Decoherence between the initial and final state radiation in a dense QCD medium

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THE OHIO STATE UNIVERSITY

N. Armesto, H. Ma, Y. Mehtar-Tani and C. A. Salgado,

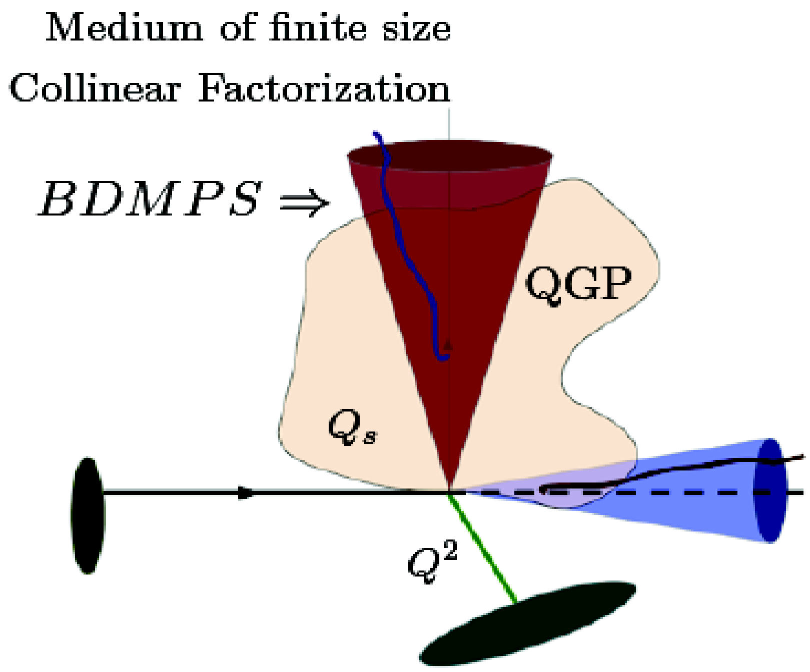
JHEP 1312 (2013) 052

T. Altinoluk, N. Armesto, G. Beuf and C. A. Salgado,

arXiv:1404.2219

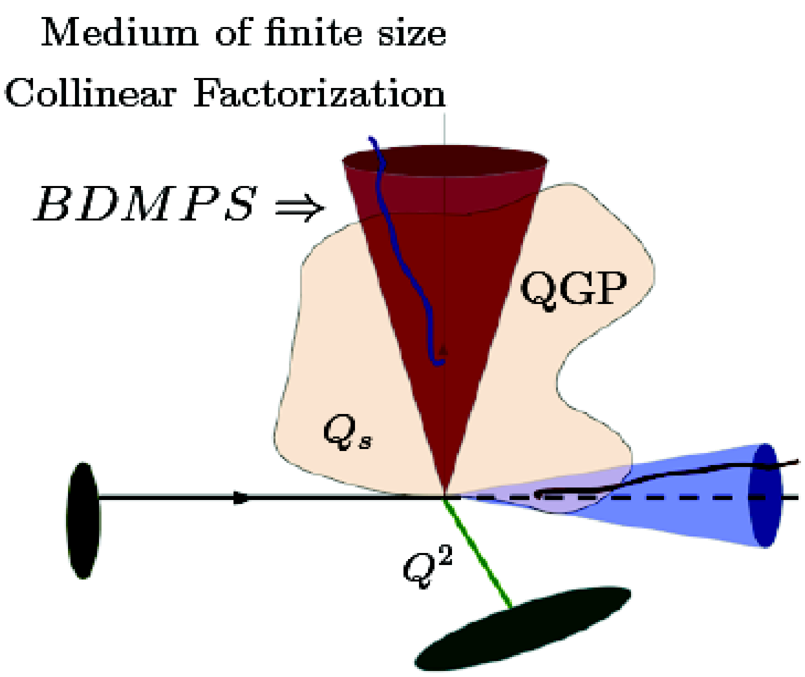
XXIV QUARK MATTER
DARMSTADT 2014

Motivation



Medium of finite size
Collinear Factorization
$BDMPS \Rightarrow$
QGP
Q_s
Q^2

Motivation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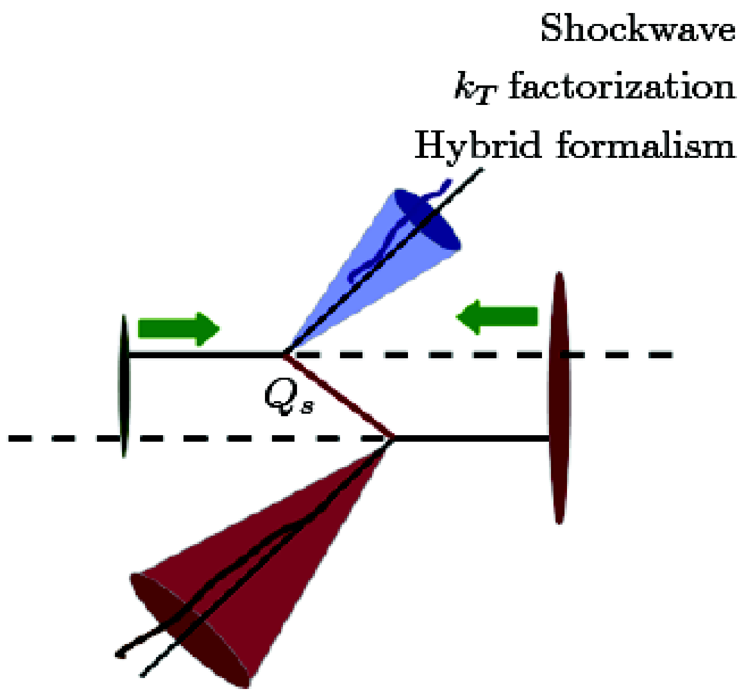
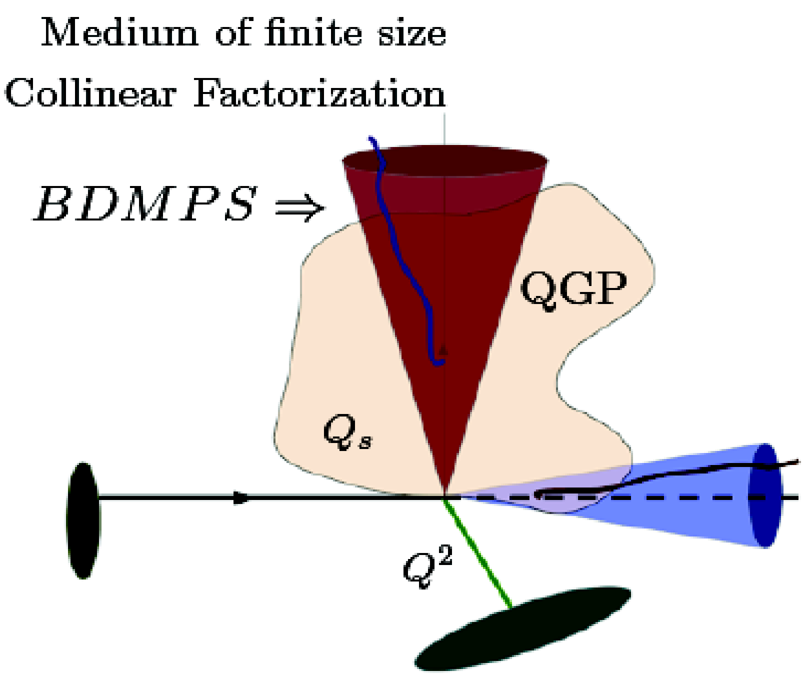
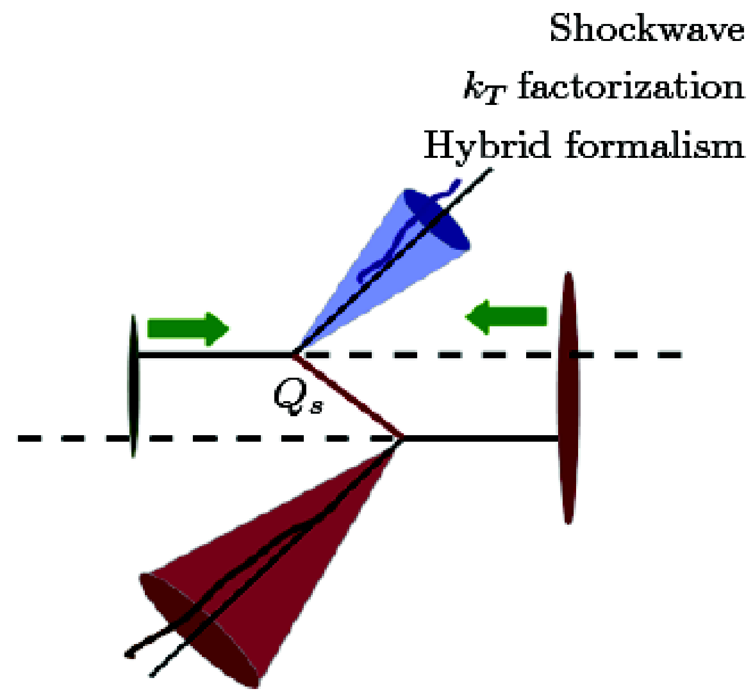

Shockwave
k_T factorization
Hybrid formalism
Q_s

Motivation



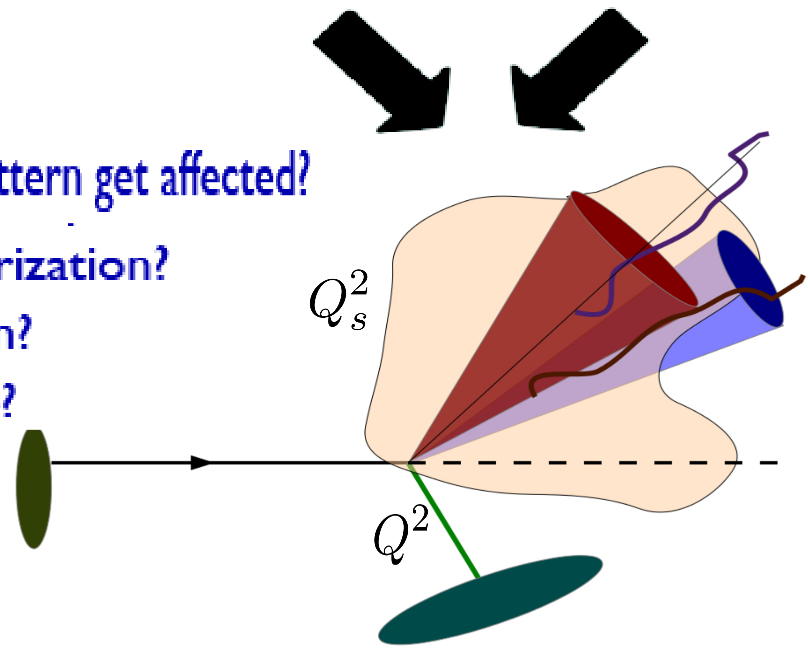


How is the vacuum coherence pattern get affected?

