Thermal fluctuations in heavy-ion collisions

Clint Young (University of Minnesota), in collaboration with Charles Gale, Sangyong Jeon, Joseph Kapusta, and Björn Schenke

Noise in relativistic hydrodynamics

The fluctuation-dissipation relation

$$G_{symm.}(\omega) = -\frac{2T}{\omega}\mathrm{Im}\,G_{retarded}(\omega)$$

relates the viscosity of a fluid to thermal fluctuations in the energy-momentum tensor. The dissipation in the non-conformal Israel-Stewart equations of relativistic hydrodynamics has therefore always implied the existence of noise whose dynamics are determined by

$$\partial_\mu \delta T^{\mu\nu} = 0 \text{ where}$$

$$\delta T^{\mu\nu} = \delta T^{\mu\nu}_{\text{ideal}} + \delta W^{\mu\nu} + \Xi^{\mu\nu},$$

$$\delta T^{\mu\nu}_{\text{ideal}} = \delta p \left[-g^{\mu\nu} + (1 + (\frac{\partial e}{\partial p}))u^\mu u^\nu\right] + (e+p)(\delta u^\mu u^\nu + u^\mu \delta u^\nu),$$
$$(u \cdot \delta W^{\mu\nu}) = -\frac{1}{\tau_\pi}(\delta W^{\mu\nu} - \eta(\partial^{\langle\mu} - u^{\langle\mu}(u \cdot \partial))\delta u^{\nu\rangle} - \Xi^{\mu\nu}),$$

and finally $\Xi^{\mu\nu}$, in the rest frame of the fluid, has the autocorrelation

$$\langle \Xi^{ij}(x)\Xi^{kl}(x')\rangle = 2\eta T[\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk} - 2\delta^{ij}\delta^{kl}/3]\delta^4(x-x').$$

The importance of the the noise is scale-dependent: $\Xi^{ij}_{\text{cell}} = \frac{1}{\Delta V}\int dV \Xi^{ij}$ has a variance $\propto \frac{1}{\Delta V}$ which suggests that the noise is important at sufficiently short distance scales.

Numerical calculation in heavy-ion collisions

The fluid formed in heavy ion collisions is small and ultrarelativistic and therefore *noisy*. Because it is relativistic, the fluctuations in density and momentum on small scales are large. A single event is noisy even though this is not always apparent when examining observables averaged over a large event class.

Calculating the evolution of thermal noise in a heavy-ion collision determines when and where thermal noise is significant:
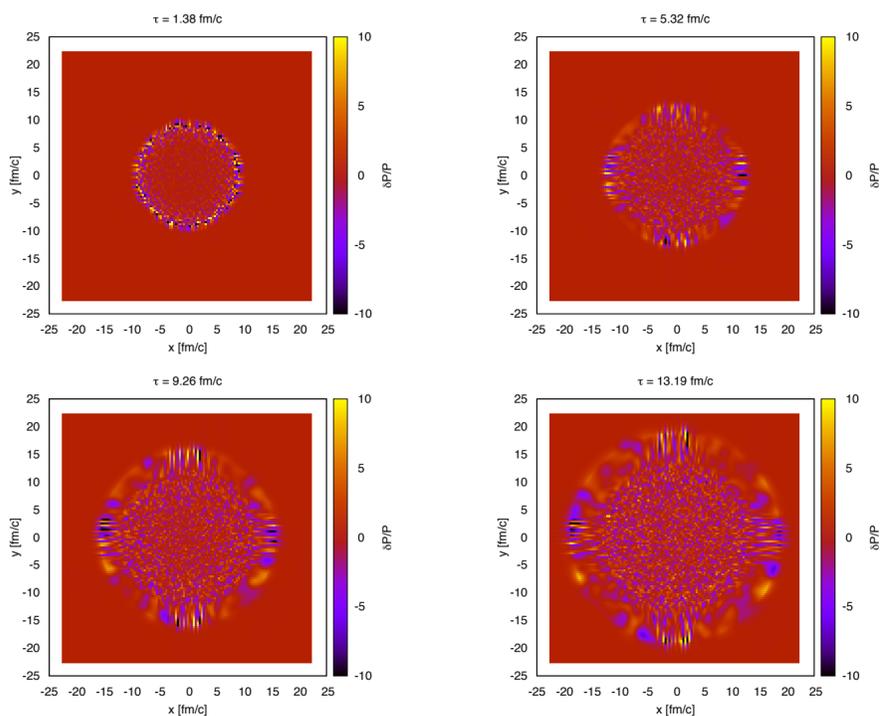
$\delta p/p$ in a central Pb+Pb collision at LHC energies. Note the relatively large fluctuations where $c_s^2 = (\frac{\partial p}{\partial e})$ is small.

