Knudsen numbers in AA and pA collisions

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We investigate the applicability of fluid dynamics in ultrarelativistic heavy ion (AA) collisions as well as in proton nuclear (pA) collisions at RHIC and LHC energies [1].

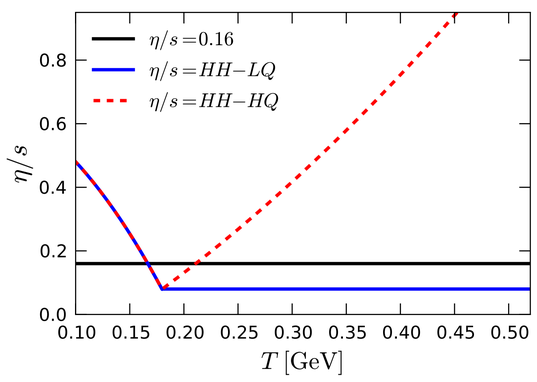
1. Model

Fluid dynamics with the equations of motion for $\pi^{\mu\nu}$ from kinetic theory [2]:

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi^{\lambda\langle\mu}\sigma_\lambda^{\nu\rangle} + \varphi_7 \pi^{\lambda\langle\mu}\pi_\lambda^{\nu\rangle} .$$

Initial state: Glauber eBC or eWN (fit $0-5$ % LHC $dN/d\eta_{\rm ch}$)
Initialization time: $\tau_0 = 0.2$ or 1.0 fm.

Shear viscosity parametrizations:



2. Knudsen number and validity of fluid dynamics

Validity of fluid-dynamics quantified by Knudsen number,

$$\mathrm{Kn} = \ell_{\rm micro}/L_{\rm macro},$$

where $\ell_{\rm micro}$ and $L_{\rm macro}$ are the microscopic and macroscopic distance/time scales, respectively.

$$\ell_{\rm micro} \sim \tau_\pi = 5\frac{\eta}{\varepsilon_0 + P_0}.$$

The macroscopic scale can be estimated from gradients of the macroscopic variables.

$$(L^\theta_{\rm macro})^{-1} = \theta\,, \quad (L^\varepsilon_{\rm macro})^{-1} = \varepsilon_0^{-1}\sqrt{\nabla_\mu\varepsilon_0\nabla^\mu\varepsilon_0}\,,$$

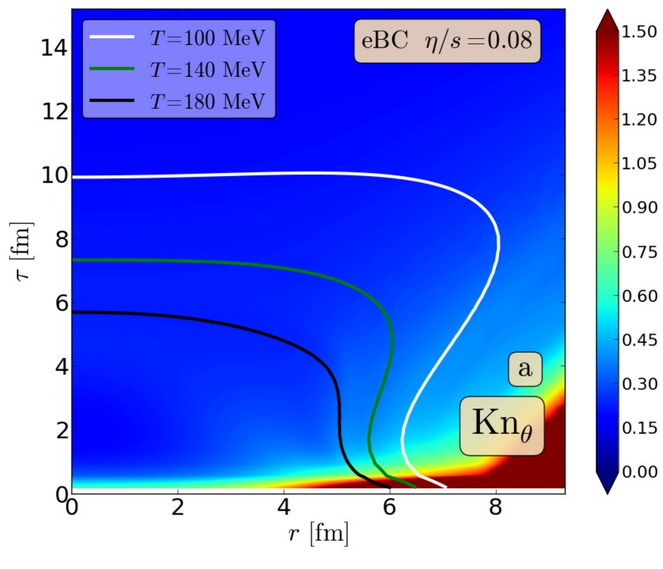
Kinetic theory vs fluid dynamics [3, 4]

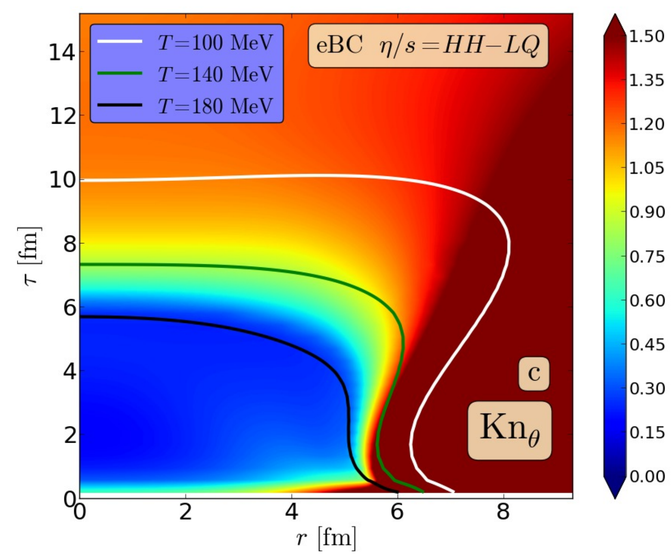
- $\mathrm{Kn} < 0.5$: fluid dynamics
- $\mathrm{Kn} < 0.5 - 1$: transient dynamics/kinetic theory
- $\mathrm{Kn} \gtrsim 1$: decoupling