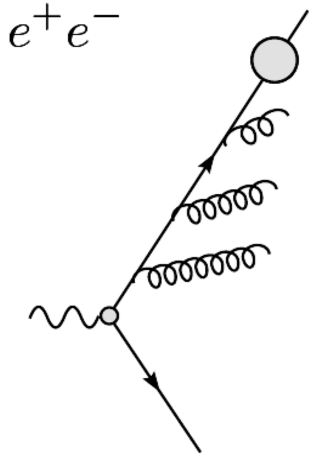
Do we have: Collinear factorization?

kT Factorization?

Something else?

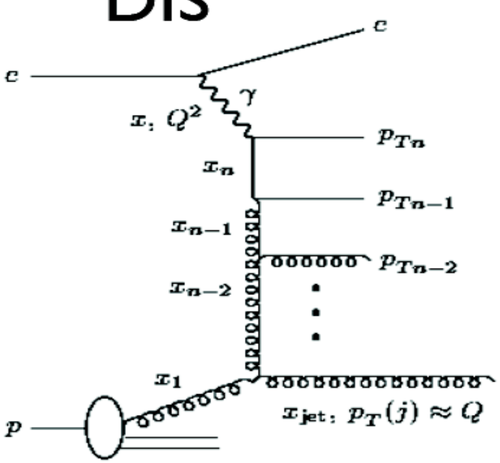


Jets in vacuum



Time-like
Fragmentation functions

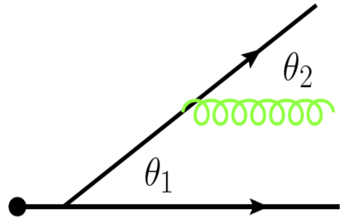
DIS



Space-like
PDF's

- **Color coherence:** destructive interferences lead to angular ordering

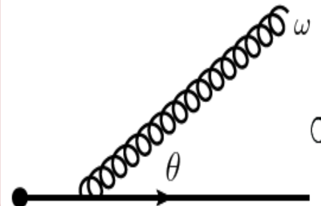
Angular ordering



$\propto \Theta(\theta_1 - \theta_2)$

- **Leading singularities**

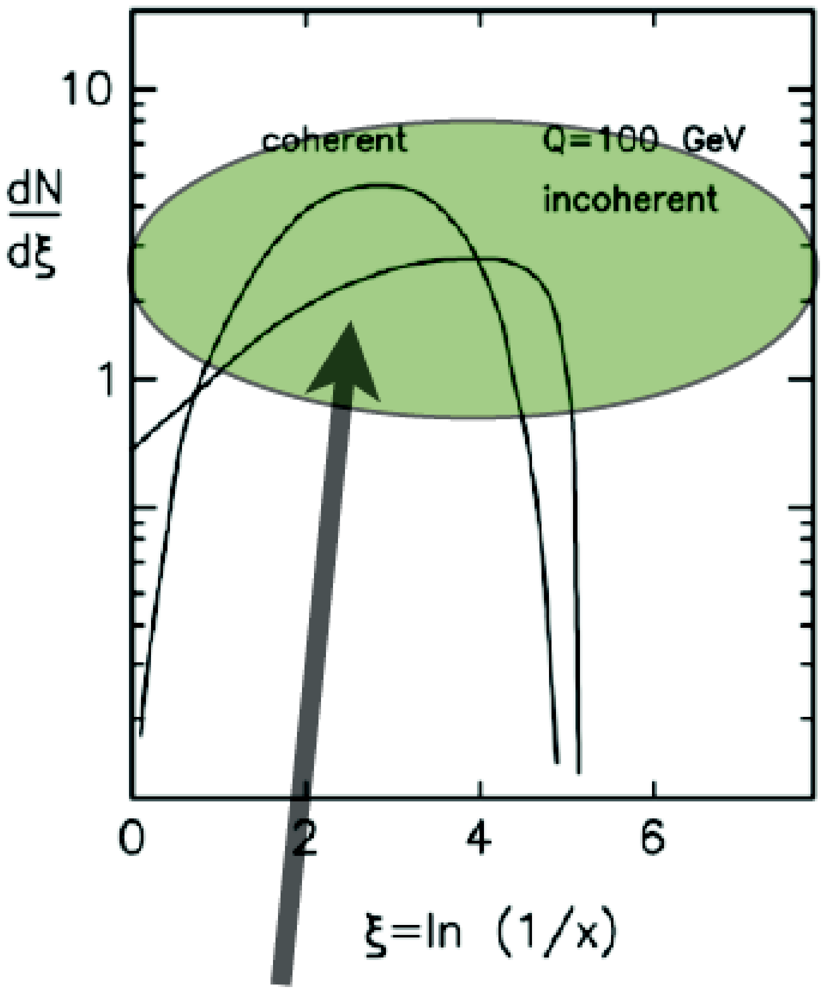
Leading singularities



$$\propto \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\theta}$$

Jets in vacuum

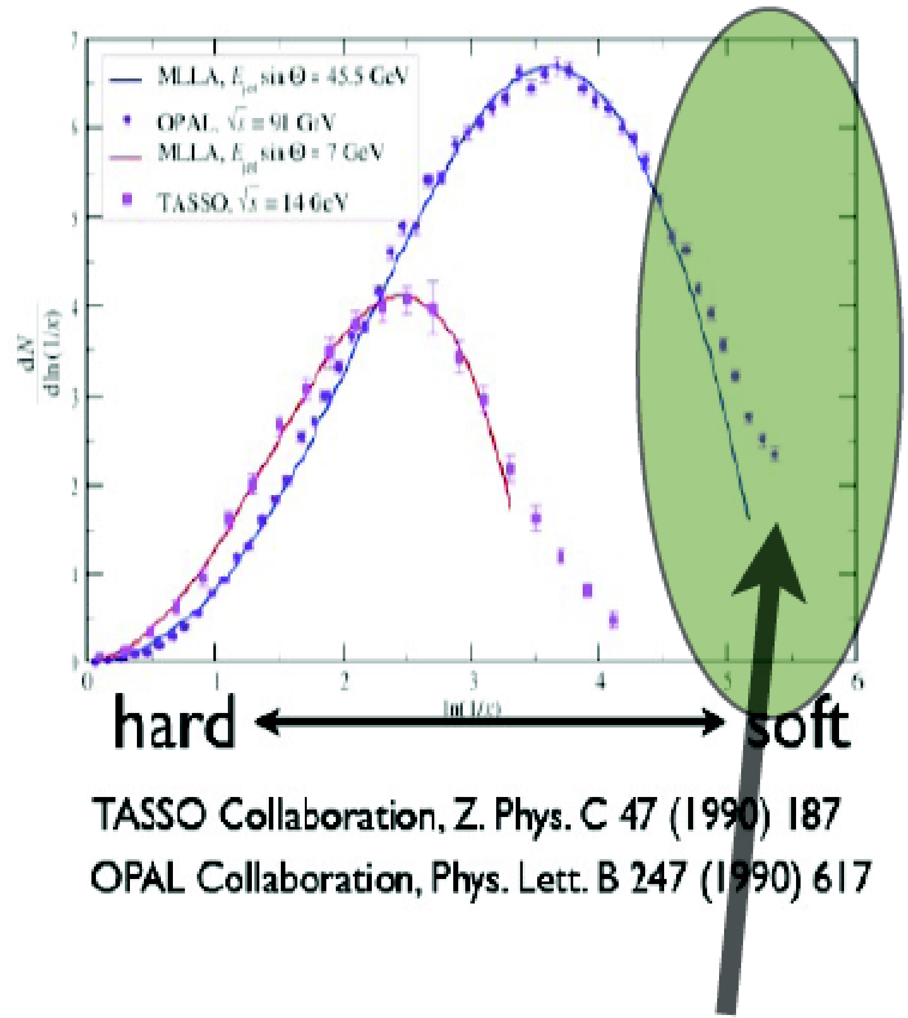
MC implementation



Sizable differences

Khoze, Ochs, Wosiek, hep-ph/0009298

Experimental evidence

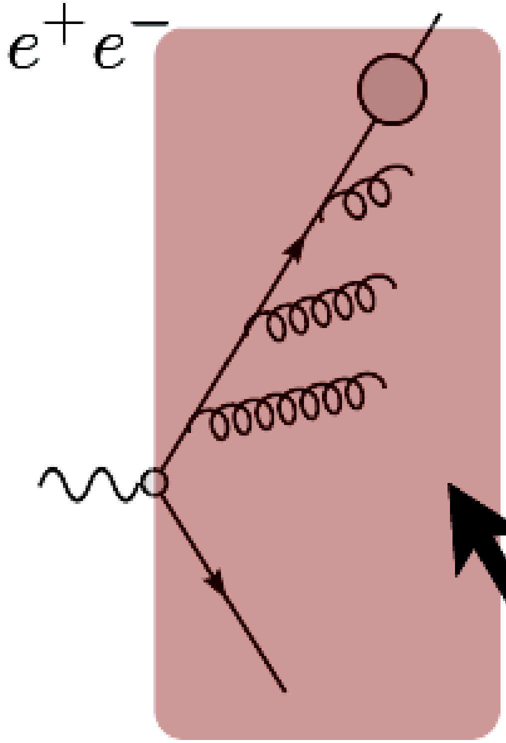


TASSO Collaboration, Z. Phys. C 47 (1990) 187

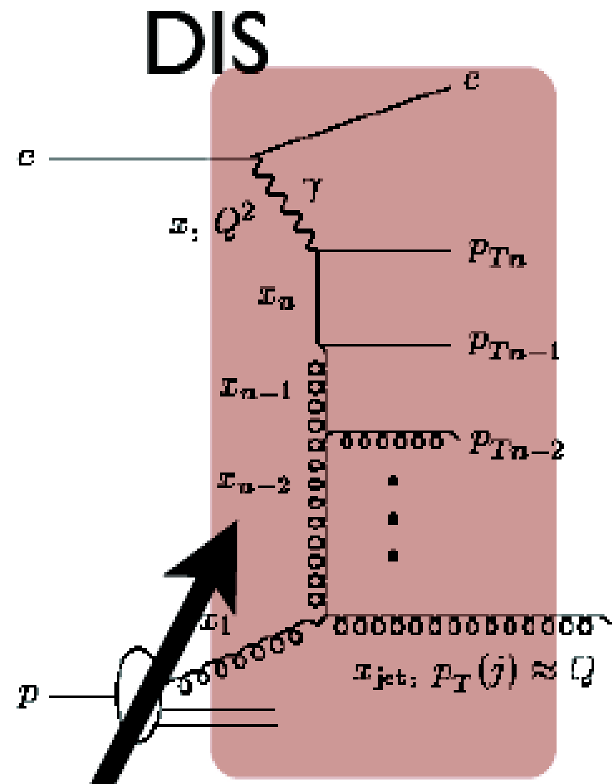
OPAL Collaboration, Phys. Lett. B 247 (1990) 617

