Projection method and new formulation of leading-order anisotropic hydrodynamics

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Motivation

- heavy-ion experimental data from RHIC and the LHC very well described by the 2nd order viscous hydrodynamics with early starting time, $\tau_0 < 1$ fm/c
- viscous corrections combined with rapid longitudinal expansion induce a substantial pressure asymmetry in the created system
- at early times the microscopic models (string models, color glass condensate, pQCD kinetic calculations) predict also a large momentum anisotropy
- AdS/CFT correspondence predicts a large difference between $\mathbf{P}_\perp$ and $\mathbf{P}_\parallel$, which slowly decays with time (Heller, Janik, Witaszczyk)

Hydrodynamic expansion

- viscous hydrodynamics is based on the linearization around an isotropic background $\mathbf{f} \simeq \mathbf{f}_{\rm eq.} + \delta\mathbf{f}$, $\delta\mathbf{f}$ should be small $\longrightarrow$ small corrections to the equilibrium stress-energy tensor $\mathbf{T}^{\mu\nu} = \int \mathbf{dP}\, \mathbf{p}^\mu \mathbf{p}^\nu \mathbf{f} = \mathbf{T}^{\mu\nu}_{\rm eq.} + \pi^{\mu\nu}$
- large shear corrections (of the order of isotropic pressure) are present, invalidating the working hypothesis of Israel-Stewart theory and leading to unphysical results at early times
- the new framework of anisotropic hydrodynamics is based on the reorganization of the hydrodynamic expansion around a non isotropic background instead of the local equilibrium $\mathbf{f} = \mathbf{f}_{\rm aniso.} + \delta\tilde{\mathbf{f}}$
- momentum (and pressure) anisotropy starting from the leading order, corrections from $\delta\tilde{\mathbf{f}}$ can be treated as perturbations

Projecion method

- Relaxation Time Approximation (RTA) for the Bolzmann equation $\mathbf{p}^\mu \partial_\mu \mathbf{f} = (\mathbf{p}^\mu \mathbf{U}_\mu)(\mathbf{f}_{\rm eq.} - \mathbf{f})/\tau_{\rm eq.}$
- space-time basis $\mathbf{U}$, $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$ to project tensorial equations and obtain scalar equations W Florkowski and R Ryblewski, *Phys. Rev. C* **85**, 044902 (2012)
- anisotropic background, **generalization** of the Romatchke-Strickland form $\mathbf{f}_{\rm aniso.} = \mathbf{k} \exp\left[-1/\lambda \sqrt{\mathbf{p}^\mu \Xi_{\mu\nu} \mathbf{p}^\nu}\right]$

1+1 dimensional flow

L Tinti, W Florkowski, *Phys Rev C* **89**, 034907 (2014)

- longitudinally boost invariant, radially symmetric on the transverse plane
- two independent anisotropy parameters for having different pressures in the transverse plane (different shear tensor, which is proportional to the pressure corrections in the Navier-Stokes limit)

$$\mathbf{f}_{\rm aniso.} = \mathbf{k} \exp\left[-\frac{1}{\lambda}\sqrt{\sum_{\mathbf{I}} (1+\xi_{\mathbf{I}})(\mathbf{p}^\mu \mathbf{I}_\mu)}\right]$$

$$\sum_{\mathbf{I}} \xi_{\mathbf{I}} = 0 \qquad \mathbf{I} \in \{\mathbf{X}, \mathbf{Y}, \mathbf{Z}\}$$

- two non trivial equations from the first moment of the Boltzmann equation (energy and momentum conservation):

$$\int \mathbf{dP}\, \mathbf{p}^\mu \mathbf{p}^\nu \partial_\mu \mathbf{f} = \mathbf{U}_\mu \frac{\int \mathbf{dP}\, \mathbf{p}^\mu \mathbf{p}^\nu \mathbf{f}_{\rm eq.} - \int \mathbf{dP}\, \mathbf{p}^\mu \mathbf{p}^\nu \mathbf{f}}{\tau_{\rm eq.}} \Rightarrow \partial_\mu \mathbf{T}^{\mu\nu} = 0$$

