Onset of cavitation in the quark–gluon plasma

Mathis Habich and Paul Romatschke

Department of Physics
UNIVERSITY OF COLORADO **BOULDER**

arXiv:1405.1978

Abstract

We study the onset of bubble formation (cavitation) in the quark–gluon plasma as a result of the reduction of the effective pressure from bulk-viscous corrections. By calculating velocity gradients in typical models for quark–gluon plasma evolution in heavy-ion collisions, we obtain results for the critical bulk viscosity above which cavitation occurs. Since present experimental data for heavy-ion collisions seems inconsistent with the presence of bubbles above the phase transition temperature of QCD, our results may be interpreted as an upper limit of the bulk viscosity. Our results indicate that bubble formation must occur at temperatures below 140 MeV, consistent with the expectation of hadronisation in low temperature QCD.

Cavitation for relativistic hydrodynamics

Cavitation is the drop of pressure below the saturated vapour pressure. This definition can be revisited for relativistic fluids in the following manner:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (p - \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu}\,, \tag{1}$$

$$p_{\rm eff} \equiv -\frac{1}{3}\Delta_{\mu\nu}T^{\mu\nu} = p - \Pi\,. \tag{2}$$

Cavitation occurs if

$$p_{\rm eff} < p_{\rm v}\,. \tag{3}$$

This indicates that the liquid undergoes a phase transition to the hadron gas phase (freeze-out of the QGP) when this condition is fulfilled.

Critical bulk viscosity

For 1st order hydrodynamics:

$$p_{\rm v} < p_{\rm eff} = p - \Pi \approx p - \zeta\nabla_\mu u^\mu\,. \tag{4}$$

The critical bulk viscosity is defined as the maximum value of ζ for which the fluid flow is still **non-cavitating**, i.e.,

$$\left.\frac{\zeta}{s}\right|_{\rm crit} \equiv \frac{(p - p_{\rm v})\,T}{(\epsilon + p)\,\nabla_\mu u^\mu}\,. \tag{5}$$

For 2nd order hydrodynamics:

$$p_{\rm v} < p_{\rm eff} = p - \zeta\nabla_\mu u^\mu + \zeta\tau_\Pi {\rm D}\,(\nabla_\mu u^\mu) + \xi_1\sigma^{\mu\nu}\sigma_{\mu\nu} + \xi_2\,(\nabla_\mu u^\mu)^2\,. \tag{6}$$

For strong coupling, these transport coefficients are [7, 5]:

$$\eta = \frac{3\zeta}{2(1-3c_s^2)}\,,\qquad \zeta\tau_\Pi = \zeta\tau_\pi = \frac{\zeta}{\epsilon+p}\eta(4-\ln 4)\,,$$
$$\xi_1 = \frac{\zeta}{\epsilon+p}\eta\,,\qquad \xi_2 = \frac{\zeta}{\epsilon+p}\eta c_s^2(4-\ln 4)\,. \tag{7}$$

Expressing these transport coefficients in terms of the speed of sound c_s^2, ζ/s, and η gives the critical bulk viscosity in second order hydrodynamics as

$$\left.\frac{\zeta}{s}\right|_{\rm crit} \equiv \frac{(p_{\rm v} - p)\,T}{(\epsilon+p)}\left[\nabla_\mu u^\mu - \frac{\eta}{s}\frac{(4-\ln 4)\left({\rm D}\nabla_\mu u^\mu + c_s^2\,(\nabla_\mu u^\mu)^2\right) + \sigma^{\mu\nu}\sigma_{\mu\nu}}{T}\right]^{-1}. \tag{8}$$

The pole for vanishing gradient terms occurs at $T \ll 100$ MeV, where one does not expect a hydrodynamic description to be applicable in the first place.