Suppression of soft gluons

A natural question



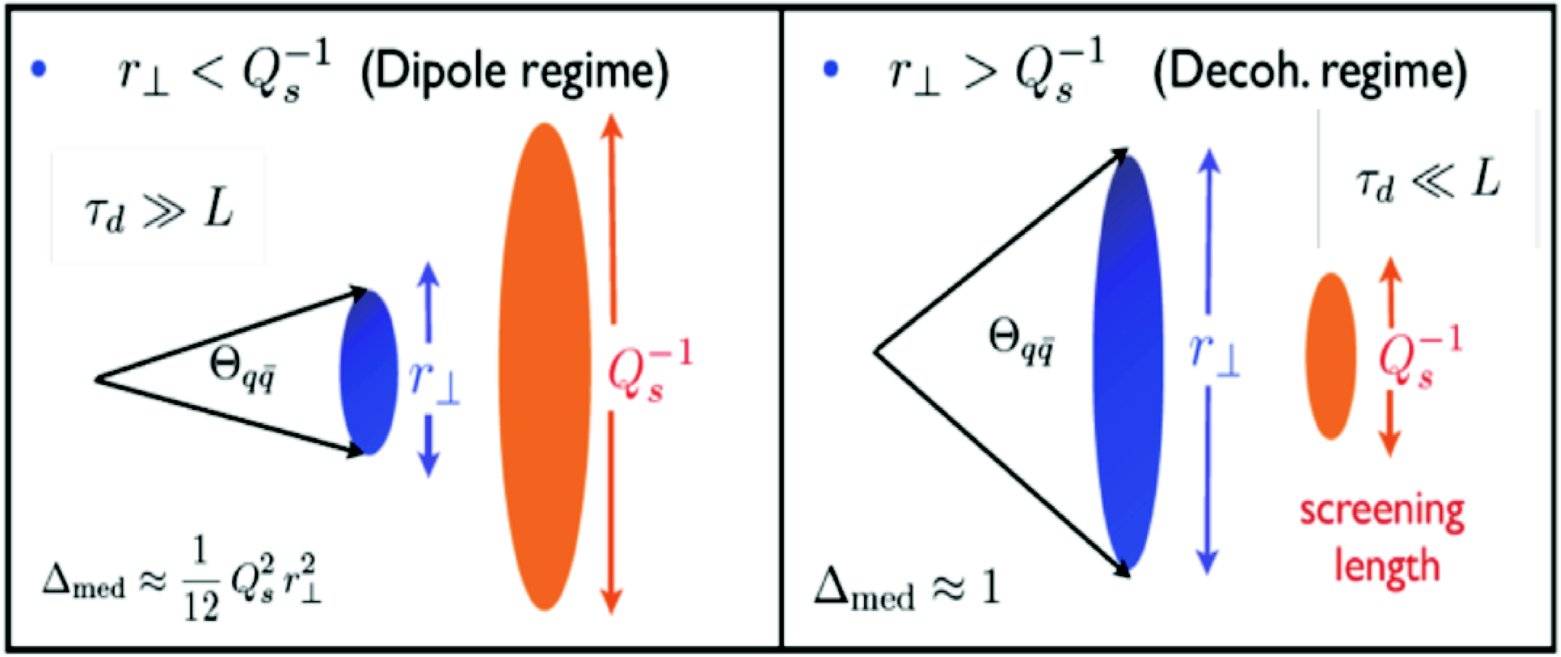
Time-like
Fragmentation functions



Space-like
PDF's

What happens in a QCD medium?

Parton cascade in a QCD medium: first steps

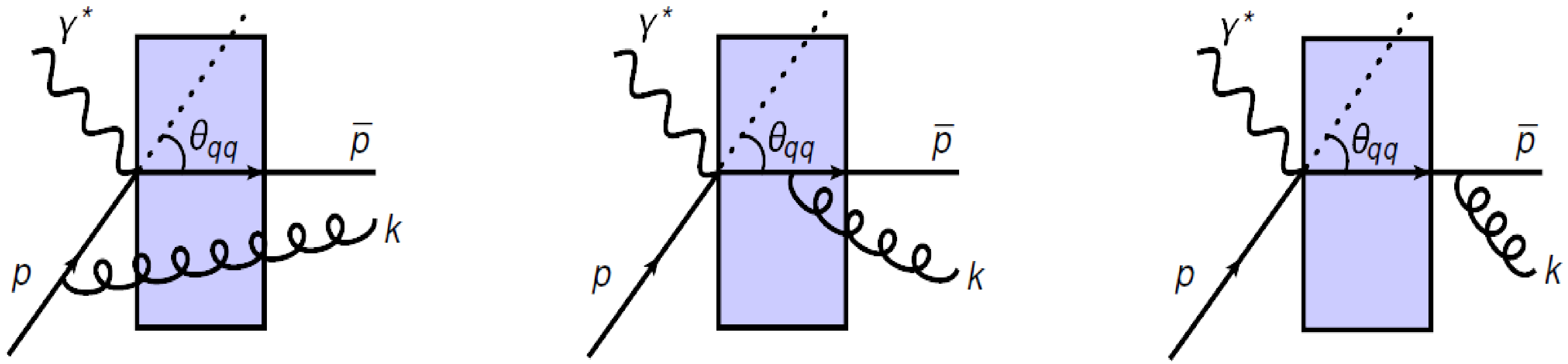


Role of decoherence effects in the ***dense-dense*** regime (2010-present): Mehtar-Tani, Tywoniuk, Salgado, Armesto, Apolinario, Milhano, Casalderrey, Iancu, Blaizot, Dominguez.

See ***Mehtar-Tani, Tywoniuk and Apolinario's* talks!!!**

What about the *dilute*-*dense* regime?

Color decoherence between the initial and final state radiation



N. Armesto, H. Ma, M. Martinez, Y. Mehtar-Tani, C. Salgado
N=1 Opacity expansion: PLB 717 (2012)280
Multiple soft scatterings: JHEP 1312(2013)052 → *Today!!*

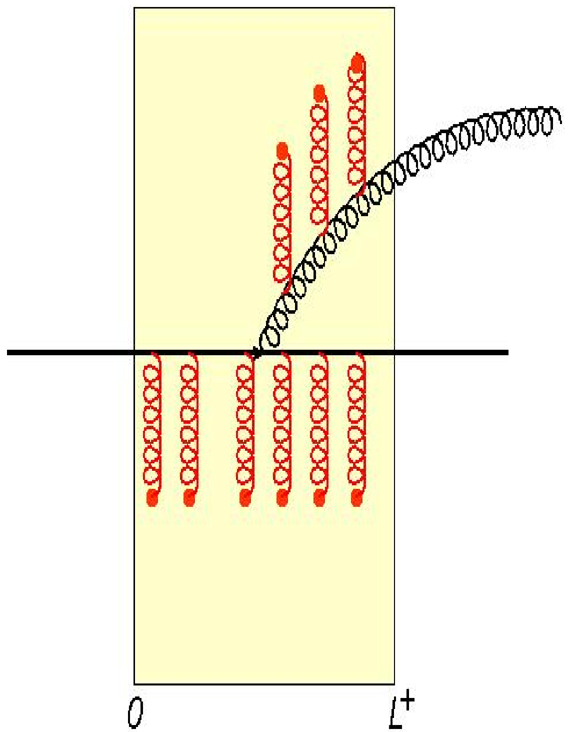
Relevant configuration to investigate:
Medium modifications to the initial state radiation
Energy Loss problem in the dilute-dense regime
Finite length/energy effects in pA collisions

Semi-classical approach to gluon production I

Our goal is to obtain the single gluon spectrum

$$(2\pi)^3 2k^+ \frac{dN}{d^3\vec{k}} = \sum_{\lambda=1,2} \left\langle \left\langle |\mathcal{M}^a_\lambda(\vec{k})|^2 \right\rangle_p \right\rangle_A$$

Scattering amplitude



Blaizot, Gelis, Venugopalan, NPA 743 (2004) 13-56

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Scattering amplitude

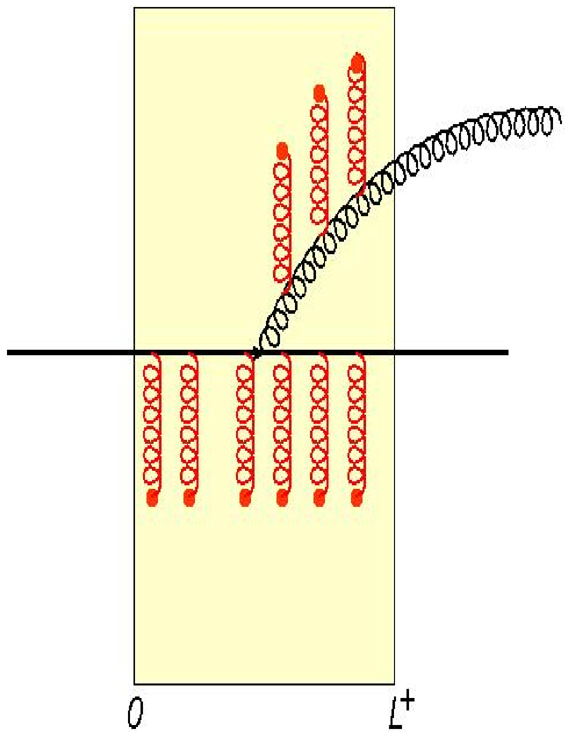
- ***Medium***: Classical background field $A^\mu_{(0)}$

$$-\partial_{\mathbf{x}} A^-_{a,(0)} = \rho_a(x)$$

$$\langle A^-_{a,(0)}(x^+,\mathbf{q}) A^{*,-}_{b,(0)}(x',\mathbf{q}')\rangle = \delta_{ab}\, n(x^+)\, \delta(x^+ - x'^+)\, \delta(\mathbf{q}-\mathbf{q}')\, \mathcal{V}^2(\mathbf{q})$$

Density of scattering centers

Debye potential



- ***Highly energetic particle:*** Classical current

$$\mathcal{J}^\mu_a(x) = g\, v^\mu\, \delta^{(3)}(\vec{x} - \vec{v}t)\, \mathcal{U}(x^+, 0, \boldsymbol{x})_{ab}\, Q_b$$

$$\mathcal{U}(x^+, 0, \boldsymbol{x}) = \mathcal{P}_+ \exp\left\{ ig \int_0^{x^+} dz^+\, T \cdot A(z^+, \boldsymbol{x}) \right\}^{ab}$$

Blaizot, Gelis, Venugopalan, NPA 743 (2004) 13-56

Semi-classical approach to gluon production II

Expand ***perturbatively*** the gluon field as $A^\mu_a \approx \underbrace{A^\mu_{(0)}}_{\mathcal{O}(g^{-1})} + \underbrace{a^\mu_a}_{\mathcal{O}(g)}$

a^μ_a is a *fluctuation* around the background field and it is a solution of the Classical Yang-Mills Eqs. with retarded boundary conditions