namely the projections along the four velocity and the radial direction $\mathbf{X}$

$$\mathbf{D}\varepsilon + \varepsilon\,\theta - \sum_{\mathbf{I}} \mathbf{P}_{\mathbf{I}} \theta_{\mathbf{I}} = 0$$

$$(\mathbf{X}\cdot\partial)\,\mathbf{P}_{\mathbf{X}} + \mathbf{P}_{\mathbf{X}}\,(\partial\cdot\mathbf{X}) - \varepsilon\,(\mathbf{X}\cdot\mathbf{DU}) + \\ -\mathbf{P}_{\mathbf{Y}}\,[\mathbf{X}\cdot(\mathbf{Y}\cdot\partial)\,\mathbf{Y}] - \mathbf{P}_{\mathbf{Z}}\,[\mathbf{X}\cdot(\mathbf{Z}\cdot\partial)\,\mathbf{Z}] = 0$$

- we close the system with the equations obtained from the second moment

$$\frac{\mathbf{D}\xi_{\mathbf{I}}}{1+\xi_{\mathbf{I}}} - \frac{1}{3}\sum_{\mathbf{J}} \frac{\mathbf{D}\xi_{\mathbf{J}}}{1+\xi_{\mathbf{J}}} + 2\sigma_{\mathbf{I}} + \frac{\xi_{\mathbf{I}}}{\tau_{\rm eq}}\left(\frac{\mathbf{T}}{\lambda}\right)^5 \sqrt{\prod_{\mathbf{J}}(1+\xi_{\mathbf{J}})} = 0$$

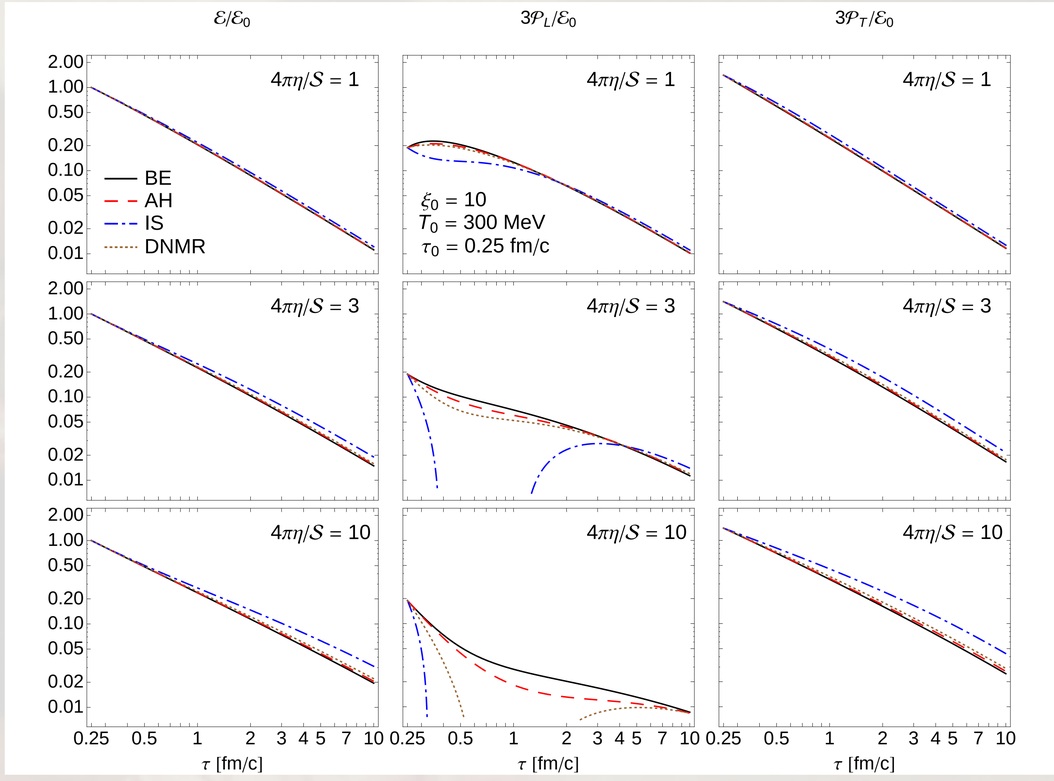
- these equations correspond, in the close to equilibrium limit to the Israel-Stewart equations, as the pressure corrections are proportional to the anisotropy parameters $\pi_{\mathbf{I}} = -\frac{32}{5}\mathbf{k}\mathbf{T}^4 \xi_{\mathbf{I}}$

