Quartetting in fermion systems with differing chemical potentials

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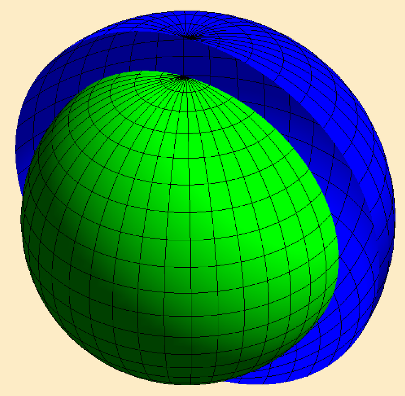
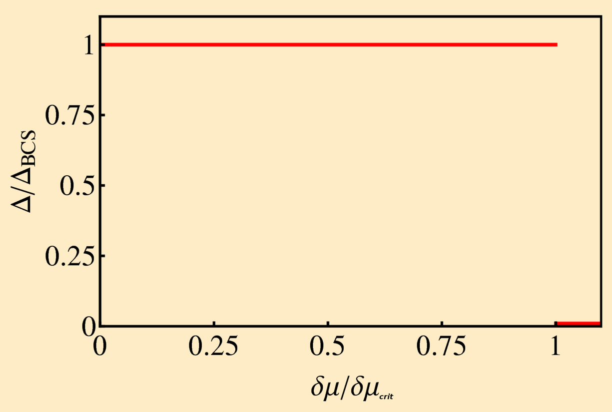
Differing Fermi Momenta - General Aspects





- ultracold atom systems
- solids
- quark matter
- neutron stars

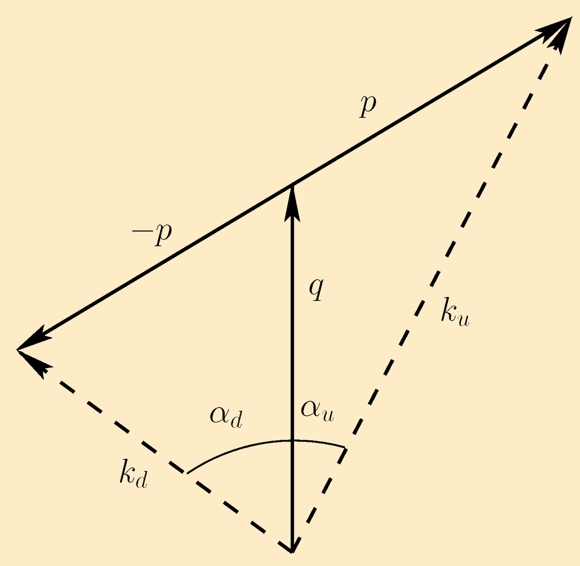
Chandrasekhar [1] and Clogston [2]:

$$\delta\mu_{\text{crit}} = \frac{\Delta_0}{\sqrt{2}}$$

$\delta\mu = \delta\mu_{\text{crit}}$: 1^{st} order

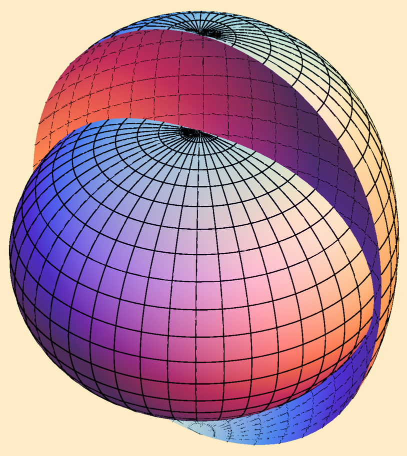
Crystalline Condensation (LOFF)

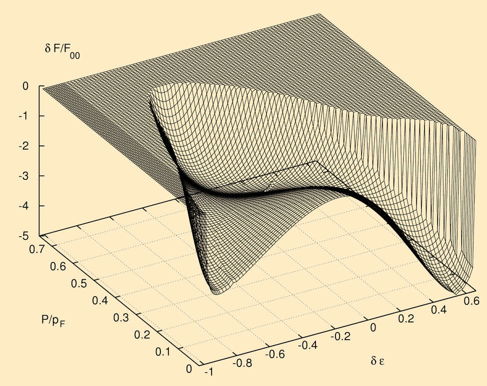
Independently published by Larkin and Ovchinnikov [5], as well as Fulde and Ferrell [6].



