Causal hydrodynamic fluctuation and its implementation in full (3+1)-D Dissipative hydrodynamic simulation

#194

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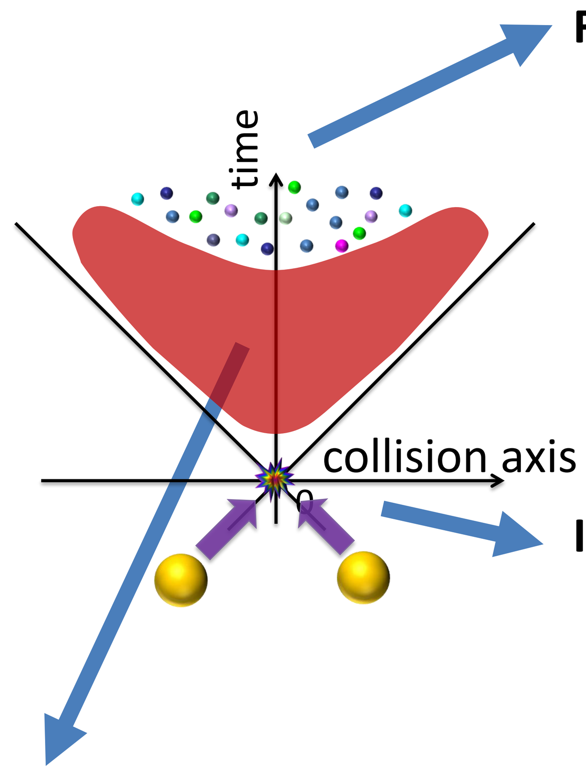
Abstract: While the initial state fluctuations are actively studied in dynamical models to investigate event-by-event fluctuation, there are other sources of fluctuations called hydrodynamic fluctuations arising in the space-time evolution of QGP. We show non-trivial natures of hydrodynamic fluctuations in relativistic system, and implement them in a numerical simulation.

1. Introduction

Event-by-event fluctuations in high-energy heavy-ion collisions

Recently in heavy-ion collisions, it is actively discussed to constrain the *initial-state models* and the *transport properties* of the created QGP using the observed *event-by-event fluctuations*.



Final state observables
– flow harmonics v_n
– two-particle correlations, ...

Response of the created matter

Initial state fluctuation
– nucleon distribution
– quantum fluctuations

Event-by-event dynamical simulations

Quantitative comparison between the results and experimental data

→ Transport properties
η, ζ, τ, λ,...
→ Initial state
Glauber/CGC

Hydrodynamic fluctuation = the thermal fluctuation of hydrodynamics

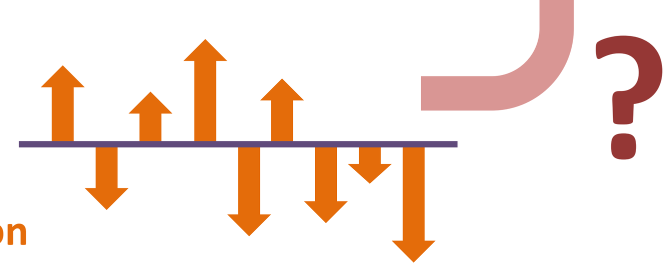
It is already revealed that the *initial-state fluctuation* play an important role in the event-by-event fluctuations. However, there is a still other source of event-by-event fluctuations called ***hydrodynamic fluctuation***.

e.g., in pressure,

$$P+\Pi = \underline{P(e,n)} - \underline{\zeta\theta} \quad + \underline{\delta\Pi},$$

EoS + Viscosity — Hydro Fluctuation = Noise term

stochastic differential equation like *Langevin equation*



Hydrodynamic fluctuation arises independently *in each space-time points*

Hydrodynamic fluctuation in an integrated dynamical model

The hydrodynamic fluctuation has an effect on observables such as the flow harmonics v_n. In order to extract the quantitative properties of the matter, the effect of the hydrodynamic fluctuation should be investigated in dynamical models.

2. Colored noise in causal hydrodynamics

In first-order dissipative hydrodynamics

$$\pi^{\mu\nu} = 2\eta\partial^{\langle\mu}u^{\nu\rangle} + \delta\pi^{\mu\nu}$$ Constitutive equation (**CE**) for shear stress

Fluctuation-Dissipation Relation (FDR)
→ Power spectrum of hydrodynamic fluctuation

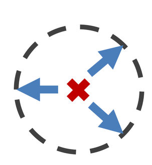
$$\langle\delta\pi^{\mu\nu}(x)\delta\pi^{\alpha\beta}(x')\rangle \propto \delta^{(4)}(x-x')$$ ***White Noise***

→ However, the equation has *acausal modes*, and *unstable modes*.