Blaizot, Gelis, Venugopalan, NPA 743 (2004) 13-56

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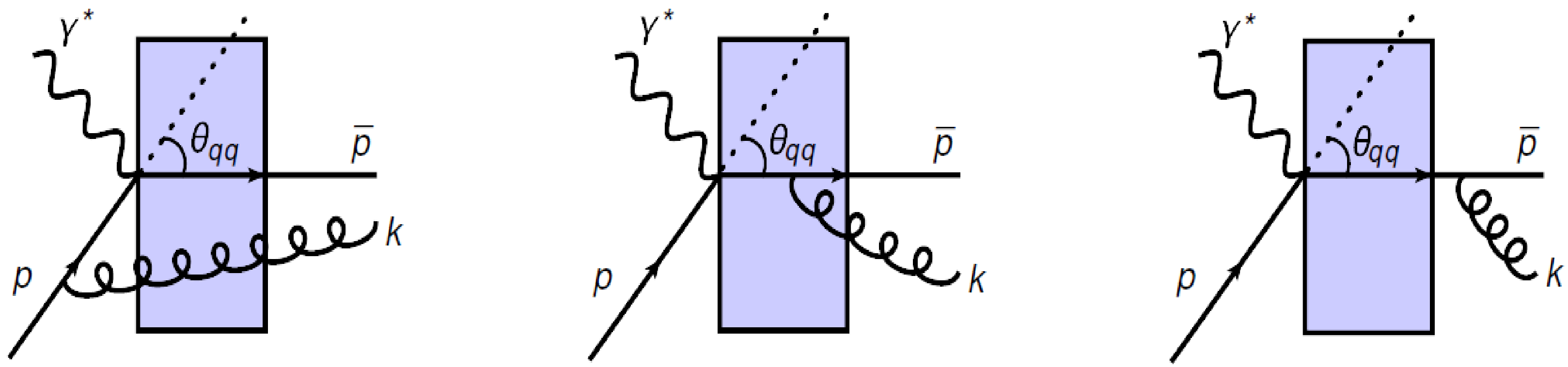
$$\mathcal{M}^a_\lambda =$$



via the LSZ reduction formula, the scattering amplitude in the LC gauge is

$$\mathcal{M}^a_\lambda = \lim_{x^+\to\infty} \int d^2\boldsymbol{x}\, d^4y\, e^{i(k^- x^+ - \boldsymbol{k}\cdot\boldsymbol{x})}\, e^{ik^+y^-} \mathcal{G}_{ab}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})\, \mathcal{J}_b(y)\, \epsilon_\lambda$$

Retarded gluon propagator in a background field

Color rotated current

Blaizot, Gelis, Venugopalan, NPA 743 (2004) 13-56

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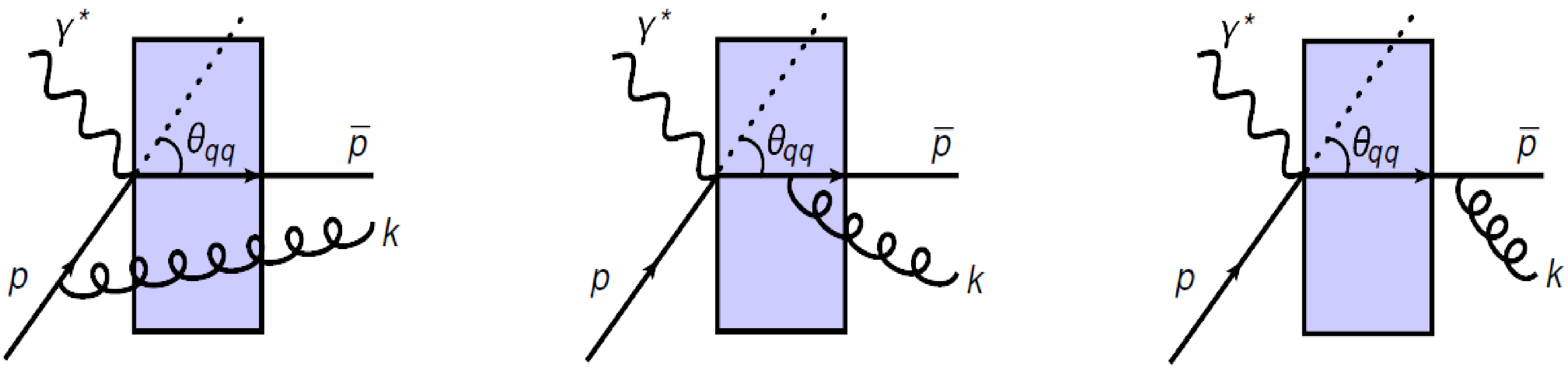

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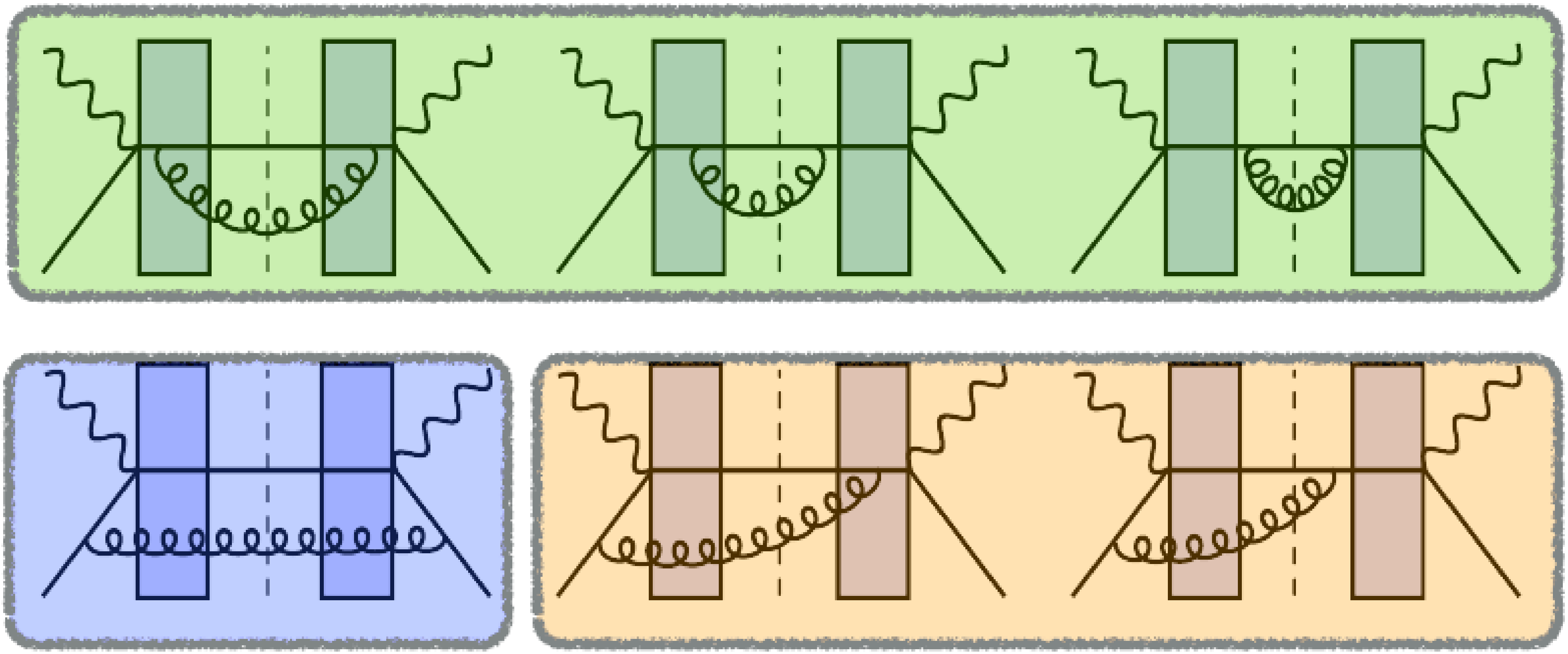
$$\mathcal{G}_{ab}(x^+,\boldsymbol{x};y^+,\boldsymbol{y}) = \int_{\boldsymbol{z}(y^+)=\boldsymbol{y}}^{\boldsymbol{z}(x^+)=\boldsymbol{x}} D\boldsymbol{z}(z^+) \exp\left[i\frac{k^+}{2}\int_{y^+}^{x^+} dz^+ \dot{\boldsymbol{z}}^2\right] \mathcal{U}_{ab}(x^+,y^+,\boldsymbol{z}(z^+))$$

Blaizot, Gelis, Venugopalan, NPA 743 (2004) 13-56

Gluon spectrum

BDMPS-Z + vacuum

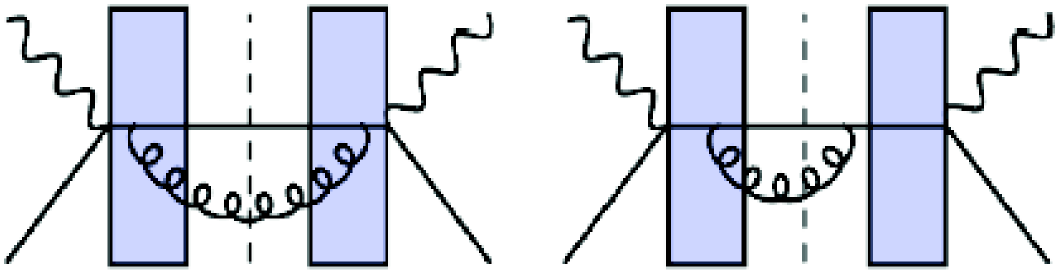


P_T broadening of ISR

Interferences in the medium: New!!

Armesto et. al, JHEP 1312 (2013) 052

Gluon spectrum I: direct emissions



$$\sim \int_0^L dt' \int \frac{d^2\boldsymbol{k}'}{(2\pi)^2} \mathcal{P}(\boldsymbol{k}-\boldsymbol{k}', L-t') \sin\left(\frac{\boldsymbol{k}'^2}{2k_f^2}\right) e^{-\frac{\boldsymbol{k}'^2}{2k_f^2}}$$

$$\mathcal{P}(\boldsymbol{k},\xi) = \frac{4\pi}{\hat{q}\xi} e^{-\frac{\boldsymbol{k}^2}{\hat{q}\xi}} \qquad k_f^2 = \sqrt{\hat{q}\omega}$$

- BDMPS-Z can be approximated by: quantum emission plus classical p_T broadening.

