Master equations of quarkonia in the Lindblad form

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Abstract: Understanding the quantum dynamics of quarkonia is essential in the description of bottomonia and charmonia in the quark-gluon plasma. So far, it has been quite naively assumed that their dynamics can be described by the Schrödinger equation with in-medium, screened potential. Such a naïve approach is not correct anymore, in particular if one wants to study their time-evolution. After the discovery of imaginary part in the in-medium potential [1], it must be recognized that quarkoia should be viewed as open quantum systems in the environment of quark-gluon plasma [2].

In the open quantum system, master equation instead of the Schrödinger equation describes the quantum dynamics of quarkonia. Open quantum system techniques, such as influence functional approach, have been applied to quarkonia [3]. In this presentation, I will summarize developments in this approach and show an additional step toward a more complete description. In particular, I will show how to obtain the Lindblad-form master equation, which preserves the complete positivity of the density matrix of the system [4].

1. Introduction

❖ We study quantum dynamics of quarkonia in the QGP.

Why is it necessary?

- Fate of quarkonia in the QGP is sensitive to the deconfinement nature.
- Heavy-ion collision creates a dynamical system with time evolution.

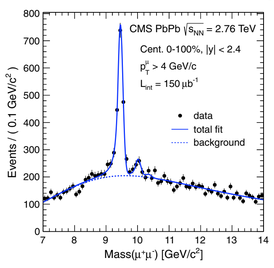
How is it formulated?

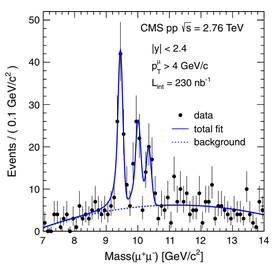
- Open quantum system approach provides a natural framework.
- By a master equation for a density matrix, instead of the Schrödinger equation for a wave function

What are the experimental signals?

- Dimuons from bottomonium decays [5] (CMS collaboration)

PbPb collisions





pp collisions

2. Open quantum system

❖ This is what we want to derive.

Total Hilbert space

- System = heavy quarks
- Environment = light quarks + gluons
- Physical state: density matrix
- Time evolution: von Neumann equation

$$\mathrm{H}_{\mathrm{tot}} = \mathrm{H}_{\mathrm{sys}} \otimes \mathrm{H}_{\mathrm{env}}$$

$$\frac{d}{dt}\hat{\rho}_{\mathrm{tot}}(t) = \frac{1}{i}\left[\hat{H}_{\mathrm{tot}}, \hat{\rho}_{\mathrm{tot}}(t)\right]$$

Effective dynamics of the system

- Reduced density matrix: Trace out the environment
- Time evolution: master equation
- Coarse graining in time is essential.
- Dynamical time scale of the system is slow.
- Reduced dynamics is approximated as Markovian.

$$\hat{\rho}_{\mathrm{red}}(t) \equiv \mathrm{Tr}_{\mathrm{env}}\left[\hat{\rho}_{\mathrm{tot}}(t)\right]$$

$$\frac{d}{dt}\hat{\rho}_{\mathrm{red}}(t) = ?$$

Lindblad form of the master equation

- Conserves positivity of ρ_{red}
- Otherwise positivity is not guaranteed.

$$\frac{d}{dt}\hat{\rho}_{\mathrm{red}}(t) = \frac{1}{i}\left[\hat{H}, \hat{\rho}_{\mathrm{red}}\right] + \sum_{i=1}^{N}\gamma_i\left(\hat{L}_i\hat{\rho}_{\mathrm{red}}\hat{L}_i^\dagger - \frac{1}{2}\hat{L}_i^\dagger\hat{L}_i\hat{\rho}_{\mathrm{red}} - \frac{1}{2}\hat{\rho}_{\mathrm{red}}\hat{L}_i^\dagger\hat{L}_i\right)$$

$$\hat{H}^\dagger = \hat{H}, \quad \gamma_i > 0$$

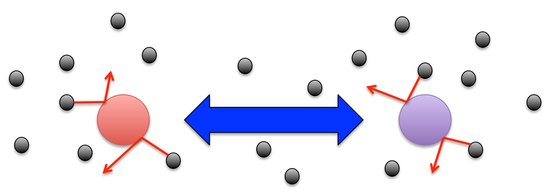


Fig.1

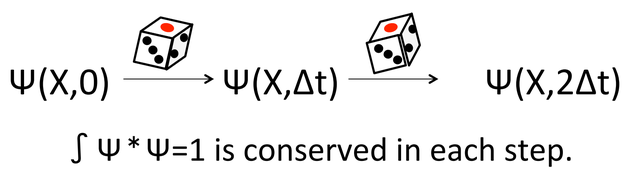
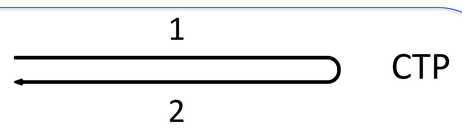


