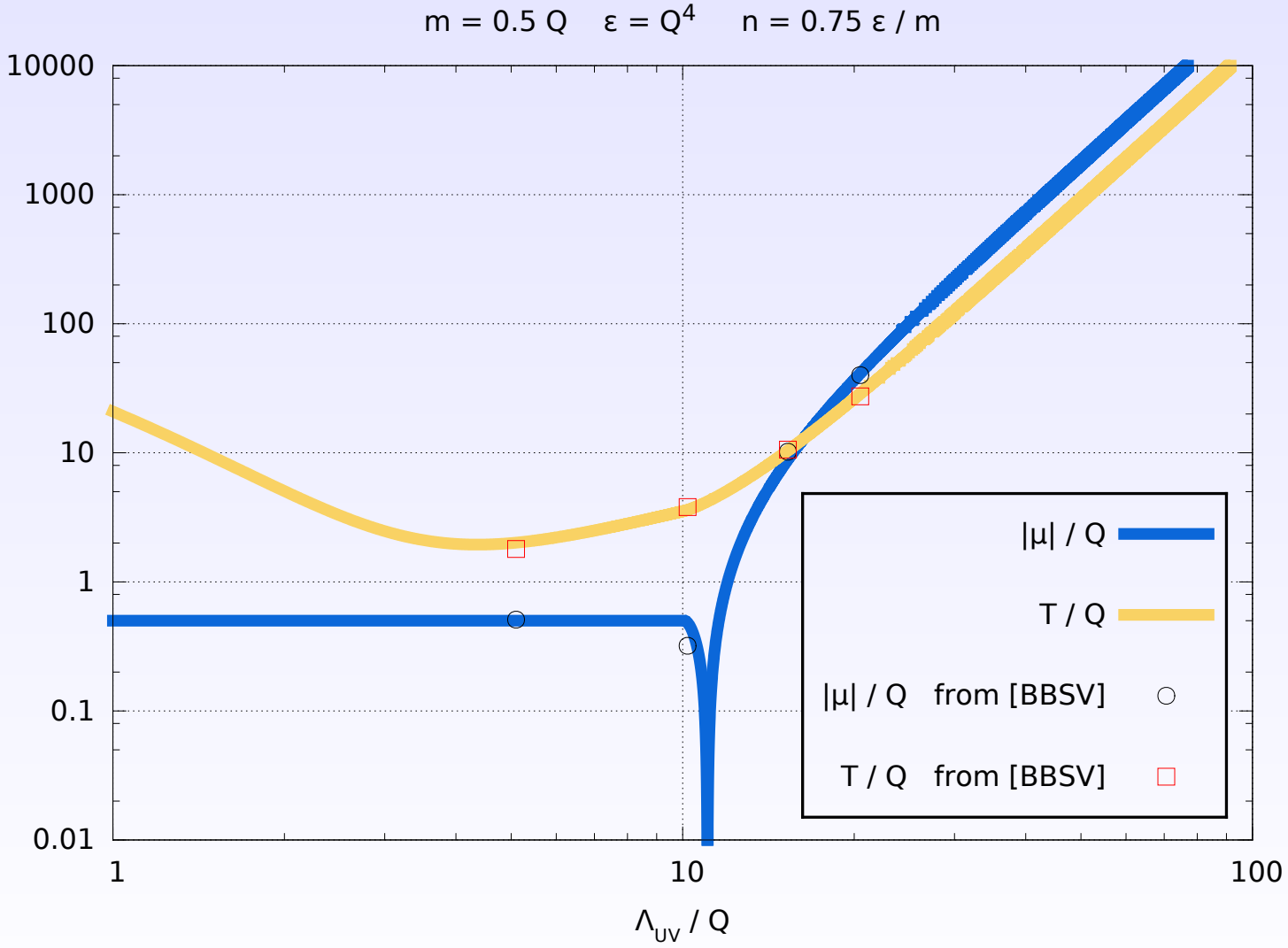
- Nucleus-nucleus collision at high energy
  $\Rightarrow$ Gluon saturation $\Rightarrow$ Color Glass Condensate
- Weakly coupled, but strongly interacting, effective theory : $g\mathcal{A} \sim 1$
- B-JIMWLK evolution equation now obtained at Next-to-Leading Log accuracy. Still early for a practical implementation