Ideal & QCD equation of state for the QGP

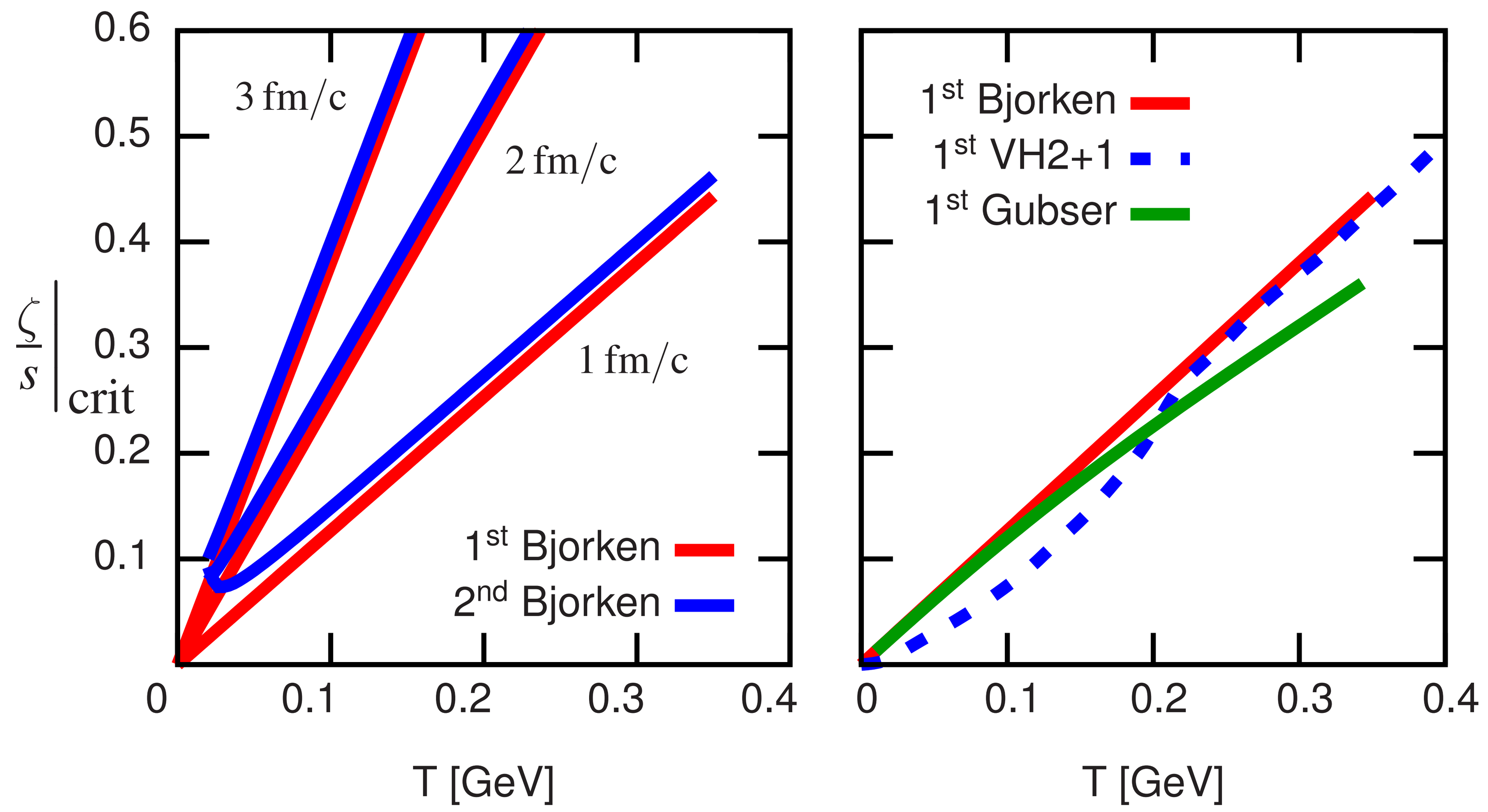

Figure 1 : Critical bulk viscosity as a function of temperature for the **ideal** equation of state [4]. Areas above respective lines of $\zeta/s|_{\rm crit}$ are regions where cavitation occurs. Left panel: comparison of 1st and 2nd order results for Bjorken flow. Right panel: comparison between Bjorken flow, Gubser flow, and numerical computations.