Event-by-event variances in the LHC's 10-20% centrality class of Pb+Pb collisions

Thermally fluctuating hydrodynamics gives results identical to calculations without noise in some circumstances. For example, because $\langle \delta T^{\mu\nu}\rangle = 0$, the single π^+ spectrum is unaffected by noise:
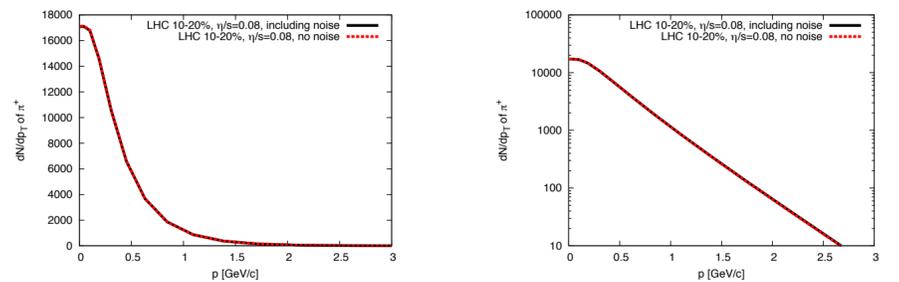
dN/dp_T for charged pions in the 10-20% centrality class

However, two-particle correlation functions and quantities dependent on the definition of an event plane are affected by thermal noise. The momentum eccentricity $\epsilon_p = \sqrt{\langle T^{xx} - T^{yy}\rangle^2 + \langle T^{xy}\rangle^2}/\langle T^{xx} + T^{yy}\rangle$ now fluctuates across events:
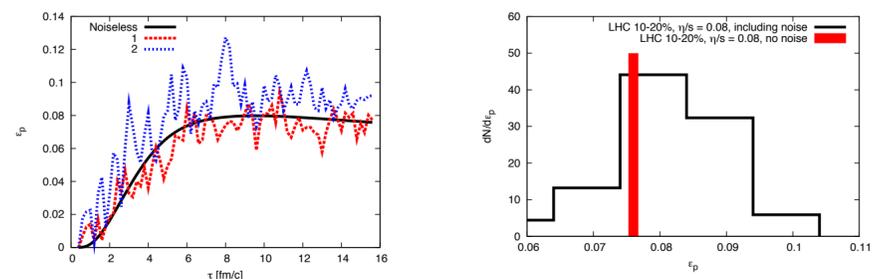
Left: the evolution of ϵ_p in proper time in a hydrodynamical simulation without noise, with its evolution in two fluctuating events. Right: the distribution of ϵ_p of an ensemble of themally fluctuating events in the 10-20% centrality class. The red impulse shows ϵ_p at the end of a hydrodynamical calculation without noise.

The effect of thermal noise on the ultra-central 0-0.2% centrality class

Thermal noise also makes variances in event classes where they might otherwise be small. This provides the best hope for an independent measurement of shear viscosity. This figure shows the distribution of ϵ_p at $\tau = 12.2$ fm/c for an ensemble of heavy-ion events at an impact parameter of 0 fm:
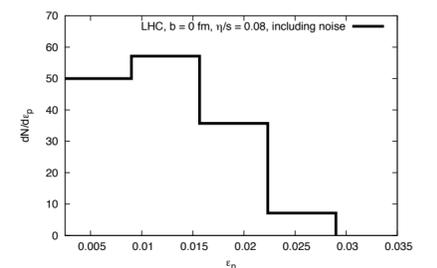
References

J. I. Kapusta, B. Muller and M. Stephanov, Phys. Rev. C 85, 054906 (2012) [arXiv:1112.6405 [nucl-th]].
C. Young, arXiv:1306.0472 [nucl-th].
B. Schenke, S. Jeon and C. Gale, Phys. Rev. C 82, 014903 (2010) [arXiv:1004.1408 [hep-ph]].