Post-collision evolution

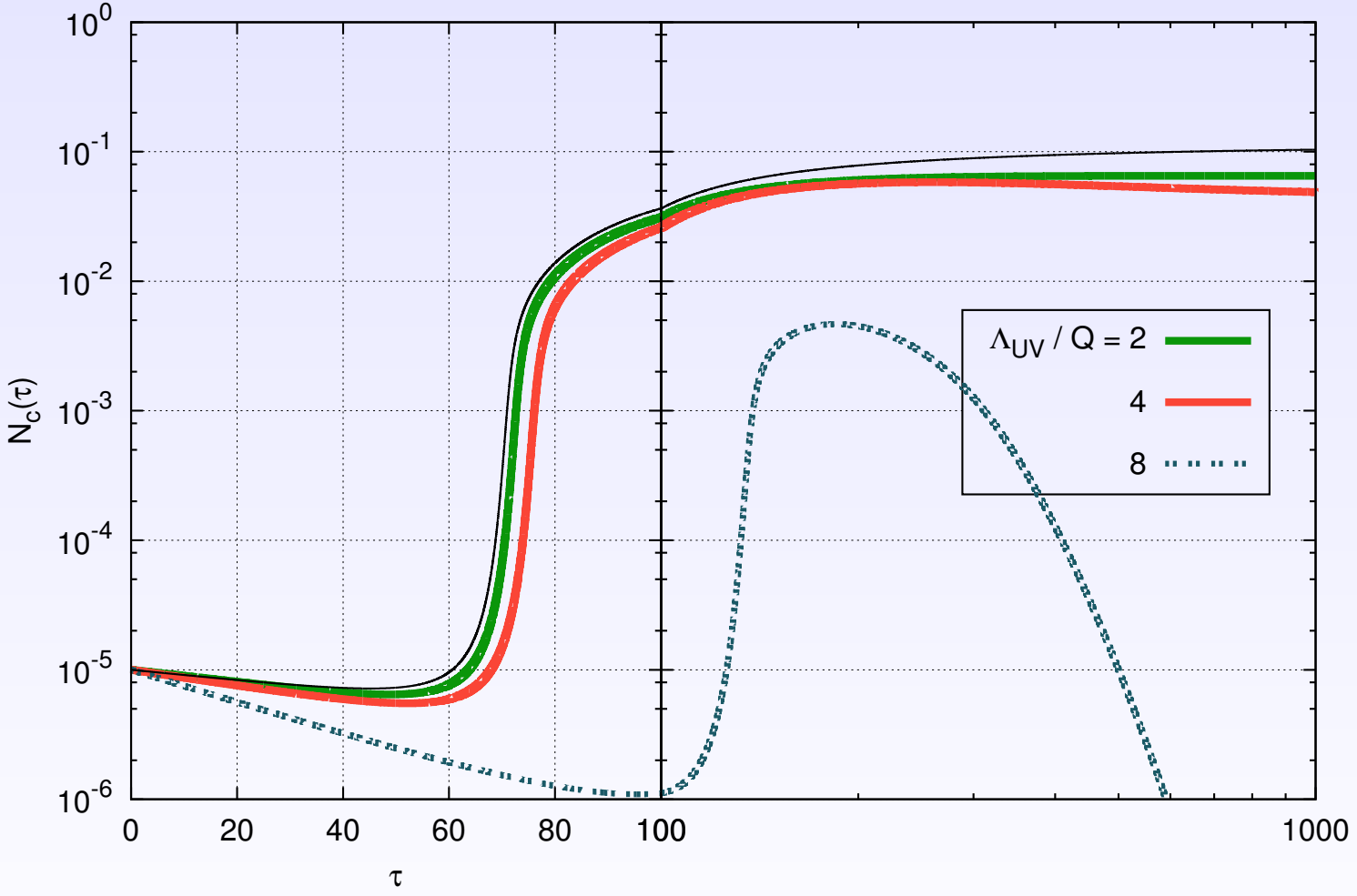
- LO : no pressure isotropization, NLO : instabilities
- Strong fields + fluctuations $\Rightarrow$ Classical statistical approximation
- Two implementations... and two different results :
  - **a.** one with a much more solid lattice setup
  - **b.** one with initial conditions under better (analytical) control
- The reason of the difficulties encountered in **(b)** is the very nature of the CGC initial conditions... Highly needed : ways to avoid the problems of the classical statistical approximation
- Bose-Einstein condensation?

