Projection method and new formulation of leading-order anisotropic hydrodynamics

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Motivation

- heavy-ion experimental data from RHIC and the LHC very well described by the 2nd order viscous hydrodynamics with early starting time, $\tau_0 < 1 \text{ fm/c}$
- viscous corrections combined with rapid longitudinal expansion induce a substantial pressure asymmetry in the created system
- at early times the microscopic models (string models, color glass condensate, pQCD kinetic calculations) predict also a large momentum anisotropy
- AdS/CFT correspondence predicts a large difference between $P_\perp$ and $P_\parallel$, which slowly decays with time (Heller, Janik, Witaszczyk)

Hydrodynamic expansion

- viscous hydrodynamics is based on the linearization around an isotropic background $f \simeq f_{\text{eq}} + \delta f$, $\delta f$ should be small --- small corrections to the equilibrium stress-energy tensor $T^{\mu\nu} = \int dP \rho(p) f(T_{\rho}) = T_{\text{eq}}^{\mu\nu} + \tau^{\mu\nu}$
- large shear corrections (of the order of isotropic pressure) are present, invalidating the working hypothesis of Israel-Stewart theory and leading to unphysical results at early times
- the new framework of anisotropic hydrodynamics is based on the reorganization of the hydrodynamic expansion around a non isotropic background instead of the local equilibrium $f = f_{\text{eq}} + \delta f$
- momentum (and pressure) anisotropy starting from the leading order, corrections from $\delta f$ can be treated as perturbations

Projection method

- Relaxation Time Approximation (RTA) for the Boltzmann equation $p^\mu \partial_\mu f = (p^\mu U_\mu)(f_{\text{eq}} - f)/\tau_{eq}$.
- space-time basis $X, Y, Z$ to project tensorial equations and obtain scalar equations W Florkowski and R Ryblewski, Phys. Rev. C 85, 044902 (2012)
- anisotropic background, generalization of the Romatchke-Strickland form $f_{\text{ansio}} = k \exp[-1/\lambda \sqrt{p^\mu p_{\mu}}]$

1+1 dimensional flow

- longitudinally boost invariant, radially symmetric on the transverse plane
- two independent anisotropy parameters for having different pressures in the transverse plane (different shear tensor, which is proportional to the pressure corrections in the Navier-Stokes limit)

\[ f_{\text{ansio}} = k \exp\left[ -\frac{1}{\lambda} \sqrt{\sum_{\alpha}(1 + \xi_\alpha)(p^\mu u_\mu)} \right] \]

\[ \sum_{\alpha} \xi_\alpha = 0 \quad \text{for} \quad \alpha \in \{X, Y, Z\} \]

- two non trivial equations from the first moment of the Boltzmann equation (energy and momentum conservation):

\[ \int dP p^\mu p_\mu \partial_\mu f = \int dP p^\mu f_{\text{eq}} \Rightarrow \partial_\mu T^{\mu\nu} = 0 \]

\[ D \varepsilon + \varepsilon \theta - \sum_{\alpha} \text{P}_\alpha \text{th}_\alpha = 0 \]

\[ (X \cdot \partial) \text{P}_X + \text{P}_X (\partial \cdot X) - \varepsilon (X \cdot \text{D}U) + \text{P}_Y (X \cdot Y) Y - \text{P}_Z (X \cdot Z) Z = 0 \]

- we close the system with the equations obtained from the second moment

\[ \frac{\text{D} \xi_\alpha}{1 + \xi_\alpha} - \frac{2}{3} \sum_{\beta} \frac{\text{D} \xi_\beta}{1 + \xi_\beta} + 2 \sigma_1 + \frac{\xi_\alpha}{\tau_{eq}} \left( \frac{T}{\lambda} \right)^5 \sum_{\alpha}(1 + \xi_\alpha) = 0 \]

\[ \varepsilon \tau_\delta D \sigma_1 + \sigma_1 = 2 \eta \sigma_1 + F_\eta \tau_\delta \quad F_\eta = -\eta T \partial \cdot \left( \frac{\alpha_1}{T} U \right) \]

- we have checked numerically that the entropy production is non-negative for any configuration

Conclusions

- TWO EQUATIONS obtained from the SECOND MOMENT of the Boltzmann equation should supplement the equations from the first moment
- CONSISTENT with the symmetries of the 1+1 radial flow, and AGREEING with Israel-Stewart equations close to equilibrium
- NEW LEADING ORDER framework for the next-to-leading order calculations

Second moment equations

W Florkowski, R Ryblewski, M Strickland, L Tinti, arXiv:1403.1223, to be printed in Phys Rev C

- even in this simple case, using the equations from the second moment improves the agreement with the exact solution
- inserting non vanishing masses the equations from the second moment provide a better approximation of the exact solutions
- bulk corrections are not described as satisfactory as the other quantities (new degree of freedom for bulk corrections? see M Nopoush, R Ryblewski, M Strickland, arXiv:1405.1355)