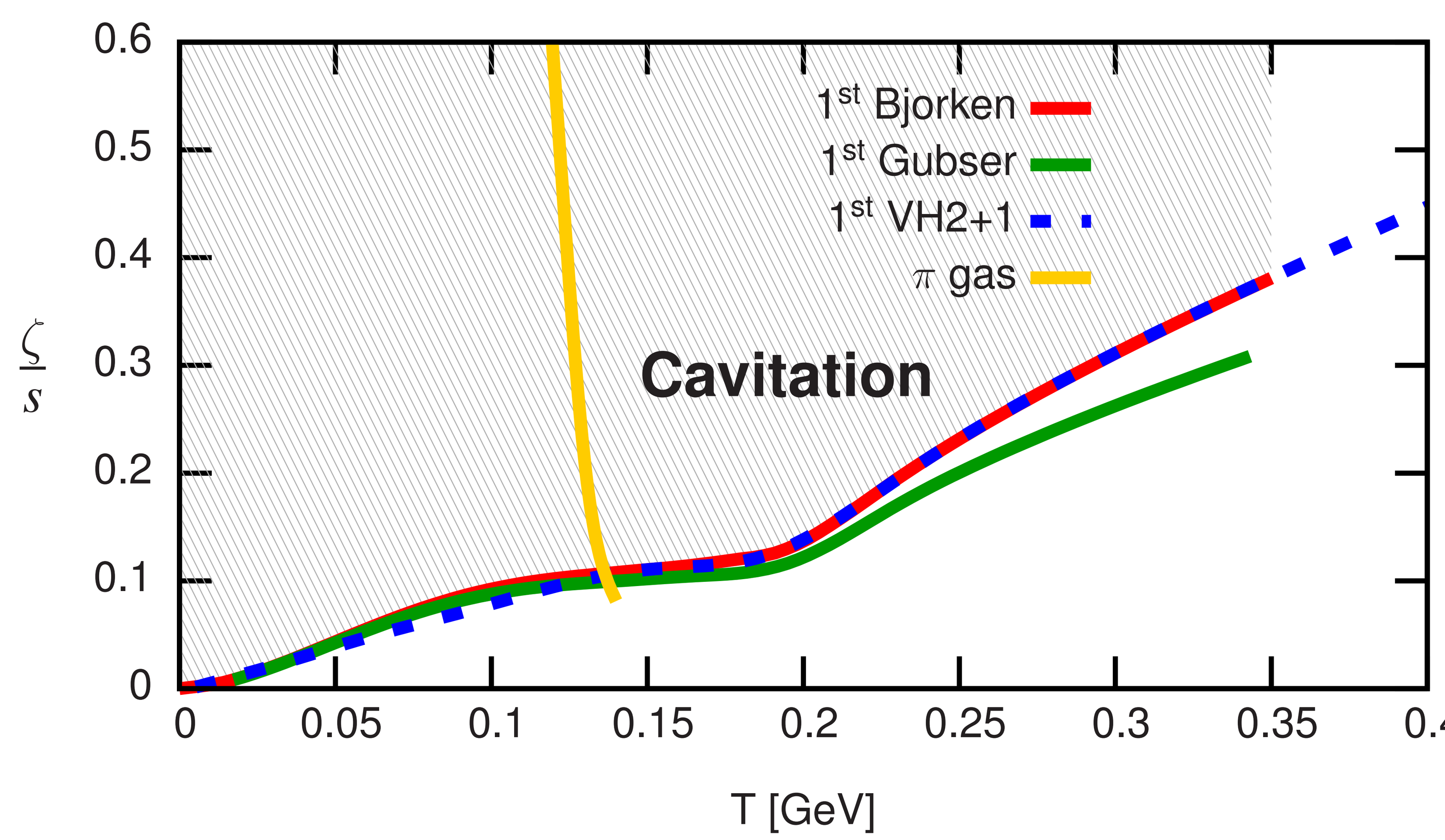

Figure 2 : Bulk viscosity over entropy density ratio as a function of temperature [4]. Shown are results for the lowest critical bulk viscosity coefficient $\zeta/s|_{\rm crit}$ using different flow profiles and a **QCD** equation of state. For higher viscosity values, we predict bubbles to form in the liquid ('cavitation'). For comparison, we also show the result of a calculation of ζ/s for a pion gas from ref. [6]. The pion gas is calculated up to $T = 140$ MeV.

Flow profiles used

The fluid gradients are calculated for:

- Bjorken flow [2]:

$$(u^\mu) = (1,0,0,0)^{\rm T}\,,\qquad \nabla_\mu u^\mu = \frac{1}{\tau}\,, \tag{9}$$

- Gubser flow [3]:

$$(u^\mu) = \left(\frac{1+q^2r^2+q^2\tau^2}{2q\tau\sqrt{1+g^2}},\ \frac{qr}{\sqrt{1+g^2}},\ 0,\ 0\right)^{\rm T} \tag{10}$$

- a numerical solver for relativistic, viscous hydrodynamics in 2+1 dimensions [1] of a **central** $Au+Au$ collision at $\sqrt{s} = 200$ GeV.

All these flow profiles are for **conformal fluids**, that is, they ignore effects of bulk viscosity in the flow itself. We choose

$$p_{\rm v} \equiv 0\,, \tag{11}$$

which will result in the most conservative estimates of cavitation since higher values of $p_{\rm v}$ would decrease $\zeta/s|_{\rm crit}$ even further (e.g., see eq. (5)).

Conclusion

We found that at temperatures $T < 140$ MeV, a bulk viscosity coefficient smaller than that expected from a pion gas leads to the formation of hadron gas bubbles in the QGP liquid. This may be interpreted as the known freeze-out phenomenon in heavy-ion collisions where the plasma undergoes a phase transition to a hadron gas. At around the QCD phase transition temperature, we predict that for values of $\zeta/s \gtrapprox 0.1$, cavitation in the QGP will occur. Furthermore, only small changes are present if 2nd order gradients are also taken into account. In particular, the 2nd order gradients of Bjorken and Gubser flow lead to higher $\zeta/s|_{\rm crit}$ while the numerical simulation's gradient exhibits the opposite behaviour for late times. Under the assumption that experimental data on the QGP from heavy-ion collisions is inconsistent with the presence of hadron gas bubbles, our results for $\zeta/s|_{\rm crit}$ may be interpreted as an upper bound on the bulk viscosity in high temperature QCD.

Acknowledgments & References

This work was supported by the **Sloan Foundation**, Award No. BR2012-038 and the **Deutsche Forschungsgemeinschaft** (DFG), Grant No. RO 4513/1-1.

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NUCLEAR THEORY GROUP • MATHIS.HABICH@COLORADO.EDU • PAUL.ROMATSCHKE@COLORADO.EDU