Extra Bits

UV cutoff dependence (with vacuum fluctuations)

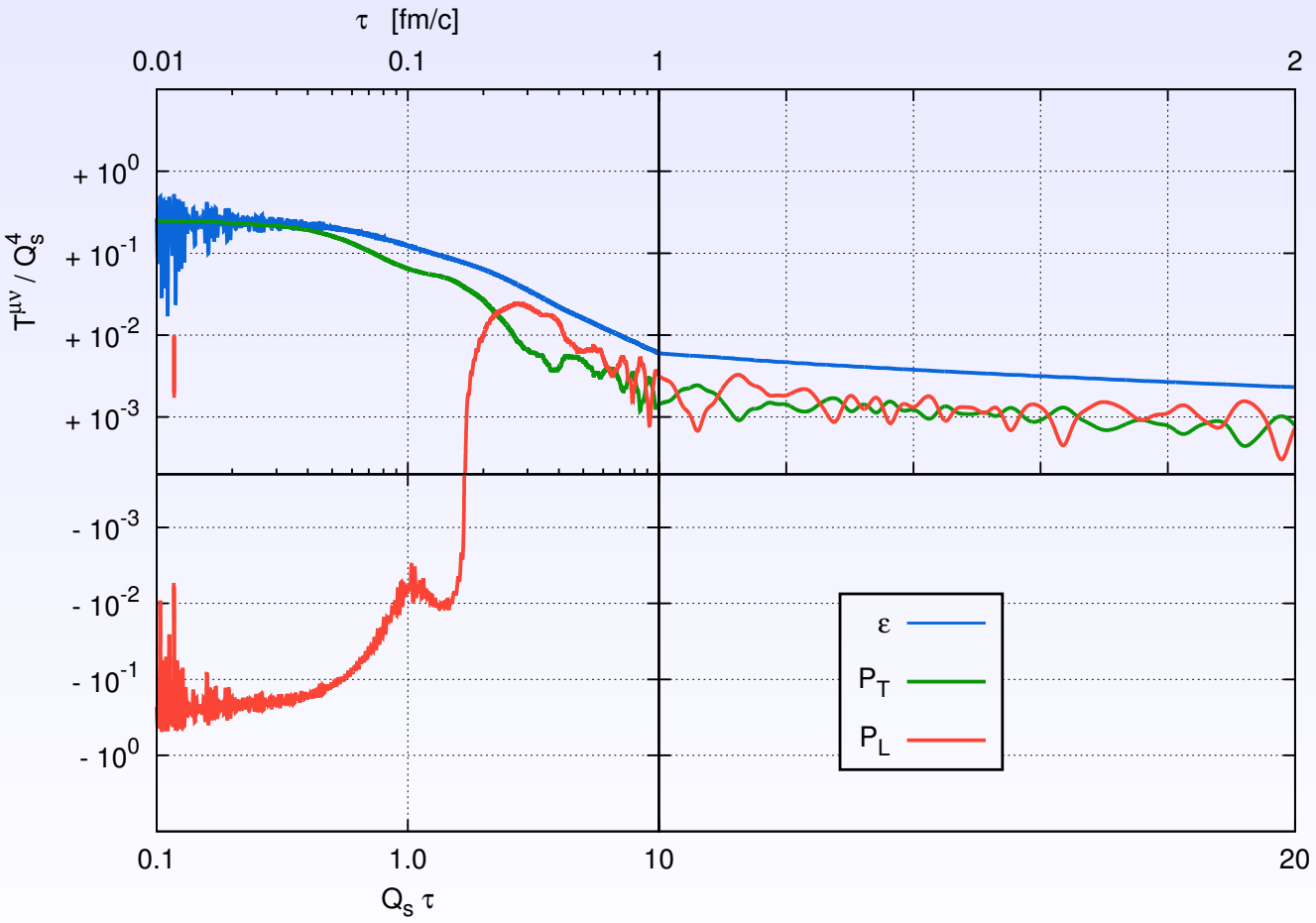


UV cutoff dependence and BEC

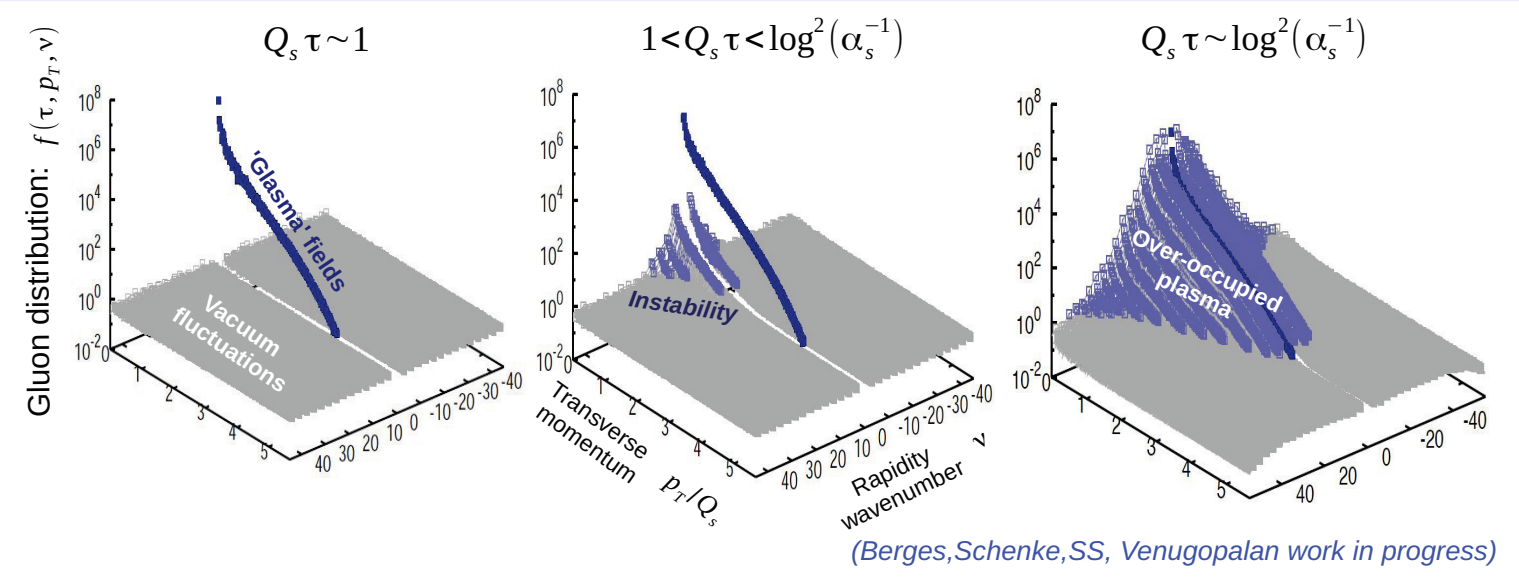


Isotropization at $g = 0.5$ revisited

MV background guaranteed “typical” by rolling the dice until it falls in the middle of the energy distribution...



BBSV's check of their initial conditions



BBSV's check of their initial conditions

1. Start with CGC initial fields at $Q_s\tau \ll 1$
2. Evolve these fields to $Q_s\tau \sim \log^2(\alpha_s^{-1})$
3. Compute some gauge-dependent occupation number from the evolved fields
4. Discard the fields
5. Make up some new fields as superposition of free plane waves, that lead to a similar $f(\mathbf{k}_\perp, \nu)$
6. Resume the YM evolution from these new fields

BBSV's check of their initial conditions

Why all this voodoo...?

Why not continue with the original fields?

3. Compute some gauge-dependent occupation number from the evolved fields
4. Discard the fields
5. Make up some new fields as superposition of free plane waves, that lead to a similar $f(\mathbf{k}_\perp, \nu)$
6. Resume the YM evolution from these new fields

Scaling laws from Kinetic theory

[BBSV (2013)]

- Assume : $f(\tau, \mathbf{k}_\perp, k_z) \equiv (Q\tau)^\alpha \, f_s\left((Q\tau)^\beta \mathbf{k}_\perp, (Q\tau)^\gamma k_z\right)$

1.	Small angle scattering :	$2\alpha - 2\beta + \gamma = -1$
2.	Particle number conservation :	$\alpha - 3\beta - \gamma = -1$
3.	Energy conservation :	$\alpha - 2\beta - \gamma = -1$

$\Rightarrow \alpha = -\frac{2}{3}, \beta = 0, \gamma = \frac{1}{3}$

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$\Rightarrow \alpha = -\frac{2}{3}, \beta = 0, \gamma = \frac{1}{3}$

- **BUT** : (**3**) is not "energy conservation". It is rather "energy conservation in a comoving volume", which is not always true