In causal dissipative hydrodynamics with non-zero relaxation times

Differential form of CE

$$\pi^{\mu\nu} = -\tau_\pi D\pi^{\langle\mu\nu\rangle} + 2\eta\partial^{\langle\mu}u^{\nu\rangle} + \cdots$$



Relaxation in a light cone

- Relaxation time τ_π: non-zero due to the causality
- Common in recent dynamical calculations

Integral form of CE

Solve

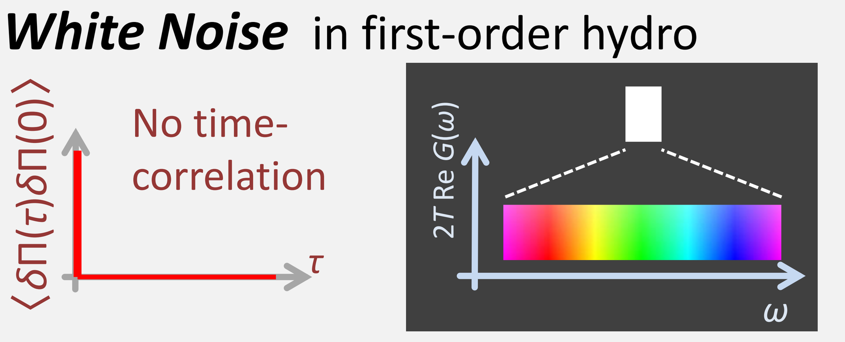
$$\pi^{\mu\nu} = \int_{x^0>x'^0} d^4x' \underline{G_\pi(x-x')^{\mu\nu\alpha\beta}}(\partial_{\langle\alpha}u_{\beta\rangle}|_{x'}) + \delta\pi^{\mu\nu}$$

Memory function G

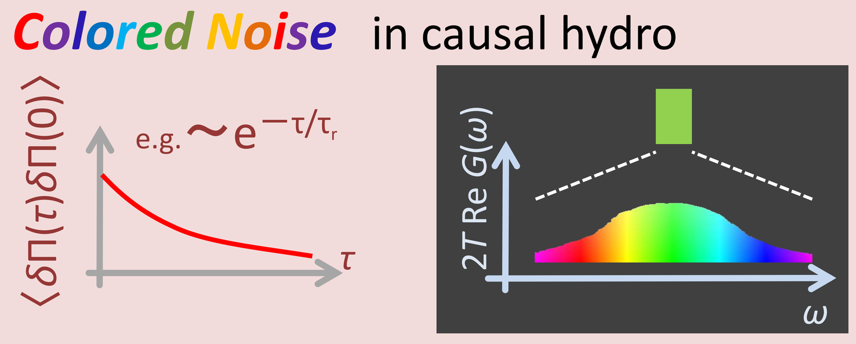
Fluctuation-Dissipation Relation (FDR)

$$\langle\delta\pi^{\mu\nu}(x)\delta\pi^{\alpha\beta}(x')\rangle = \underline{TG'_\pi(x-x')^{\mu\nu\alpha\beta}}$$ always ***Colored Noise***

White Noise in first-order hydro



Colored Noise in causal hydro



3. White noise in differential form of CE

Causal hydrodynamic fluctuation in differential form of CE

For practical calculations, the differential form of CE is useful.
General form of differential form of CE for dissipative current J:

$$\mathcal{L}J = \kappa F + \xi, \qquad \mathcal{L} = 1 + \tau_R D + \cdots$$

1st-order term — Noise term — Linear operator (higher-order terms)

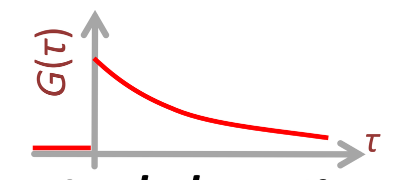
Noise term $$\xi = \mathcal{L}\delta J \qquad \delta J$$ Noise in integral form ***Colored Noise***

Q: Power spectrum of ξ → *colored* or *white*?

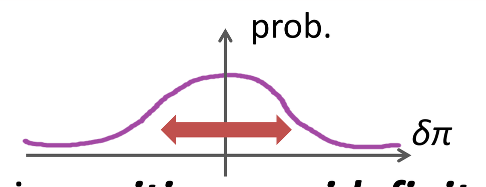
Using the general assumption:

$$\mathcal{L} = 1 + D^1 + \cdots + D^N$$

L is truncated to ***finite order*** in derivatives



retarded: G=0 if t<0
relaxing: G→0 as t→∞
timelike: G=0 if x^2<0



G is ***positive-semidefinite*** since it is a covariance matrix of the noise (FDR)

