Jet quenching from the lattice

Marco Panero

Instituto de Física Teórica, Universidad Autónoma de Madrid/CSIC

Quark Matter 2014, Darmstadt, 20 May 2014

Based on: M.P., K. Rummukainen and A. Schäfer, PRL 112 (2014) 162001
M. Panero

Jet quenching from the lattice
Jet quenching is related to *energy loss* and *momentum broadening* experienced by a hard parton moving in deconfined medium. 

Bjorken, 1982
Jet quenching is related to *energy loss* and *momentum broadening* experienced by a hard parton moving in deconfined medium [Bjorken, 1982].
Outline

1 Introduction

2 Theoretical approach

3 Soft physics contribution from a Euclidean setup

4 Lattice implementation

5 Results

6 Conclusions
Hard parton propagation in QGP

Multiple soft-scattering description, in the \textit{eikonal approximation} \cite{Baier:1997mu, 1997}

M. Panero
Jet quenching from the lattice
Multiple soft-scattering description, in the *eikonal approximation* Baier et al., 1997

Average momentum broadening described by jet quenching parameter:

\[ \hat{q} = \frac{\langle p_\perp^2 \rangle}{L} \]
Hard parton propagation in QGP

Multiple soft-scattering description, in the \textit{eikonal approximation} \cite{Baier:1997},

\begin{equation*}
\hat{q} = \frac{\langle p_{\perp}^2 \rangle}{L} = \int \frac{d^2 p_{\perp}}{(2\pi)^2} p_{\perp}^2 C(p_{\perp})
\end{equation*}

Can be evaluated in terms of a \textit{collision kernel} $C(p_{\perp})$ (differential parton-plasma constituents collision rate)
Hard parton propagation in QGP

Multiple soft-scattering description, in the *eikonal approximation* Baier et al., 1997

Average momentum broadening described by jet quenching parameter:

\[
\hat{q} = \frac{\langle p^2 \rangle}{L} = \int \frac{d^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp)
\]

Can be evaluated in terms of a *collision kernel* \( C(p_\perp) \) (differential parton-plasma constituents collision rate)

\( C(p_\perp) \) is related to two-point correlator of *light-cone Wilson lines*
Computing $\hat{q}$ at RHIC and LHC temperatures

Perturbative expansions may not be quantitatively reliable at RHIC or LHC temperatures
Perturbative expansions may not be quantitatively reliable at RHIC or LHC temperatures.

Holographic computations are not directly based on the microscopic formulation of QCD.
Computing $\hat{q}$ at RHIC and LHC temperatures

Perturbative expansions may not be quantitatively reliable at RHIC or LHC temperatures.

Holographic computations are not directly based on the microscopic formulation of QCD.

Monte Carlo lattice computations rely *crucially* on a Euclidean setup, so are generally unsuitable for real-time phenomena.
Computing \( \hat{q} \) at RHIC and LHC temperatures

Perturbative expansions may not be quantitatively reliable at RHIC or LHC temperatures.

Holographic computations are not directly based on the microscopic formulation of QCD.

Monte Carlo lattice computations rely *crucially* on a Euclidean setup, so are generally unsuitable for real-time phenomena.

A direct lattice evaluation of light-cone Wilson line correlators *very impractical*.
Outline

1 Introduction
2 Theoretical approach
3 Soft physics contribution from a Euclidean setup
4 Lattice implementation
5 Results
6 Conclusions
Key idea

Energy scale hierarchy in high-temperature, perturbative QCD:

\[ g^2 T/\pi \text{ (ultrasoft)} \ll gT \text{ (soft)} \ll \pi T \text{ (hard)} \]

IR divergences accounted for by 3D effective theories \( \text{Braaten and Nieto, 1995} \)
\( \text{Kajantie et al., 1995} \):

- electrostatic QCD (3D Yang-Mills + adjoint scalar field) for soft scale
- magnetostatic QCD (3D pure Yang-Mills) for ultrasoft scale