- Near $\delta\mu_{\text{crit}}$: LOFF condensate ($\mathbf{q}+\mathbf{p}$, $\mathbf{q}-\mathbf{p}$) favored
- translational and rotational not invariant
- Condensate varies as plane wave with $2\mathbf{q}$
- crystalline structure, $\mathbf{\Delta}(\mathbf{r}) = \cos(2\mathbf{q}\cdot\mathbf{r})$

Deformed Fermi Surfaces (DFS)





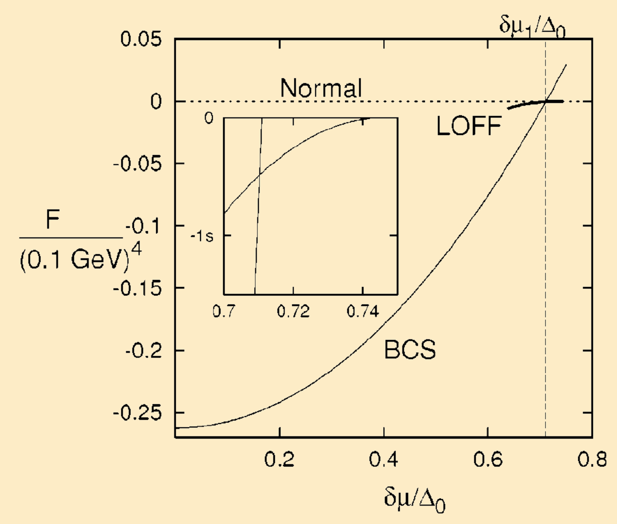
Proposed by Müther and Sedrakian, see [3] and [4].

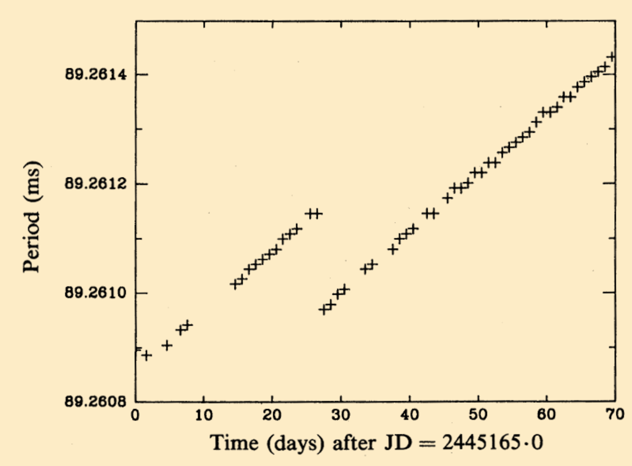
Left: Deformed fermi surfaces, $\mu_{\mathbf{f}} = \bar{\mu}_{\mathbf{f}}(1 \pm \varepsilon_{\mathbf{A}} \sin^2 \vartheta)$

Right: Free energy DFS vs. LOFF. *Is DFS the true ground state?*

LOFF in QCD

Left: The role of LOFF in QCD has been investigated by Alford, Bowers and Rajagopal [7]. *Right:* A pulsar glitch [8].

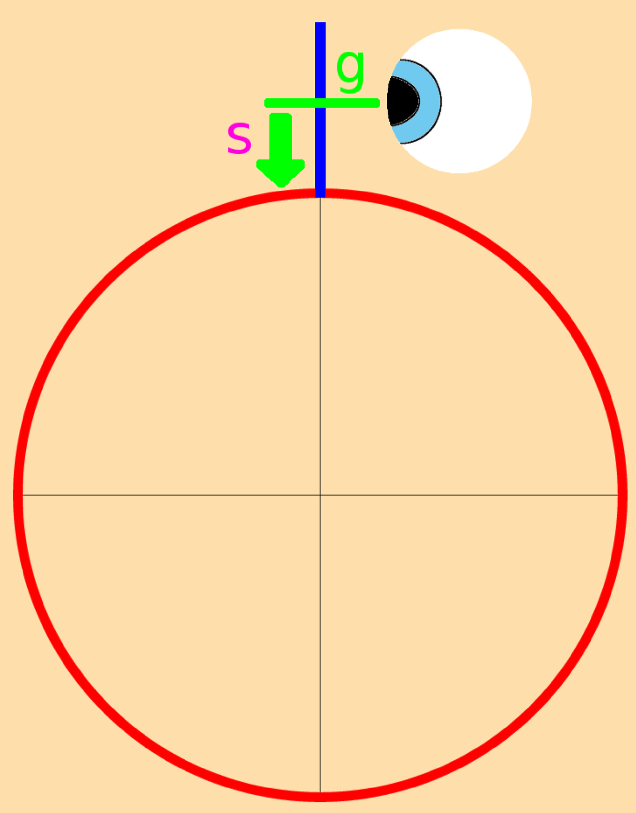




Crystalline LOFF condensation might indicate the presence of quark matter in the interior of neutron stars by offering an explanation for glitches (observable sudden spin-up of a star).