Fig.2

3. Influence functional for heavy quarks

❖ Path integral on the closed-time path

Effective action on CTP



- Influence functional at leading order in g

$$\exp\left[iS^{\mathrm{IF}}[j_1, j_2]\right] \approx \exp\left[-g^2/2\int j_1 G_A^{\mathrm{F}} j_1 + j_2 G_A^{\tilde{\mathrm{F}}} j_2 - j_1 G_A^{>} j_2 - j_2 G_A^{<} j_1\right]$$

$$G_A^{\mathrm{F}}(x-y) = \left\langle \mathrm{T}\hat{A}(x)\hat{A}(y)\right\rangle_T,$$
$$G_A^{<}(x-y) = \left\langle \hat{A}(y)\hat{A}(x)\right\rangle_T,$$
$$G_A^{>}(x-y) = \left\langle \hat{A}(x)\hat{A}(y)\right\rangle_T,$$
$$G_A^{\tilde{\mathrm{F}}}(x-y) = \left\langle \tilde{\mathrm{T}}\hat{A}(x)\hat{A}(y)\right\rangle_T.$$

Approximations

- Non-relativistic limit: color density interaction only
- Time coarse graining: Heavy quark dynamics is much slower than medium time scale.
- How to choose "time t" from x^0 and y^0 is subtle but very important.

$$\left(x^0, y^0\right) \Leftrightarrow t \equiv \max\left(x^0, y^0\right), s \equiv \left|x^0 - y^0\right|$$

- Medium correlation lasts for much shorter than heavy quark time scale.
- Time coarse graining by derivative expansion in t

$$-g^2 G^R_{ab,00}(\omega = 0, r) \equiv V(r)\delta_{ab}$$
$$-g^2 G^>_{ab,00}(\omega = 0, r) \equiv D(r)\delta_{ab}$$

Influence functional (Fig.1 for classical picture)

- Zeroth order: screened potential and decoherence (Lindblad form)
- First order: momentum dissipation (not Lindblad form)
- Second order: momentum dissipation (Lindblad form)

$$S^{\mathrm{IF}}[j_1, j_2] \cong -\frac{1}{2}\int_{t,\vec{x},\vec{y}}\left(\rho_1^a, \rho_2^a\right)_{(t,\vec{x})}\begin{bmatrix} V + iD & -iD \\ -iD & -V + iD \end{bmatrix}_{(\vec{x}-\vec{y})}\begin{pmatrix}\rho_1^a \\ \rho_2^a\end{pmatrix}_{(t,\vec{y})} \quad \text{Zeroth order}$$

$$-\frac{1}{4T}\int_{t,\vec{x},\vec{y}}\left(\rho_1^a, \rho_2^a\right)_{(t,\vec{x})}\begin{bmatrix} -D & -D \\ D & D \end{bmatrix}_{(\vec{x}-\vec{y})}\begin{pmatrix}\dot{\rho}_1^a \\ \dot{\rho}_2^a\end{pmatrix}_{(t,\vec{y})} \quad \text{First order}$$

$$-\frac{i}{24T^2}\int_{t,\vec{x},\vec{y}}\left(\rho_1^a, \rho_2^a\right)_{(t,\vec{x})}\begin{bmatrix} -D & D \\ D & -D \end{bmatrix}_{(\vec{x}-\vec{y})}\begin{pmatrix}\ddot{\rho}_1^a \\ \ddot{\rho}_2^a\end{pmatrix}_{(t,\vec{y})} \quad \text{Second order}$$

❖ Stochastic potential and decoherence

Dissociation of quarkonium bound states

- Decoherence and screening are the essential physics (zeroth order).
- Stochastic potential is a handy wave function description of decoherence.

$$i\frac{\partial}{\partial t}\Psi_{Q\bar{Q}}(t,\vec{r}) = \begin{bmatrix} -\frac{\nabla_r^2}{M} + iC_{\mathrm{F}}D(0) + \left\{-V(r) - iD(r)\right\}(t^a \otimes t^{a*}) \\ +\theta^a(t,\vec{r}/2)(t^a \otimes 1) - \theta^a(t,-\vec{r}/2)(1 \otimes t^{a*}) \end{bmatrix}\Psi_{Q\bar{Q}}(t,\vec{r}),$$

$$\left\langle \theta^a(t,\vec{x})\theta^a(s,\vec{y})\right\rangle = -D(\vec{x}-\vec{y})\delta(t-s)\delta^{ab}$$

(Fig.2 for stochastic quantum evolution)

➡ See also Poster F-47 by Alexander Rothkopf

4. Summary

- Open quantum system approach is applied to quantum dynamics of quarkonium in the QGP.
- Taking up to the 0th or 2nd order in derivative expansion in t (time coarse graining), master equations are obtained in the Lindblad form.
- Quarkonium dissociation can be described by the 0th order, with decoherence and screening.

References:
[1] M. Laine et al., JHEP 0703, 054 (2007); JHEP 0705, 028 (2007); A. Beraudo, J. P. Blaizot and C. Ratti, Nucl. Phys. A806, 312 (2008); N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky, Phys. Rev. D78, 014017 (2008); A. Rothkopf, T. Hatsuda and S. Sasaki, Phys. Rev. Lett. 108, 162001 (2012).
[2] Y. Akamatsu and A. Rothkopf, Phys.Rev. D85, 105011 (2012).
[3] Y. Akamatsu, Phys. Rev. D87, 045016 (2013).
[4] Y. Akamatsu, arXiv:1403.5783 [hep-ph].
[5] S.Chatrchyan et al. (CMS Collaboration), Phys. Rev. Lett. 109, 222301 (2012).