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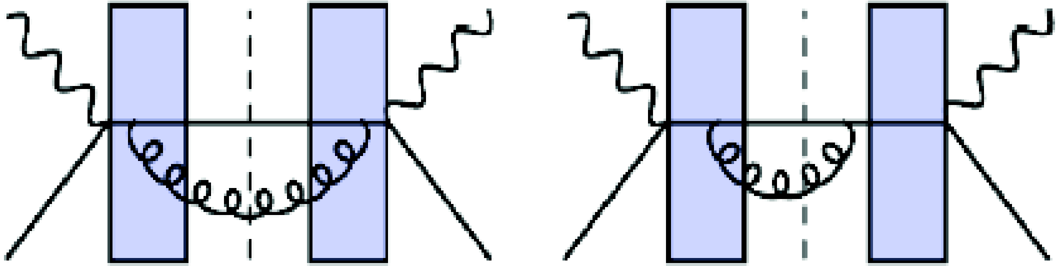

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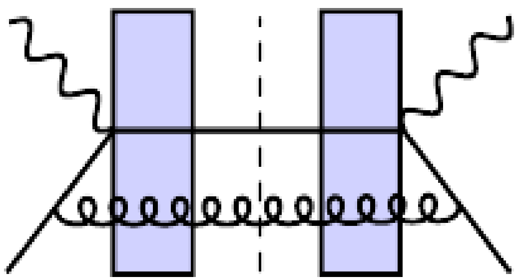


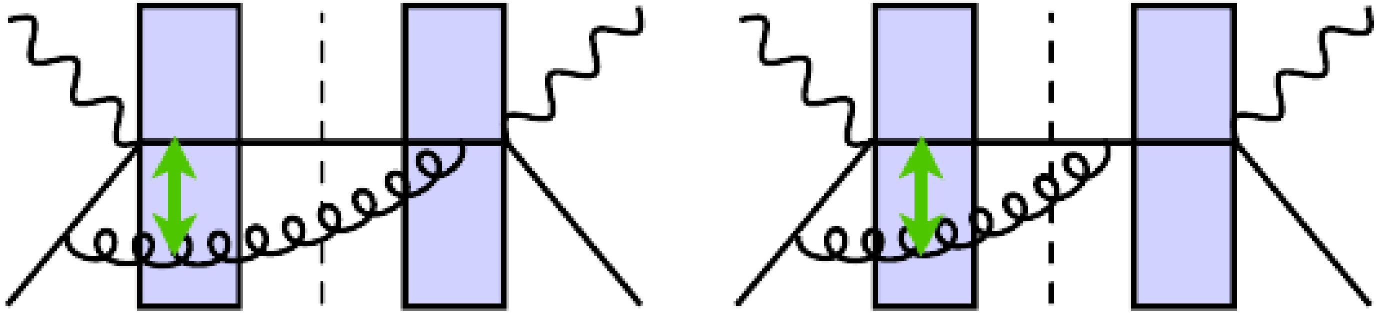
$$\sim \int \frac{d^2\boldsymbol{k}'}{(2\pi)^2} \frac{\mathcal{P}(\boldsymbol{k}'-\bar{\boldsymbol{\kappa}}, L^+)}{\boldsymbol{k}'^2}$$

- p_T broadening of ISR contains classical p_T broadening plus collinear divergence.

$$\langle k_T \rangle \sim Q_s = \sqrt{\hat{q}L^+}$$

Armesto et. al, JHEP 1312 (2013) 052

Guon spectrum II: Interferences



Transverse size of the Quark-gluon system

- If hard scattering is the largest scale:

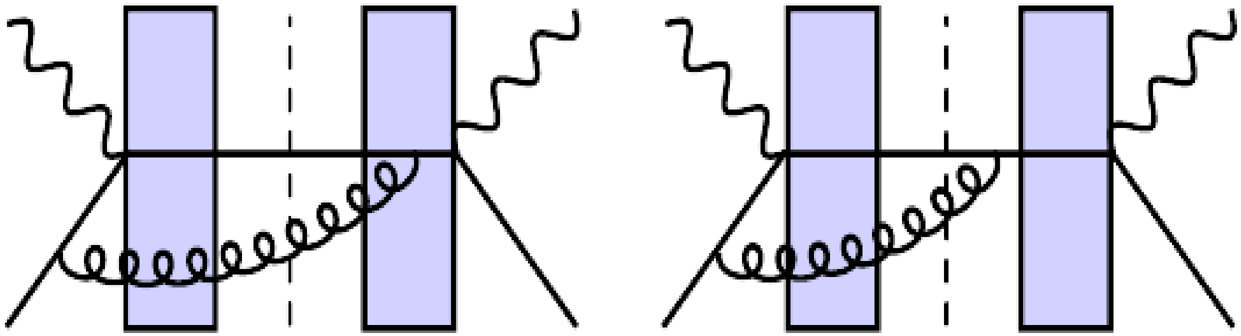
⇒ Insensitive to the medium

- If typical medium induced momentum is the largest scale

⇒ Medium is able to resolve the qg system

Armesto et. al, JHEP 1312 (2013) 052

Guon spectrum II: Interferences



The Color correlation of the Quark-gluon system is measured by

$$\mathcal{K}(x^+,\boldsymbol{x};y^+,\boldsymbol{y}|k^+) = \int_{\boldsymbol{r}(y^+)=\boldsymbol{y}}^{\boldsymbol{r}(x^+)=\boldsymbol{x}} \mathcal{D}\boldsymbol{r}\, \exp\left[\int_{y^+}^{x^+} d\xi \left(i\frac{k^+}{2}\dot{\boldsymbol{r}}^2(\xi) - \frac{1}{2}n(\xi)\sigma\left(\boldsymbol{r}(\xi)\right)\right)\right]$$

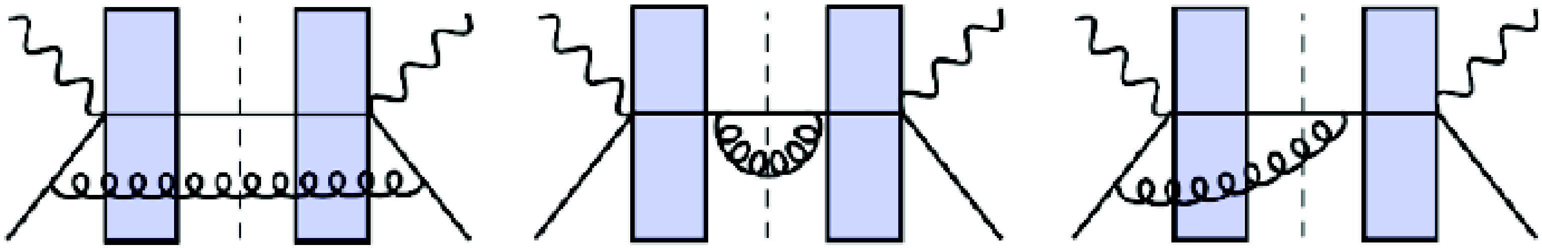
- Describes the Brownian motion of the gluon
- Harmonic oscillator approximation: $n\sigma(\mathbf{r}) \approx \hat{q}\mathbf{r}^2$
- Two extreme limits

⇒ High Energy Limit (Shockwave) $\tau_f \sim \sqrt{\omega/\hat{q}} \gg L^+$

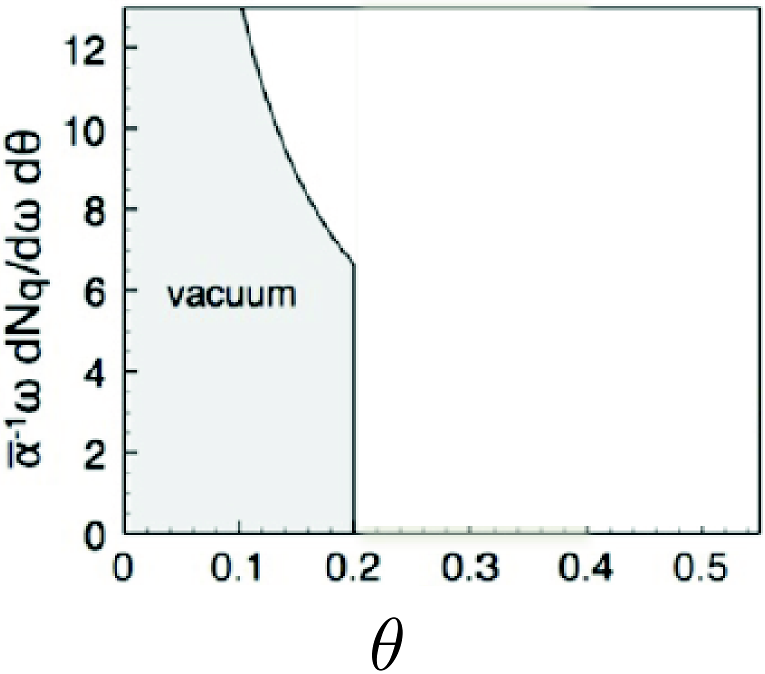
⇒ "Infinite" medium length $\tau_f \sim \sqrt{\omega/\hat{q}} \ll L^+$

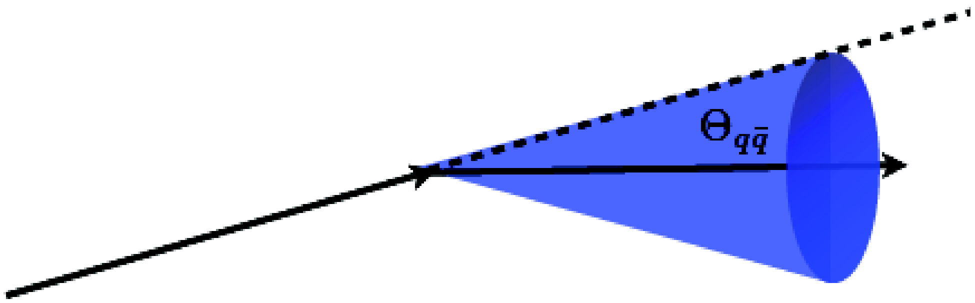
Armesto et. al, JHEP 1312 (2013) 052

Gluon spectrum: High energy limit (shockwave)



In the absence of a medium we recover the **vacuum** *coherence* pattern





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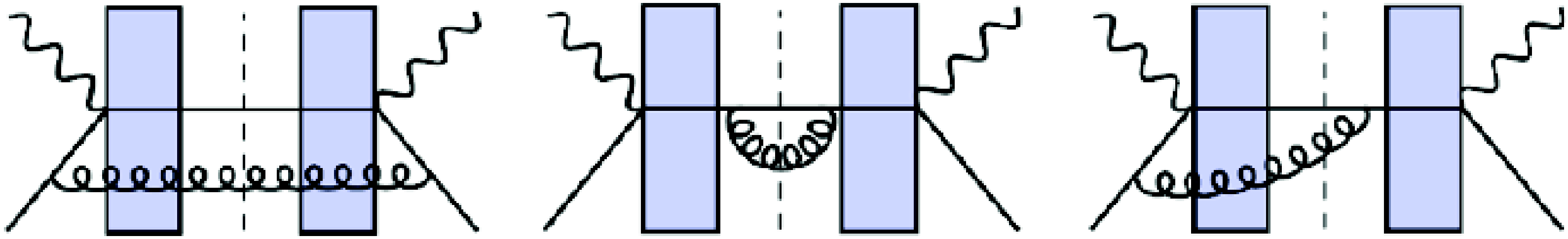
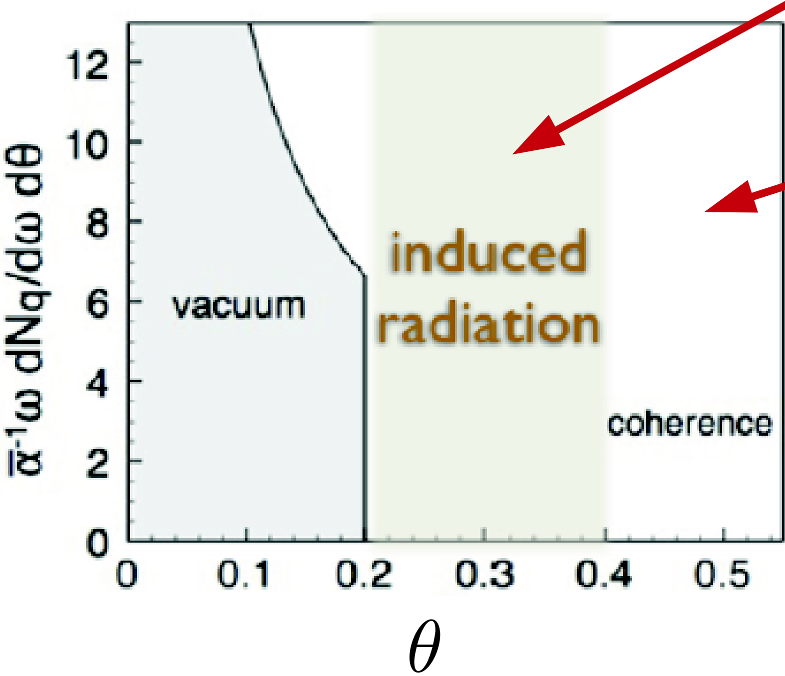
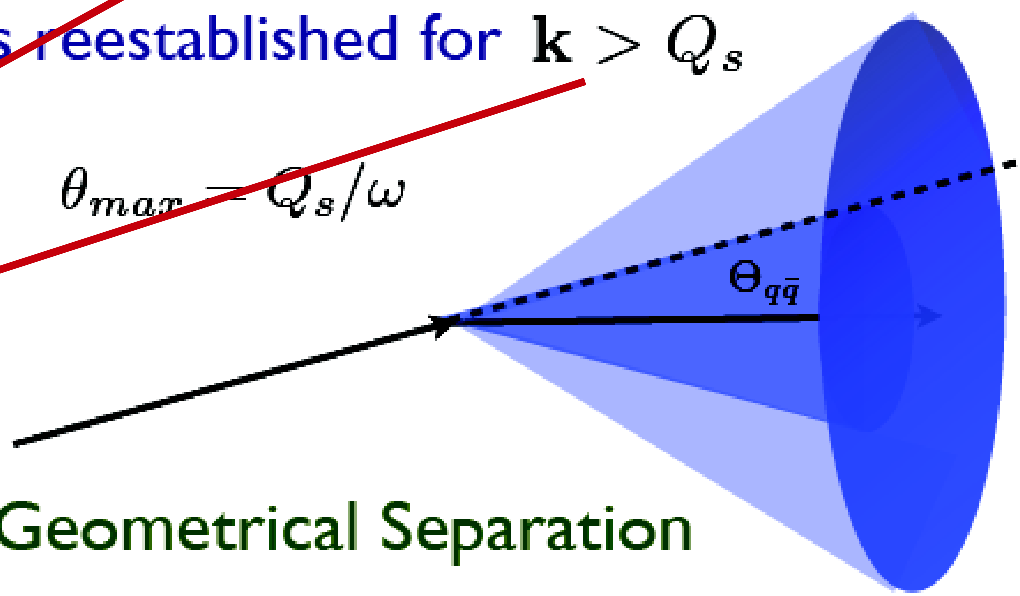