Large NLO corrections hindering PT due to soft, essentially classical fields

Observation: Soft contributions to physics of light-cone partons insensitive to parton velocity \( \rightarrow \) Can turn the problem Euclidean! \( \text{Caron-Huot, 2008} \)
Spatially separated ($|t| < |z|$) light-like Wilson lines \cite{Ghiglieri:2013}\n
\[ G^<(t, x_\perp, z) = \int d\omega d^2p_\perp dp_\perp \tilde{G}^<(\omega, p_\perp, p_z) e^{-i(\omega t - x_\perp \cdot p_\perp - zp_z)} \]

\[ = \int d\omega d^2p_\perp dp_\perp \left[ \frac{1}{2} + n_B(\omega) \right] \left[ \tilde{G}_R(\omega, p_\perp, p_z) - \tilde{G}_A(\omega, p_\perp, p_z) \right] e^{-i(\omega t - x_\perp \cdot p_\perp - zp_z)} \]

Shift $p' \cdot z = p_z - \omega t / z$, integrate over frequencies by analytical continuation into upper (lower) half-plane for retarded (advanced) contribution $\longrightarrow$ sum over Matsubara frequencies

\[ G^<(t, x_\perp, z) = T \sum_{n \in \mathbb{Z}} \int d^2p_\perp dp'_\perp \tilde{G}_E(2\pi n T, p_\perp, p'_\perp + 2\pi i n T t / z) e^{i(x_\perp \cdot p_\perp + zp'_z)} \]

- $n \neq 0$ contributions: exponentially suppressed at large separations
- Soft contribution: from $n = 0$ mode. Time-independent: evaluate in EQCD
Electrostatic QCD on the lattice

Super-renormalizable EQCD Lagrangian

\[ \mathcal{L} = \frac{1}{4} F_{ij}^a F_{ij}^a + \text{Tr} \left( (D_i A_0)^2 \right) + m_E^2 \text{Tr} \left( A_0^2 \right) + \lambda_3 \left( \text{Tr} \left( A_0^2 \right) \right)^2 \]

Parameters chosen (by matching) to reproduce soft physics of high-\(T\) QCD

- 3D gauge coupling: \( g_E^2 = g^2 T + \ldots \)
- Debye mass parameter: \( m_E^2 = \left( 1 + \frac{n_f}{6} \right) g^2 T^2 + \ldots \)
- 3D quartic coupling: \( \lambda_3 = \frac{9-n_f}{24\pi^2} g^4 T + \ldots \)

Standard Wilson lattice regularization \( \text{Hietanen et al., 2008} \)

Our setup: QCD with \( n_f = 2 \) light flavors, two temperature ensembles:

- \( T \approx 398 \text{ MeV} \)
- \( T \approx 2 \text{ GeV} \)

Closely related studies in MQCD \( \text{Laine, 2012 Benzke et al., 2012} \)
Effective theory: purely spatial

but

Operator describes *real time* evolution
Light-cone Wilson line correlator

\[ \langle W(\ell, r) \rangle = \left\langle \text{Tr} \left( L_3 L_1 L_3^{-1} L_1^\dagger \right) \right\rangle \sim \exp \left[ -\ell V(r) \right] \]

with

\[
L_3 = \prod U_3 H \\
L_1 = \prod U_1 \\
H = \exp(-a g_E^2 A_0)
\]

Well-defined renormalization properties \cite{D'Onofrio et al., 2014}
Outline

1. Introduction
2. Theoretical approach
3. Soft physics contribution from a Euclidean setup
4. Lattice implementation
5. Results
6. Conclusions
Contribution to $\hat{q}$ related to the curvature of $V(r)$ near the origin
Contribution to $\hat{q}$ related to the curvature of $V(r)$ near the origin