$$\tau_\pi \mathbf{D}\pi_{\mathbf{I}} + \pi_{\mathbf{I}} = 2\eta\sigma_{\rm i} + \mathbf{F}_\eta \pi_{\mathbf{I}} \qquad \mathbf{F}_\eta = -\eta\mathbf{T}\partial\cdot\left(\frac{\alpha_1}{\mathbf{T}}\mathbf{U}\right)$$

- we have checked numerically that the entropy production is non-negative for any configuration

Exactly solvable case

W Florkowski, R Ryblewski, and M Strickland, *Phys Rev C* **88**, 024903 (2013)

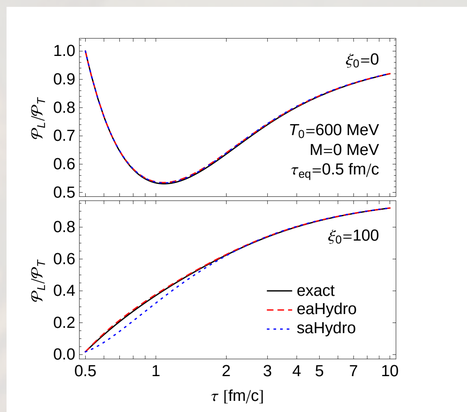


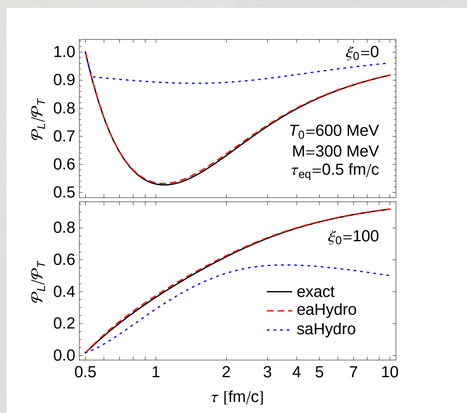
- 0+1 dimension longitudinally boost invariant, homogeneous in the transverse plane.
- anisotropic hydrodynamics using the zeroth moment for closing the system of equations, as previously proposed

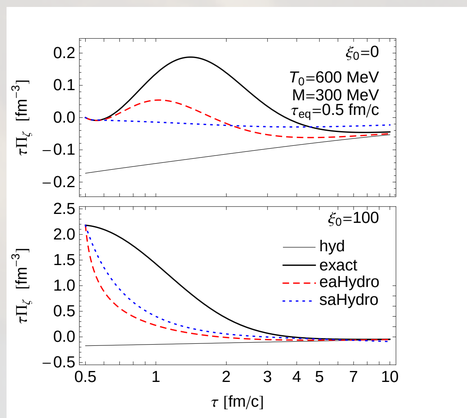
Second moment equations

W Florkowski, R Ryblewski, M Strickland, L Tinti, arXiv:1403.1223, to be printed in *Phys Rev C*

- even in this simple case, using the equations from the second moment improves the agreement with the exact solution
- inserting non vanishing masses the equations from the second moment provide a better approximation of the exact solutions
- bulk corrections are not described as satisfactory as the other quantities (new degree of freedom for bulk corrections? see M Nopoush, R Ryblewski, M Strickland, arXiv:1405.1355)







Conclusions

- TWO EQUATIONS obtained from the SECOND MOMENT of the Boltzmann equation should supplement the equations from the first moment
- CONSISTENT with the symmetries of the 1+1 radial flow, and AGREEING with Israel-Stewart equations close to equilibrium
- NEW LEADING ORDER framework for the next-to-leading order calculations

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