- Medium acts as a unique scattering center
- Interferences are suppressed if $\mathbf{k} < Q_s$
- Vacuum color coherence is reestablished for $\mathbf{k} > Q_s$



$\theta_{max} = Q_s/\omega$

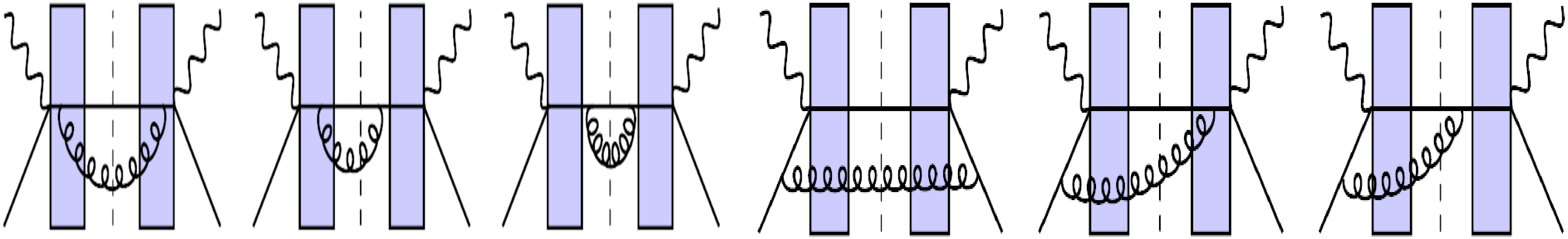


Geometrical Separation

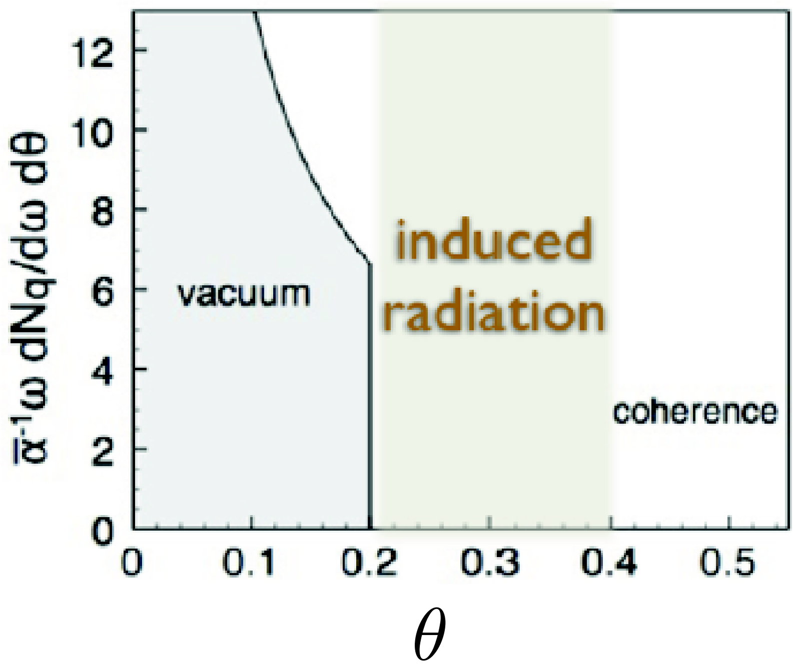
Contact with high energy limit: Kovchegov-Mueller (1998)

Armesto et. al, JHEP 1312 (2013) 052

Gluon spectrum: Infinite medium lenght



- Similar results in the incoherent regime: medium opens the phase space of emissions up to a maximum angle.



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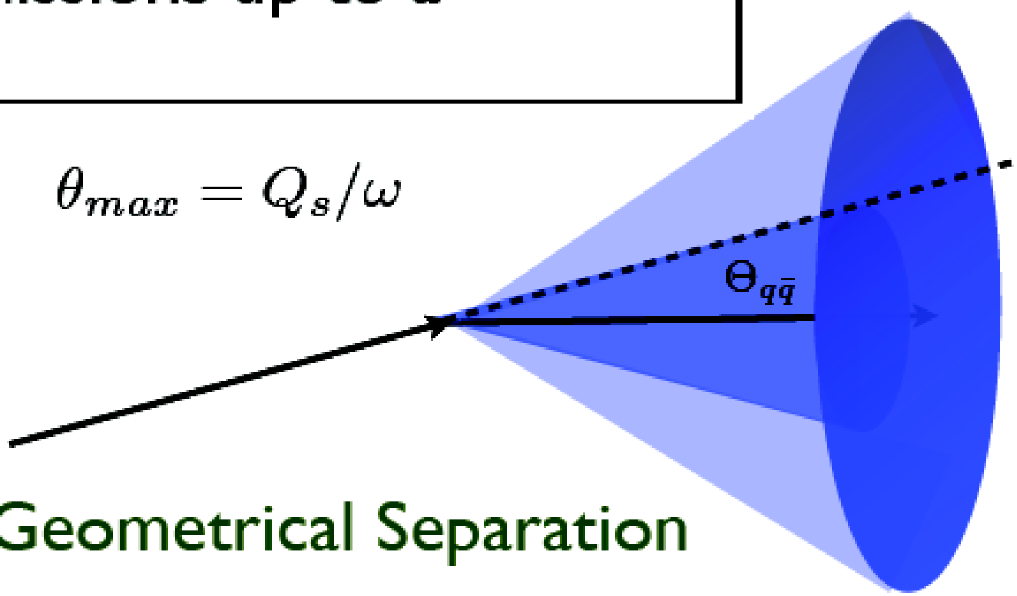


Geometrical Separation

Armesto et. al, JHEP 1312 (2013) 052

Generalizing to pA collisions: first results

- We perform a systematic eikonal expansion to the gluon propagator in the background field.
- We study soft gluon production in pA collisions beyond eikonal accuracy.

$$k^+ \frac{d\sigma}{dk^+ \, d^2\mathbf{k}} = \frac{1}{\mathbf{k}^2} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \, \varphi_p(\mathbf{q}) \, (\mathbf{k}-\mathbf{q})^2 \int_{\mathbf{b},\mathbf{r}} e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}} \; S_A(\mathbf{r},\mathbf{b}) + \; O\left(\left(\frac{L^+}{k^+}\partial_\perp^2\right)^2\right)$$

Recover the k_T factorization formula

- Some spin asymmetries

$$A_N(\boldsymbol{k}) \equiv \frac{\frac{d\sigma^\uparrow}{d^2k\,dy} - \frac{d\sigma^\downarrow}{d^2k\,dy}}{\frac{d\sigma^\uparrow}{d^2k\,dy} + \frac{d\sigma^\downarrow}{d^2k\,dy}} = \frac{\frac{d\sigma^\uparrow}{d^2k\,dy}(\boldsymbol{k}) - \frac{d\sigma^\uparrow}{d^2k\,dy}(-\boldsymbol{k})}{\frac{d\sigma^\uparrow}{d^2k\,dy}(\boldsymbol{k}) + \frac{d\sigma^\uparrow}{d^2k\,dy}(-\boldsymbol{k})}$$

Polarized target: $p + A^\uparrow \to g + X$

Polarized gluon production from unpolarized pA: $p + A \to g^\pm + X$

The eikonal contribution ***vanishes*** exactly while the leading dominant terms are the ***next to eikonal*** terms (finite size/medium effects)!!!

Altinoluk, Armesto, Beuf, Martinez, Salgado, arXiv:1404.2219

Conclusions

- We investigate medium modifications to the color coherence pattern between the initial and final state radiation.
- There is a gradual onset of ***decoherence*** between both emitters due to multiple scatterings with the medium

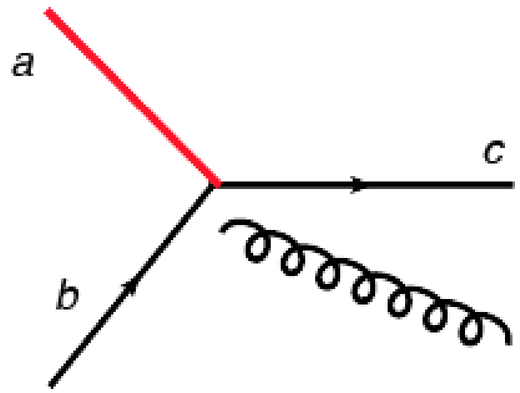
  $\Rightarrow$ Opening of phase space for ***large angle*** emissions
- First phenomenological consequences in pA collisions beyond the eikonal approximation:
  - *Soft gluon production:* non eikonal corrections to the CGC.
  - *Some spin asymmetries:* non eikonal corrections are the dominant contribution.

Outlook (keep tuned!!!)