Data fitted with a procedure similar to Laine, 2012

$$V/g_E^2 = Arg_E + B(rg_E^2)^2 + C(rg_E^2)^2 \ln(rg_E^2) + \ldots$$
Contribution to $\hat{q}$ related to the curvature of $V(r)$ near the origin

Data fitted with a procedure similar to \cite{Laine, 2012}

$$V/g_E^2 = Arg_E^2 + B(r g_E^2)^2 + C(r g_E^2)^2 \ln(r g_E^2) + \ldots$$

Purely NP soft contribution to $\hat{q}$ is quite large

$$\hat{q}_{\text{EQCD}}^{\text{NP}} \sim 0.5 g_E^6$$
Contribution to $\hat{q}$ related to the curvature of $V(r)$ near the origin. Data fitted with a procedure similar to Laine, 2012

$$V/g_E^2 = Arg_E^2 + B(r g_E^2)^2 + C(r g_E^2)^2 \ln(r g_E^2) + \ldots$$

Purely NP soft contribution to $\hat{q}$ is quite large:

$$\hat{q}_{\text{EQCD}}^{\text{NP}} \sim 0.5 g_E^6$$

Approximate estimate $\hat{q} \sim 6 \text{ GeV}^2/\text{fm}$ at RHIC temperatures.
Lattice versus perturbation theory

“Naïve” comparison with NLO PT

Coordinate-space collision kernel from EQCD

\( n_f = 2, T \sim 2 \text{ GeV}, \text{perturbative } m_D \)

\[ V / m_D \]

\[ r m_D \]

\( \beta = 12 \)
\( \beta = 14 \)
\( \beta = 16 \)
\( \beta = 18 \)
\( \beta = 24 \)
\( \beta = 32 \)
\( \beta = 40 \)
\( \beta = 54 \)
\( \beta = 67 \)
\( \beta = 80 \)

NLO PT
Lattice versus perturbation theory

Discrepancy reduced if data are plotted in terms of non-perturbative $m_D$

_Laine and Philipsen, 2008_

(For a discussion of screening masses and real-time rates, see also _Brandt et al., 2014_}_{IfI}
Lattice versus perturbation theory

Discrepancy reduced if data are plotted in terms of non-perturbative $m_D$

Laine and Philipsen, 2008

(For a discussion of screening masses and real-time rates, see also Brandt et al., 2014)
Discrepancy reduced if data are plotted in terms of non-perturbative $m_D$

Laine and Philipsen, 2008

Coordinate-space collision kernel from EQCD
($n_f = 2, T \approx 398$ MeV, nonperturbative $m_D$)

(For a discussion of screening masses and real-time rates, see also Brandt et al., 2014)

Using NP value for $m_D$ in

$$\hat{q}_{\text{EQCD}}^{\text{NLO}} = g^4 T^2 m_D C_f C_a \frac{3\pi^2 + 10 - 4 \ln 2}{32\pi^2}$$

yields again $\hat{q} \sim 6$ GeV$^2$/fm at RHIC temperatures
Outline

1. Introduction
2. Theoretical approach
3. Soft physics contribution from a Euclidean setup
4. Lattice implementation
5. Results
6. Conclusions
Conclusions

- Lattice approach possible for certain real-time problems—see also Majumder, 2012 Ji, 2013
- Here: focus on soft contributions Laine and Rothkopf, 2013 Cherednikov et al., 2013
- A systematic approach
- Our results for jet quenching parameter give evidence of large non-perturbative effects
- Results in ballpark of
  - holographic computations Liu, Rajagopal and Wiedemann, 2006 Armesto, Edelstein and Mas, 2006 Gürsoy, Kiritsis, Michalogiorgakis and Nitti, 2009 ✓
  - estimates from phenomenological models Dainese et al., 2004 Eskola et al., 2004 Bass et al., 2008 ✓—but see also Burke et al., 2013 ❌
- Generalization to other observables (e.g. photon production rate, as suggested in Ghiglieri et al., 2013) in progress