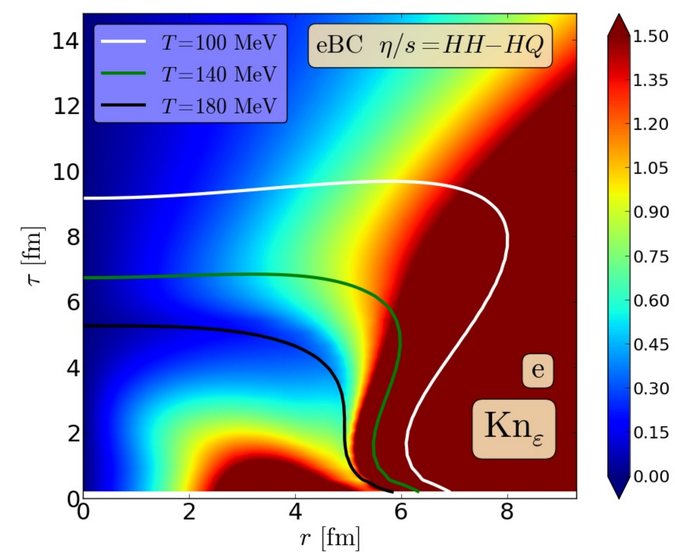
3. Spacetime evolution of Kn

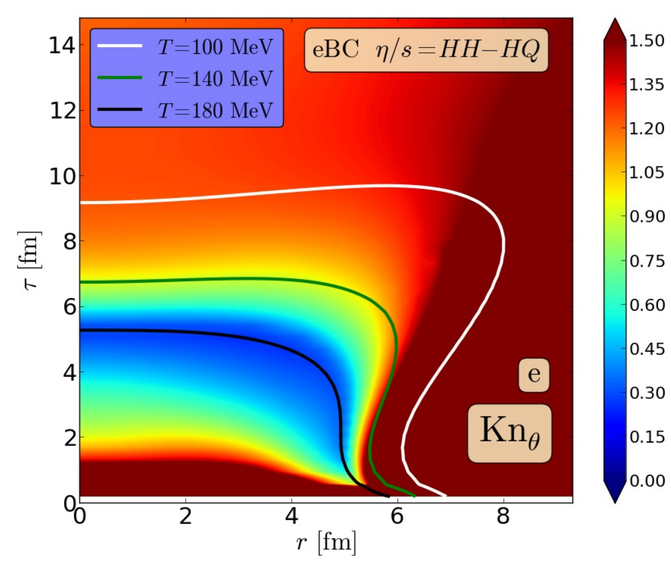
- $20-30$ % Pb+Pb collisions at the LHC
- Spacetime evolution along $x = y$ diagonal