- **Energy loss in high energy forward processes in pA collisions:** Kopeliovich et. Al, Strickman et. al., Kaidalov et. al., Peigne & Arleo, Liou & Mueller. ***Forthcoming***
- **Single inclusive gluon production in the hybrid formalism beyond eikonal accuracy.** ***Forthcoming***

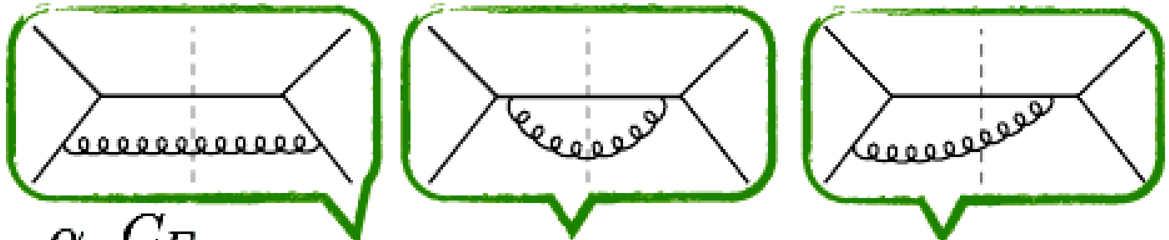
Backup

Coherence pattern between Initial-Final State



$Q_a = 0 \Rightarrow Singlet$

$Q_a \neq 0 \Rightarrow Octet$

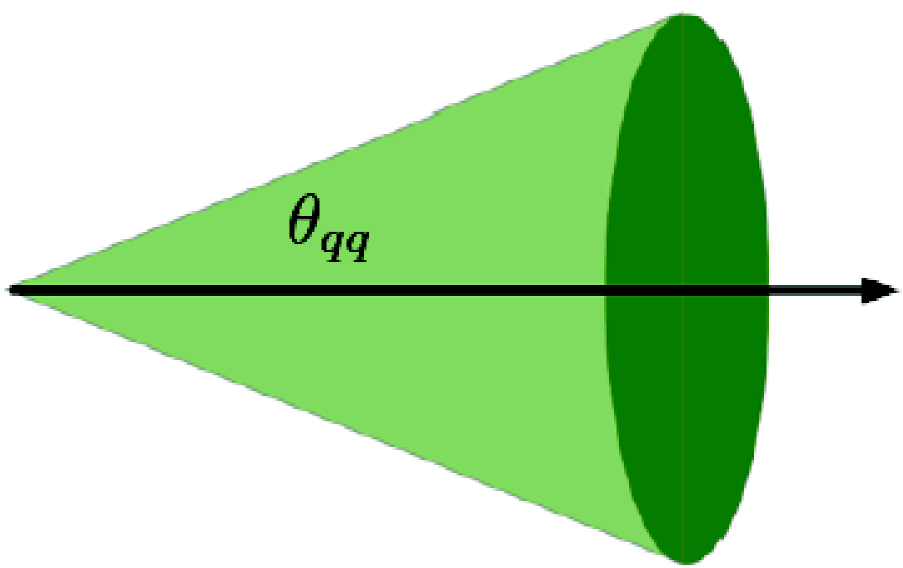


$$\omega \frac{dN}{d^3\vec{k}} = \frac{\alpha_s C_F}{(2\pi)^2\,\omega^2}\left[Q_b^2 \mathcal{R}_b + Q_c^2 \mathcal{R}_c + 2\,Q_b \cdot Q_c \mathcal{J}\right]$$

$$= \frac{\alpha_s C_F}{(2\pi)^2\,\omega^2}\left[Q_b^2(\mathcal{R}_b - \mathcal{J}) + Q_c^2(\mathcal{R}_c - \mathcal{J}) + Q_a^2 \mathcal{J}\right]$$

Coherent radiation

Total charge

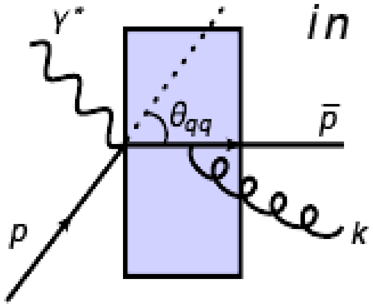


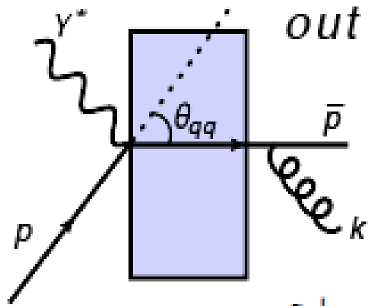
Coherent spectrum

$$\langle dN_i \rangle_\phi = \frac{\alpha_s C_F}{\pi} \boxed{\frac{d\omega}{\omega}\,\frac{d\theta_i}{\theta_i}}\,\Theta\left(\theta_{qq} - \theta_i\right)$$

Collinear and soft divergence

Scattering amplitude from CYM Eqs.





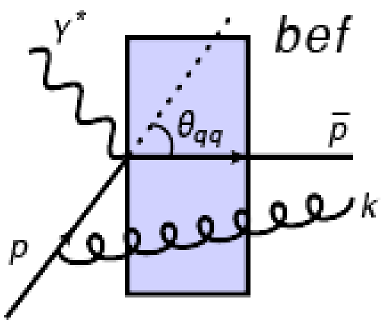
Outcoming parton

$$\mathcal{M}^a_{\lambda,in}(\vec{k}) = \frac{g}{k^+}\int d^2\boldsymbol{x} e^{i(k^- L^+ - \boldsymbol{k}\cdot\boldsymbol{x})}\int_0^{L^+} dy^+ e^{ik^+\bar{u}^- y^+}$$
$$\times\ \epsilon_\lambda \cdot \left(i\boldsymbol{\partial}_y + k^+\bar{\boldsymbol{u}}\right)\mathcal{G}_{ab}\left(L^+, \boldsymbol{x}, y^+, \boldsymbol{y} = \bar{\boldsymbol{u}}y^+ | k^+\right)\mathcal{U}_{bc}(y^+, 0) Q^{out}_c$$

$$\mathcal{M}^a_{\lambda,out}(\vec{k}) = -2i\frac{\epsilon_\lambda \cdot \bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\kappa}}^2} e^{i(\boldsymbol{k}\cdot\bar{\boldsymbol{u}})L^+}\,\mathcal{U}_{ab}(L^+, 0) Q^{out}_b\ ,$$

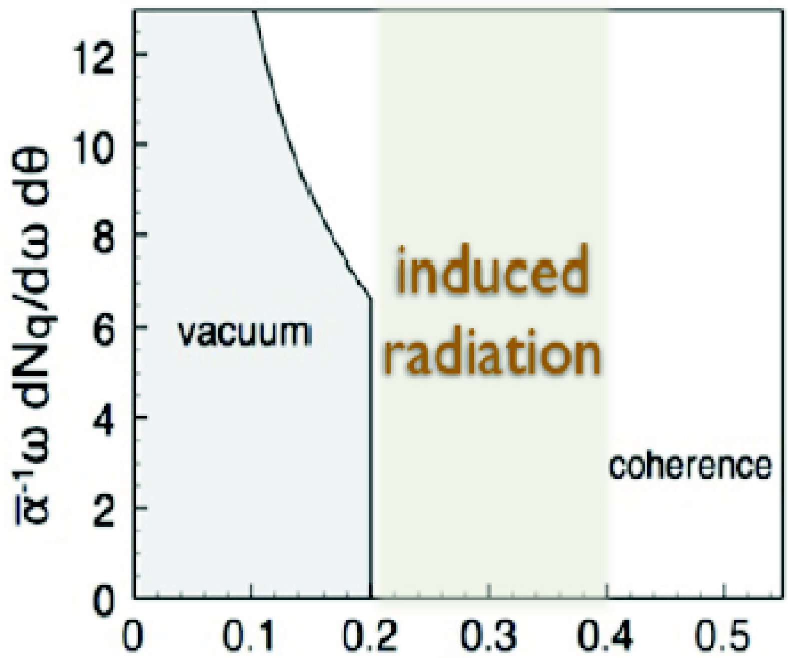
Incoming parton

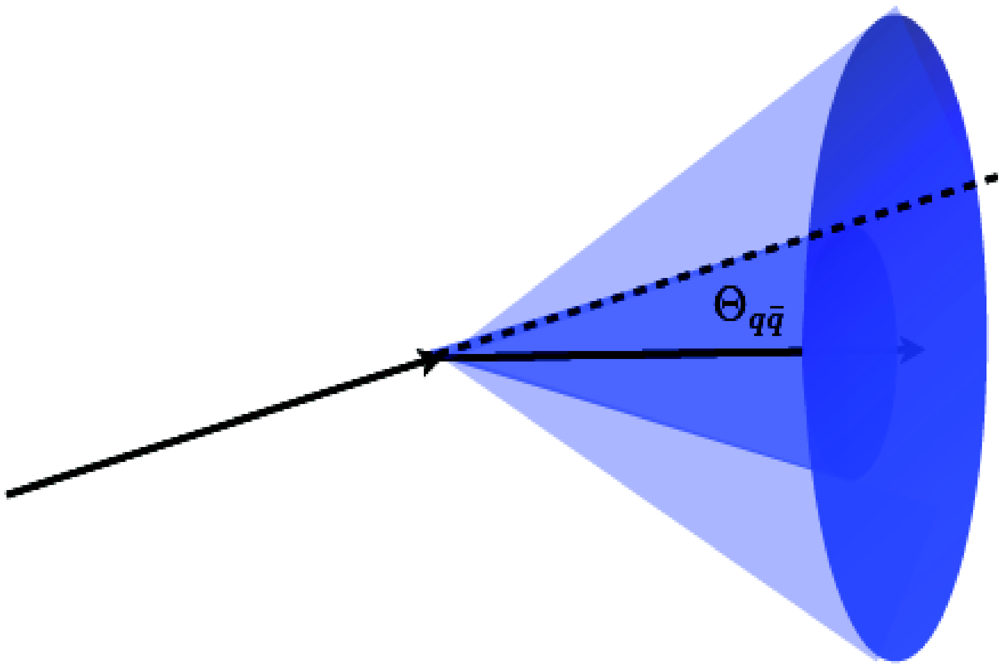
$$\mathcal{M}^a_{\lambda,bef}(\vec{k}) = \frac{g}{k^+}\int_{x^+=\infty} d^2\boldsymbol{x}\, e^{i(k^- x^+ - \boldsymbol{k}\cdot\boldsymbol{x})}\int_{-\infty}^{0} dy^+ e^{ik^+ u^- y^+}$$
$$\times\ \epsilon_\lambda \cdot \left(i\boldsymbol{\partial}_y + k^+\boldsymbol{u}\right)\mathcal{G}_{ab}\left(x^+, \boldsymbol{x}, y^+, \boldsymbol{y} = \boldsymbol{u}y^+ | k^+\right) Q^{in}_b$$