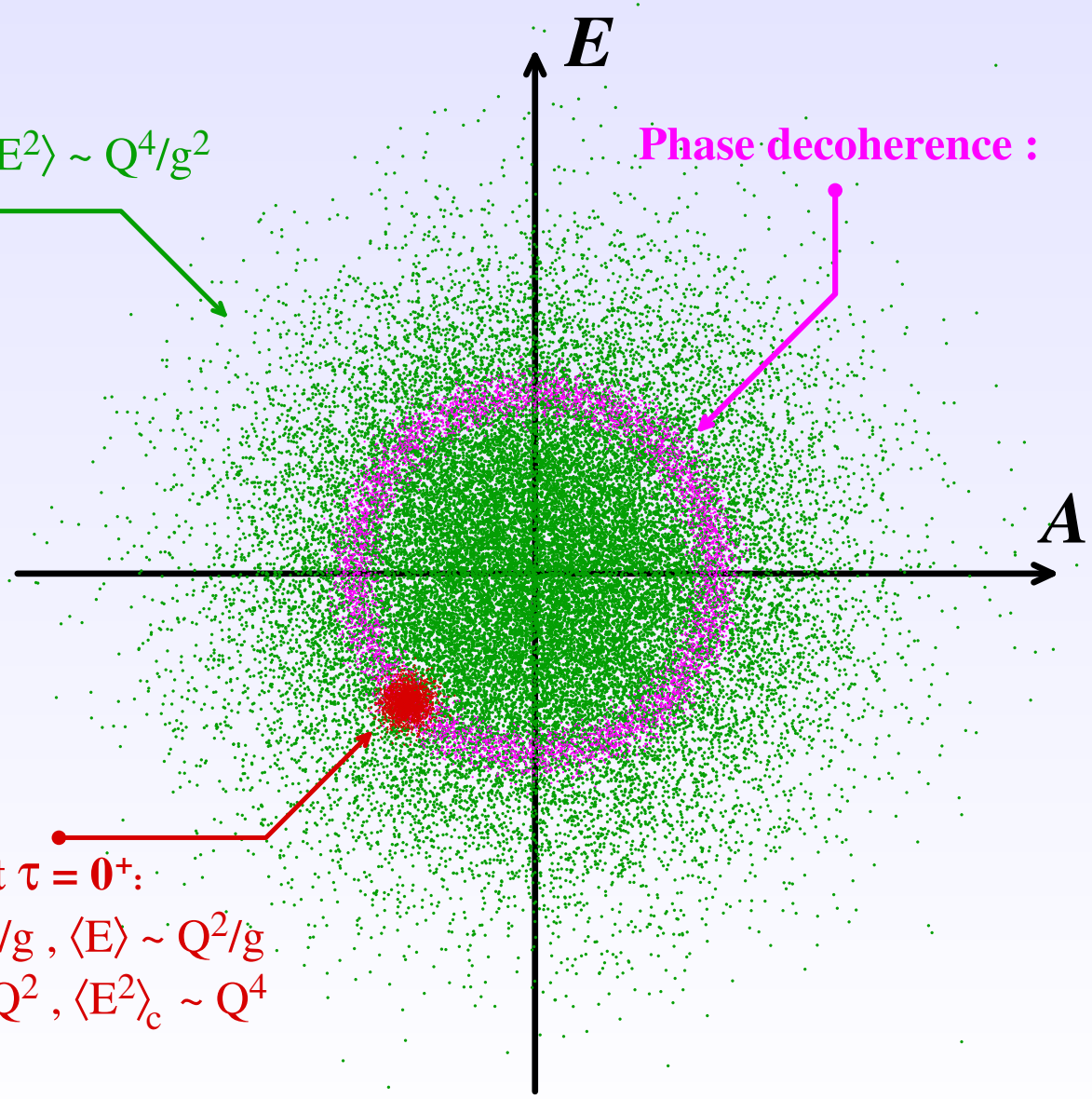
  Only true if $P_L \approx 0$. By relaxing this hypothesis, other scaling solutions with different exponents α, β, γ are possible

Phase decoherence versus Full decoherence

BBSV :

$\langle A \rangle$, $\langle E \rangle = 0$

$\langle A^2 \rangle \sim Q^2/g^2$, $\langle E^2 \rangle \sim Q^4/g^2$



Phase decoherence :

CGC at $\tau = 0^+$:

$\langle A \rangle \sim Q/g$, $\langle E \rangle \sim Q^2/g$

$\langle A^2 \rangle_c \sim Q^2$, $\langle E^2 \rangle_c \sim Q^4$