Four fermion condensation?

RG-scaling of couplings.



weak coupling, $\delta\mu \ll \Delta$

- $\langle qq \rangle$: marginal
- $\langle qqqq \rangle$: irrelevant

weak coupling, $\delta\mu \gtrsim \Delta$

- $\langle qq \rangle$: irrelevant
- $\langle qqqq \rangle$: irrelevant

strong coupling, $\delta\mu \gtrsim \Delta$

- $\langle qq \rangle$: suppressed
- $\langle qqqq \rangle$: ?

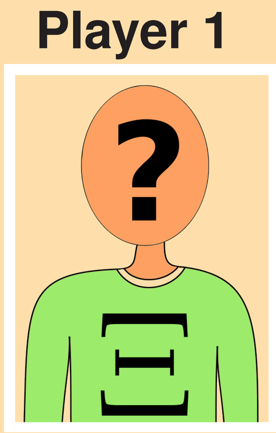
Why quartetting? At large mismatch ($\delta\mu$) BCS pairing is kinematically suppressed, while a quartet can overcome this restriction.

Fermion Quartetting: Bosonized Theory

$$\begin{aligned}\mathscr{L} \;=\; & \bar{\psi}^\alpha_A \left(\not{\partial}_\mu - (\mu + \delta\mu\sigma_3)\gamma^4 + m\right)\psi^\alpha_A + \tfrac{1}{2}\left(|\partial_\mu \Xi|\right)^2 + \tfrac{1}{2}\left(|\partial_\mu \Theta|\right)^2 \\ & + \frac{m_\Theta^2}{2}\Theta^{\alpha\beta}_{AB}\varepsilon_{ijkl}c^i_{\alpha A}c^j_{\beta B}c^k_{\gamma C}c^l_{\delta D}\Theta^{\gamma\delta}_{CD} \\ & + \frac{g^\gamma_\Theta}{2}\sqrt{\Xi^*}\,\varepsilon_{ijkl}c^i_{\alpha A}c^j_{\beta B}c^k_{\gamma C}c^l_{\delta D}\Theta^{\alpha\beta}_{AB}\psi^\gamma_C\psi^\delta_D \\ & + \frac{g^\gamma_\Theta}{2}\sqrt{\Xi}\,\varepsilon_{ijkl}c^i_{\alpha A}c^j_{\beta B}c^k_{\gamma C}c^l_{\delta D}\Theta^{\alpha\beta}_{AB}\bar{\psi}^\gamma_C\bar{\psi}^\delta_D \\ & + U(|\Xi|) + g|\Xi||\Theta|^2 + m|\Theta|^2\end{aligned}$$

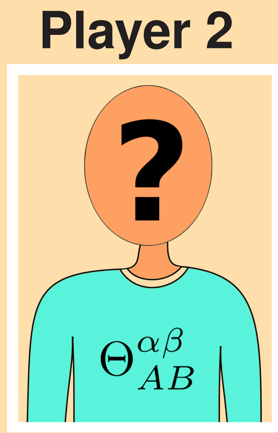
New fields after bosonization

Player 1



- *Name:* Ξ
- *Species:* Boson
- *Occupation:* complex scalar
- *Represents:* 4-fermion condensate

Player 2



- *Name:* $\Theta^{\alpha\beta}_{AB}$
- *Species:* Ghost-like tensor field
- *Occupation:* real, 0 baryon number
- *Represents:* 2-fermion pairing

Fermion Quartetting: Toy model $SU(2)_c \otimes SU(2)_f$

$$\mathscr{L}_{int} = g_8\left(\psi^\alpha_A\psi^\beta_B T^{\alpha\beta\gamma\delta}_{ABCD}\psi^\gamma_C\psi^\delta_D\right)\left(\bar{\psi}^\epsilon_E\bar{\psi}^\zeta_F T^{\epsilon\zeta\eta\theta}_{EFGH}\bar{\psi}^\eta_G\bar{\psi}^\theta_H\right),$$

$$T^{\alpha\beta\gamma\delta}_{ABCD} = \varepsilon_{ijkl}c^i_{A\alpha}c^j_{B\beta}c^k_{C\gamma}c^l_{D\delta},$$

where each $c^i_{A\alpha}$ picks one representative out of $\underbrace{SU(2)_c}_{\text{index } A} \otimes \underbrace{SU(2)_f}_{\text{index } \alpha}$.

Flow equation

$$\frac{\partial}{\partial k}\Gamma_k = \frac{1}{2}\mathbf{Tr}\left\{\left[\Gamma^{(2)}_k + R_k\right]^{-1}\frac{\partial}{\partial k}R_k\right\}$$

Acknowledgments

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