Armesto et. al, JHEP 1312 (2013) 052

Gluon spectrum: incoherent regime





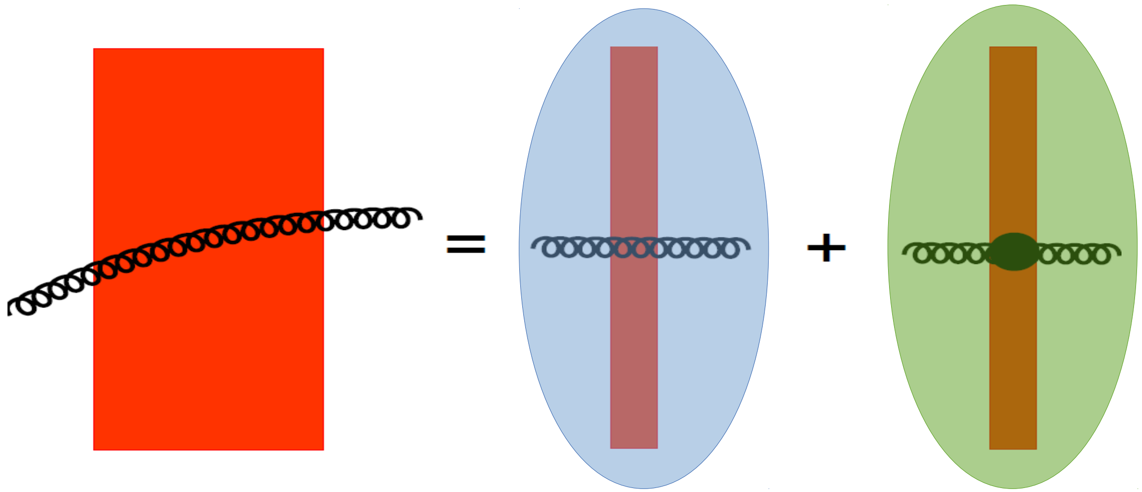
- Interferences play a role at early-times
- Gluon loses vacuum coherence

  ⇒ Open phase space at large angle emissions up to $\theta_{max} = Q_s/\omega$

- Typical ``medium induced'' gluon momentum ~ $Q_s = \hat{q}L$

Armesto et. al, JHEP 1312 (2013) 052

Next-to-Eikonal exp. to the gluon propagator



Wilson line
(shockwave)

$$\int d^2\mathbf{x}\, e^{-i\mathbf{k}\cdot\mathbf{x}}\, \mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \theta(x^+-y^+)\, e^{-i\mathbf{k}\cdot\mathbf{y}}\, e^{-ik^-(x^+-y^+)} \left\{ \mathcal{U}(x^+,y^+,\mathbf{y}) \right.$$

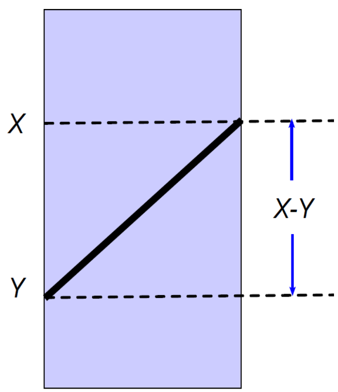
$$\left. + \frac{(x^+-y^+)}{k^+}\mathbf{k}^i\, \mathcal{U}^i_{(1)}(x^+,y^+,\mathbf{y}) + i\frac{(x^+-y^+)}{2k^+}\mathcal{U}_{(2)}(x^+,y^+,\mathbf{y}) \right.$$

"Decorated operators"

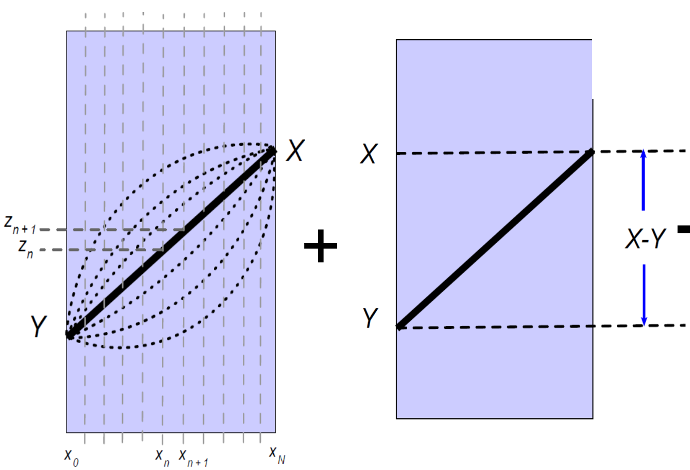
Non Eikonal Corrections

Altinoluk, Armesto, Beuf, Martinez, Salgado, arXiv:1404.2219

Next-to-Eikonal exp. to the gluon propagator



$$\mathcal{U}^{i,ab}_{(1)}(x^+,y^+,\mathbf{y}) = \int_{y^+}^{x^+} dz^+ \frac{1}{(x^+-y^+)} \left\{ [\partial_{y^i}\mathcal{U}(x^+,z^+,\mathbf{y})]\mathcal{U}(z^+,y^+,\mathbf{y}) \right\}^{ab}$$



$$\mathcal{U}^{ab}_{(2)}(x^+,y^+,\mathbf{y}) = \int_{y^+}^{x^+} dz^+ \frac{1}{(x^+-y^+)} \left\{ [\partial_{y}^2\mathcal{U}(x^+,z^+,\mathbf{y})]\mathcal{U}(z^+,y^+,\mathbf{y}) \right\}^{ab} .$$

Altinoluk, Armesto, Beuf, Martinez, Salgado, arXiv:1404.2219

KT factorization beyond Eikonal accuracy

$$\frac{1}{N_c^2-1}\sum_{\lambda\,\text{phys.}}\left\langle \overline{\mathcal{M}}_{\lambda}^{ab}(\underline{k},\boldsymbol{q})^{\dagger}\,\overline{\mathcal{M}}_{\lambda}^{ab}(\underline{k},\boldsymbol{q})\right\rangle_A = \frac{1}{\boldsymbol{k}^2\,\boldsymbol{q}^2}\int d^2\boldsymbol{b}\int d^2\boldsymbol{r}\; e^{-i(\boldsymbol{k}-\boldsymbol{q})\cdot\boldsymbol{r}}$$

$$\times\Bigg\{4\,(\boldsymbol{k}-\boldsymbol{q})^2 S_A(\boldsymbol{r},\boldsymbol{b}) + 2\frac{L^+}{k^+}\left[(\boldsymbol{k}-\boldsymbol{q})^2\,\boldsymbol{k}^j + \boldsymbol{k}^2(\boldsymbol{k}^j-\boldsymbol{q}^j)\right]\left[\mathcal{O}^j_{(1)}(\boldsymbol{r},\boldsymbol{b}) + \mathcal{O}^j_{(1)}(-\boldsymbol{r},\boldsymbol{b})\right]$$

$$+2i\frac{L^+}{k^+}\boldsymbol{k}\cdot(\boldsymbol{k}-\boldsymbol{q})\left[\mathcal{O}_{(2)}(\boldsymbol{r},\boldsymbol{b}) - \mathcal{O}_{(2)}(-\boldsymbol{r},\boldsymbol{b})\right] + O\left(\left(\frac{L^+}{k^+}\partial_\perp^2\right)^2\right)\Bigg\}\,.$$

where

$$S_A(r,b) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^{\dagger}\left(L^+,0;b-\frac{r}{2}\right)\mathcal{U}\left(L^+,0;b+\frac{r}{2}\right)\right]\right\rangle_A\,,$$

→ Dipole amplitude
Shockwave contribution

$$\mathcal{O}^j_{(1)}(r,b) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^{\dagger}\left(L^+,0;b-\frac{r}{2}\right)\mathcal{U}^j_{(1)}\left(L^+,0;b+\frac{r}{2}\right)\right]\right\rangle_A\,,$$

$$\mathcal{O}_{(2)}(r,b) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^{\dagger}\left(L^+,0;b-\frac{r}{2}\right)\mathcal{U}_{(2)}\left(L^+,0;b+\frac{r}{2}\right)\right]\right\rangle_A\,.$$

→ Non Eikonal
Corrections

Altinoluk, Armesto, Beuf, Martinez, Salgado, arXiv:1404.2219

SSA: Polarized Target

$$k^+\left(\frac{d\sigma^\uparrow}{dk^+\,d^2\mathbf{k}}-\frac{d\sigma^\downarrow}{dk^+\,d^2\mathbf{k}}\right)=\frac{2}{\mathbf{k}^2}\frac{L^+}{k^+}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\,\varphi_p(\mathbf{q};x_{\rm cut})$$

$$\times\left\{\left[(\mathbf{k}-\mathbf{q})^2\,\mathbf{k}^j+\mathbf{k}^2(\mathbf{k}^j-\mathbf{q}^j)\right]\int d^2\mathbf{r}\,\cos\Big(\mathbf{r}\cdot(\mathbf{k}-\mathbf{q})\Big)\int d^2\mathbf{b}\,\mathcal{O}^j_{(1)}(\mathbf{r},\mathbf{b},\mathbf{s})\right.$$

$$\left.+\mathbf{k}\cdot(\mathbf{k}-\mathbf{q})\int d^2\mathbf{r}\,\sin\Big(\mathbf{r}\cdot(\mathbf{k}-\mathbf{q})\Big)\int d^2\mathbf{b}\,\mathcal{O}_{(2)}(\mathbf{r},\mathbf{b},\mathbf{s})\right\}+O\left(\left(\frac{L^+}{k^+}\partial_\perp^2\right)^2\right)$$

- **Eikonal corrections cancel exactly due to the rotational symmetry around the center of the target.**
- **First subleading Non-Eikonal corrections turn out to be the dominant terms.**

Final interactions play an important role

- **Similar behavior observed with higher twist contributions**

Altinoluk, Armesto, Beuf, Martinez, Salgado, arXiv:1404.2219

SSA: Longitudinal Polarized Gluon Production

$$k^+ \frac{d\sigma^+}{dk^+\, d^2\mathbf{k}} - k^+ \frac{d\sigma^-}{dk^+\, d^2\mathbf{k}} = \frac{L^+}{k^+} \int \frac{d^2\mathbf{q}}{(2\pi)^2}\, \varphi_p(\mathbf{q}; x_{\rm cut})\, \mathbf{q}^2 \int d^2\mathbf{b} \int d^2\mathbf{r}\, e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}}$$

$$\times \Bigg\{ -i \left[\left(\frac{\mathbf{k}^i}{\mathbf{k}^2} - \frac{\mathbf{q}^i}{\mathbf{q}^2} \right) \epsilon^{ij} - 2\, \frac{\left(\epsilon^{il}\, \mathbf{k}^i\, \mathbf{q}^l\right)}{\mathbf{k}^2\, \mathbf{q}^2}\, \mathbf{k}^j \right] \mathcal{O}^j_{(1)}(\mathbf{r}, \mathbf{b})$$

$$- \frac{\left(\epsilon^{ij}\, \mathbf{k}^i\, \mathbf{q}^j\right)}{\mathbf{k}^2\, \mathbf{q}^2}\, \mathcal{O}_{(2)}(\mathbf{r}, \mathbf{b}) \Bigg\} + O\left(\left(\frac{L^+}{k^+} \partial_\perp^2 \right)^2 \right) .$$

- **Shockwave contribution vanishes exactly again!!!.**
- **Longitudinal polarization of the gluon (via polarized hadrons) is a good observable to study the structure of the next to eikonal corrections.**

Altinoluk, Armesto, Beuf, Martinez, Salgado, arXiv:1404.2219