3.1 Kn from expansion rate

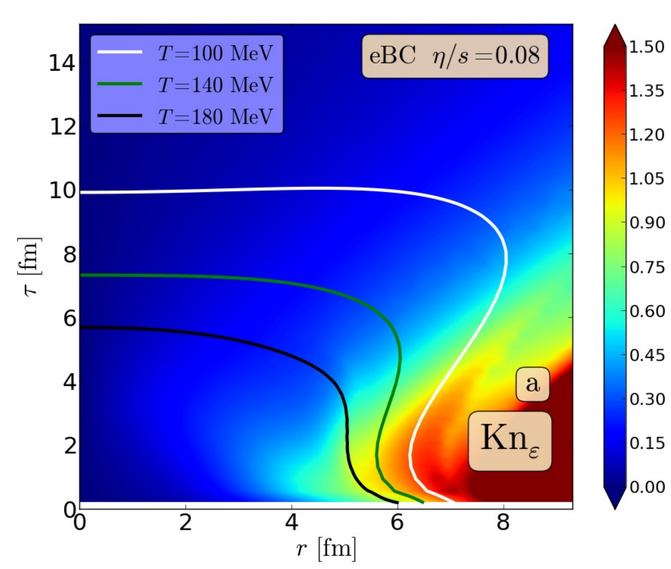


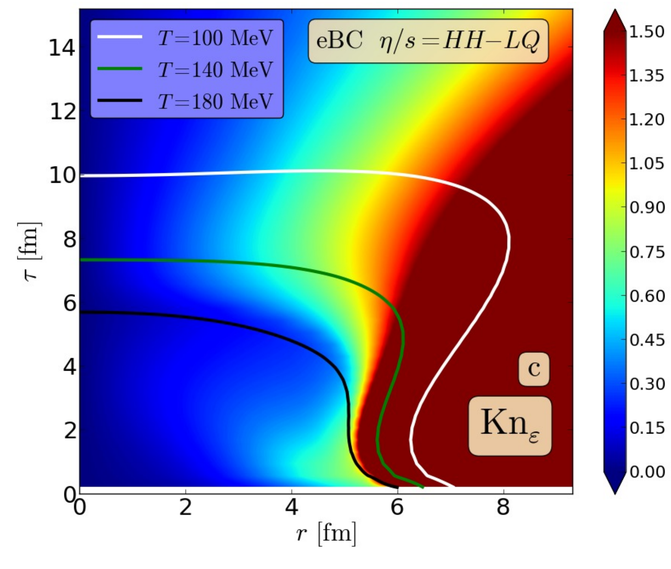






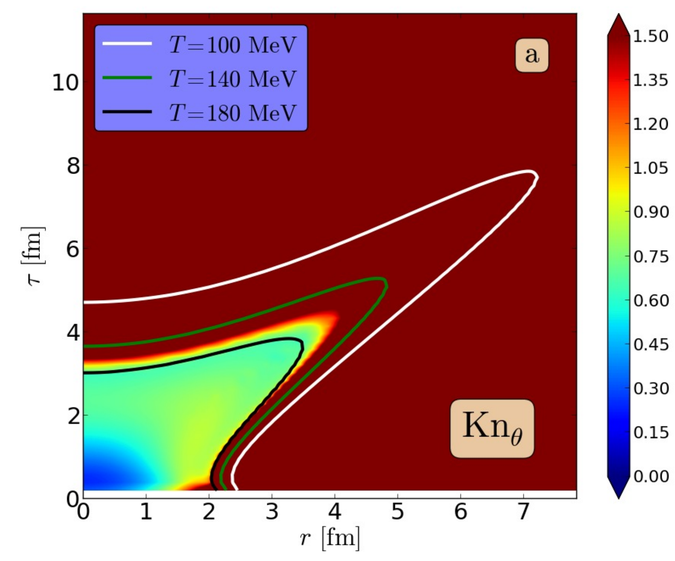
3.2 Kn from energy density gradient





4. Knudsen numbers in pA collisions

η/s = HH-LQ



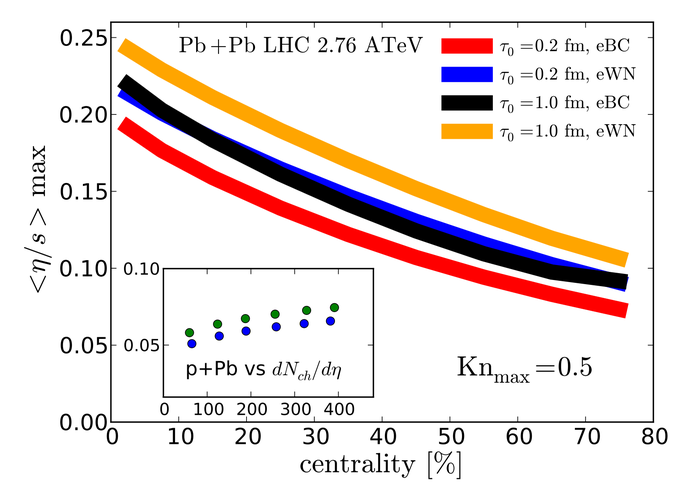
5. Maximum effective η/s

Local maximum allowed η/s for fluid-dynamical evolution,

$$\eta/s|_{\rm max} = \mathrm{Kn}_{max}\frac{(\varepsilon_0 + p_0)L_i}{5s}.$$

Space-time averaged maximum η/s

$$\left\langle\frac{\eta}{s}\right\rangle_{\rm max} = \frac{\int_{T>T_f} d\tau dx dy \tau s(\eta/s|_{\rm max})}{\int_{T>T_f} d\tau dx dy \tau s},$$



6. Conclusions

- Estimated maximum $\eta/s \sim 0.1 - 0.2$ in AA collisions.
- Similar to estimates of QGP viscosity
- For pA collisions the limit is even lower $\lesssim 0.08$

References

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[2] G. S. Denicol, H. Niemi, E. Molnar and D. H. Rischke, Phys. Rev. D **85**, 114047 (2012).
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[4] I. Bouras, E. Molnar, H. Niemi, Z. Xu, A. El, O. Fochler, C. Greiner and D. H. Rischke, Phys. Rev. C **82**, 024910 (2010).