Constraints

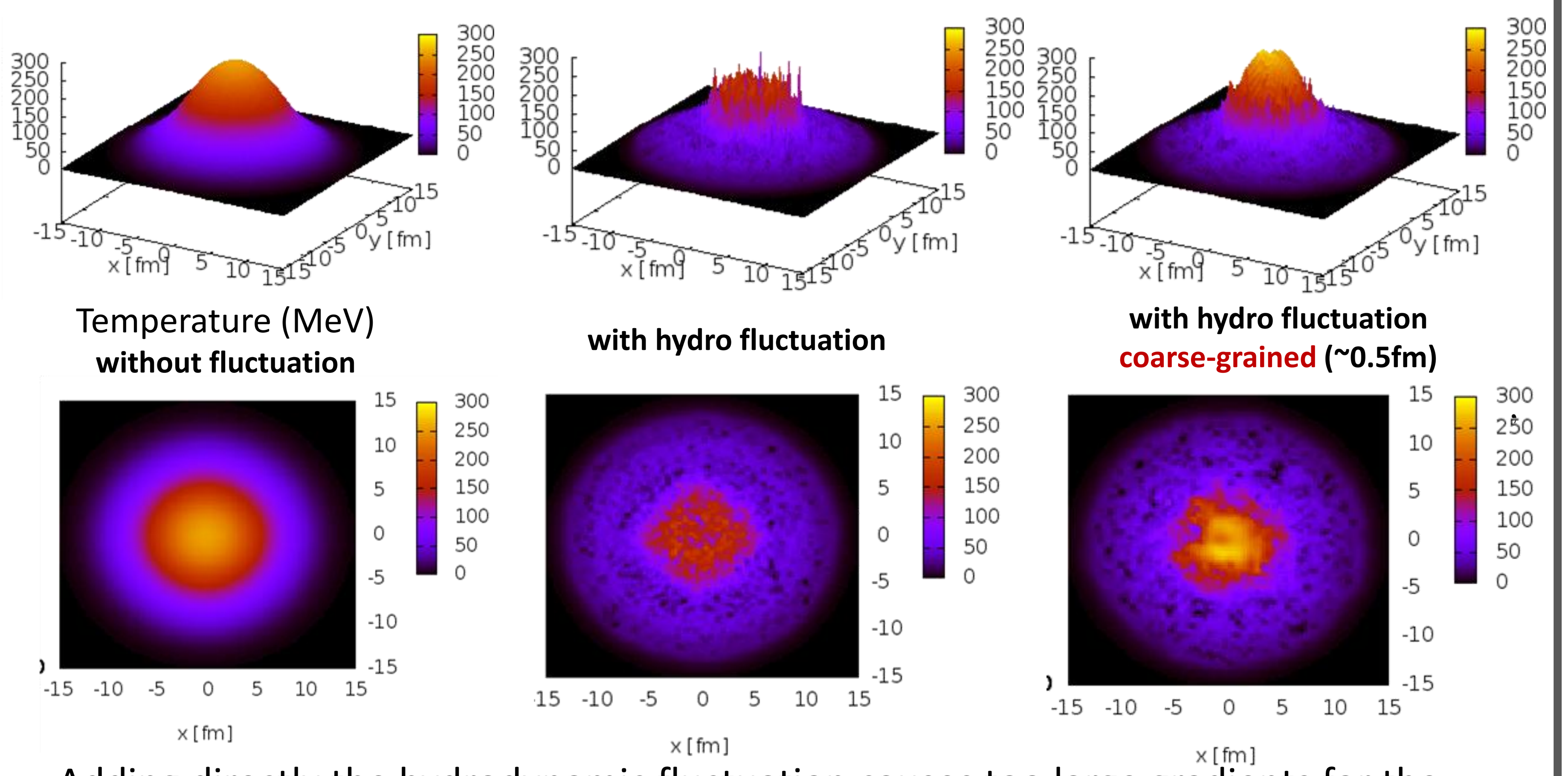
$$\mathcal{L}_{\omega,\mathbf{k}} = 1 + i\omega A_{\mathbf{k}},$$
$$\langle\xi(x)\xi(x')\rangle = 2T\kappa\delta^{(4)}(x-x').$$ **White**

A: Noise in integral form of CE δJ: always ***Colored***
Noise in differential form of CE ξ: always ***White***

3. Hydro fluctuation in dynamical models

We implemented hydrodynamic fluctuations in relativistic dissipative hydrodynamics and are testing the behavior of the hydrodynamic fluctuation.

Simulations from smooth initial condition (η/s = 1/4π, t = 5.0 fm, ideal gas)



Temperature (MeV) without fluctuation — with hydro fluctuation — with hydro fluctuation coarse-grained (~0.5fm)

Adding directly the hydrodynamic fluctuation causes too large gradients for the gradient expansion of CE

→ *Coarse-graining scale of the fluctuation is needed* (~ microscopic scale)

Hydrodynamic fluctuations and the observables

$$\delta\pi^{\mu\nu}/2\eta\partial^{\langle\mu}u^{\nu\rangle} \sim 1/\sqrt{(\eta/s)(V/\mathrm{fm}^4)}$$ T ~ 200 MeV, length ~ 1fm

The effect is important in small η/s or small V

- peripheral collisions, central pp, pA collisions
- higher harmonics v_n

typical volume of higher harmonics

V ~ 1/n

4. Summary

- Hydrodynamic fluctuation should be colored/white noise in the integral/differential form of the constitutive equation due to the causality.
- In numerical simulations, some coarse-graining scale of the fluctuation should be introduced.
- Hydrodynamic fluctuation are more important in small systems and higher orders of the flow harmonics.