Abstract
Early equilibration of a QCD medium is an important issue in collider physics. We develop a phenomenological model of early thermalization and chemical equilibration based on collinear parton splitting and recombination. We find that both could be achieved in a short time but quark thermal/chemical equilibration is slower than gluon thermalization.

1. Introduction
A standard model of relativistic heavy-ion collisions:

- $t > 10$ fm/c: Hadronic gas
- $t = 1-10$ fm/c: DGP/hadron-fluid
- $t < 0.1$ fm/c: Equilibration
- $t < 0$ fm/c: “Little bang”

The pre-equilibrium dynamics is a next high-energy frontier in QCD physics after the success of fluid/hadron cascade dynamics.

2. Collinear parton splitting model
- Splitting processes: the distributions $f_q$ and $f_g$ evolve so that the total parton number is conserved.

$$\frac{df_q}{dt} = \int d\alpha \left[ f_{qg}(\alpha) f_q(1-\alpha) - f_q(\alpha) f_q(1-\alpha) \right]$$

Gluon splitting

$$\frac{df_g}{dt} = \int d\alpha \left[ f_{gg}(\alpha) f_g(1-\alpha) - f_g(\alpha) f_g(1-\alpha) \right]$$

Gluon emission

$$\frac{df_{qg}}{dt} = \int d\alpha \left[ f_{qg}(\alpha) f_q(1-\alpha) - f_q(\alpha) f_g(1-\alpha) \right]$$

Quark pair production

where $f_{qg}(\alpha) = \alpha^0 (1-\alpha)^2$ and $f_{gg}(\alpha) = (1-\alpha)^2 f_{qg}(\alpha)$. The splitting functions are $r_{qg}(z) = \frac{1}{2} (1-\alpha) [1-\alpha^2] z$, $r_{gg}(z) = \frac{1}{2} (1-\alpha) [1-\alpha^2] z$, and $r_{qg}(\alpha) = N_v^2 z + (1-\alpha^2) z$ for $N_v = 3$.

- Recombination processes: the 2nd law of thermodynamics requires the system to be stable at local equilibrium.

$$\frac{df_q}{dt} = -\frac{1}{2} \int d\alpha \left[ f_{qg}(\alpha) f_{qg}(1-\alpha) - f_{qg}(\alpha) f_{qg}(1-\alpha) \right]$$

Gluon recombination

$$\frac{df_g}{dt} = -\frac{1}{2} \int d\alpha \left[ f_{gg}(\alpha) f_{gg}(1-\alpha) - f_{gg}(\alpha) f_{gg}(1-\alpha) \right]$$

Quark recombination

where $f_{qg}(\alpha) = 16N_v^2 \exp(-2\frac{\alpha}{\tau})$ and $f_{gg}(\alpha) = 12N_v^2 \exp(-2\frac{\alpha}{\tau})$ with thermal mass $\omega_{th} \sim \alpha_0^1/2$ representing finite density effects.

- Parton-medium interaction: Fokker-Planck equation encodes off-shell conditions for the splitting and recombination processes.

$$\frac{df_q}{dt} = \frac{d\sigma}{dt} [N_f - \frac{d\sigma}{d^2p} B^2(\alpha^2)]$$

where $N_f$ and $B^2$ characterize drag and diffusion effects.

3. Numerical analyses
- Model parameters for one-dimensional non-expanding systems (color glass condensate-like initial conditions):
  - Gluon distribution $f_g(p < Q_s) \sim 1/\alpha_s$ and $f_g(p > Q_s) \sim 0$.
  - Quark distribution $f_q(p) = 0$.
  - Splitting rate $\Gamma \sim \alpha_s^2 (q/p)^{1/2}$ and $\Gamma_{th} \sim \alpha_s^{2/3} T$.
  - Drag and diffusion coefficients $D_{th} \sim m_0 T$.

Results: gluonic system ($N_f = 0$).

Results: quark-gluon system ($N_f = 1$).

4. Summary and outlook
- We developed a simplified description of early thermal and chemical equilibration of quarks and gluons for the transition from glasma to QGP.
- Gluon thermalization is fast; quark equilibration can be slower and the effects of recombination become more important.
- Future prospects include three-dimensional analyses for the discussion of expansion and isotropization.

5. Related work
- Thermal photon elliptic flow can be enhanced by late production of quarks because photon emission mainly occurs after sizable anisotropy has developed.
- Numerical analyses indicate the effect may explain the “photon $v_1$ problem”.

AM, B. Müller, arXiv:1403.4225 [nucl-th]