Medium Effects on the transport coefficients of an interacting pion gas

XXIV QUARK MATTER DARMSTADT 2014

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The transport coefficients of a pion gas has been evaluated as a function of temperature at finite chemical potential. A theoretical framework for solving the relativistic transport equation called Chapman-Enskog approximation is presented here for calculating viscosities and thermal conductivity. The ππ scattering cross-section is evaluated by exchanging the in medium ρ and σ meson propagators through which the medium effects are introduced. The effect of early chemical freeze-out in heavy ion collisions is implemented through a temperature dependent pion chemical potential. These are found to effect the temperature dependence of the transport coefficients in a significant way.

Relativistic Boltzmann transport equation

$$p^\mu \partial_\mu f^{(0)}(x,p) = C[f]$$ *Collision term*

$$C[f] = \int d\Gamma_k d\Gamma_{p'} d\Gamma_{k'} \left[f(x,p')f(x,k')\{1+f(x,p)\}\{1+f(x,k)\} - f(x,p)f(x,k)\{1+f^{(0)}(x,p')\}\{1+f^{(0)}(x,k')\} \right] W$$

The Uehling-Uhlenbeck collision term for a binary elastic collision p+k → p′+k′ — *Reaction rate*

$$W = \frac{s}{2}\frac{d\sigma}{d\Omega}(2\pi)^6 \delta^4(p+k-p'-k')$$ *Differential scattering cross-section*

$$d\Gamma_p = \frac{d^3p}{(2\pi)^3 p^0}$$ → *Phase-space factor*

Solving Boltzmann transport equation by Chapman-Enskog approximation

$$f(x,p) = f^0(x,p) + f^0(x,p)\{1+f^0(x,p)\}\varphi(x,p)$$

The equilibrium distribution function for a Bosonic system ,

$$f_0 = \frac{1}{\exp\left\{\frac{p^\nu u_\nu(x) - \mu(x)}{K_B T(x)}\right\} - 1}$$

A small deviation of distribution function from the equilibrium resulting from the correspondence between the non-equilibrium kinetic theory and viscous hydrodynamics.

$$\xi = -\int d\Gamma_p f^0(p)(1+Af^0(p))QA$$ **Expression for bulk viscosity**

$$\lambda = -\frac{1}{3T}\int d\Gamma_p f^0(p)(1+f^0(p))(p^\sigma u_\sigma - h)B_\mu p_\nu \Delta^{\mu\nu}$$ **Expression for thermal conductivity**

$$\eta = -\frac{1}{10}\int d\Gamma_p f^0(p)(1+f^0(p))\langle p^\mu p^\nu \rangle C_{\mu\nu}$$ **Expression for shear viscosity**

Integral equation solved by A

$$\Re[A] = -\frac{1}{T} f^{(0)}(p)\{1+f^{(0)}(p)\}Q$$

$$\Re[B_\mu] = -\frac{1}{T} f^{(0)}(p)\{1+f^{(0)}(p)\}\Delta_{\mu\sigma}p^\sigma(p_\nu u^\nu - h)$$

$$\Re[C_{\mu\nu}] = -\frac{1}{T} f^{(0)}(p)\{1+f^{(0)}(p)\}\langle p_\mu p_\nu \rangle$$

Transport coefficients using Chapman-Enskog Approximation

- Bulk viscosity: $\xi = T\frac{\alpha_2^2}{a_{22}}$ $\quad a_{22} = 2z^2 I_3(z)$
- Thermal conductivity: $\lambda = \frac{T}{3m_\pi}\frac{\beta_1^2}{b_{11}}$ $\quad b_{11} = 8z(I_2(z) + I_3(z))$
- Shear viscosity: $\eta = \frac{T}{10}\frac{\gamma_0^2}{c_{00}}$ $\quad c_{00} = 16\left[I_1(z) + I_2(z) + \frac{1}{3}I_3(z)\right]$

Integral form of the bracket expression

$$I_\alpha(z) = \frac{z^4}{[S_2^{-1}(z)]^2} e^{(-2\mu_\pi/T)} \int_0^\infty d\psi \cosh^3\psi \sinh\psi^7 \int_0^\pi d\Theta \sin\Theta \frac{1}{2}\frac{d\sigma}{d\Omega}(\psi,\Theta)$$

$$\int_0^\infty d\chi \sinh^{2\alpha}(\chi) \int_0^\infty d\varphi \int_0^\pi d\theta \sin\theta \frac{e^{2z\cosh\psi\cosh\chi}}{(e^E-1)(e^F-1)(e^G-1)(e^H-1)} M_\alpha(\theta,\Theta)$$

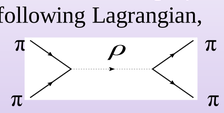
$$\downarrow$$

$$f^{(0)}(p)f^{(0)}(k)\{1+f^{(0)}(p')\}\{1+f^{(0)}(k')\}$$

Evaluating the ππ cross section

The invariant amplitude for the ππ scattering is evaluated using a ρ meson exchange between the pions using the following Lagrangian,

$$\mathcal{L}_{\rho\pi\pi} = g_\rho \bar{\rho}_\mu \cdot (\bar{\pi} \times \partial^\mu \bar{\pi})$$



g_ρ =6.05 is fixed from ρ ππ decay width.

In order to describe ππ scattering at low energies the σ exchange diagrams are also included using the interaction

$$\mathcal{L}_{\sigma\pi\pi} = \frac{1}{2} g_\sigma m_\sigma \bar{\pi} \cdot \bar{\pi}\sigma$$

Iso-spin averaged amplitudes from effective field theory

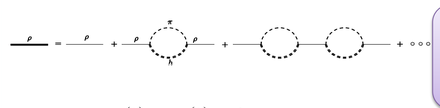
$$M_{I=0} = 2g_\rho^2\left[\frac{s-u}{t-m_\rho^2} + \frac{s-t}{u-m_\rho^2}\right] + g_\sigma^2 m_\sigma^2\left[\frac{3}{s-m_\sigma^2+im_\sigma\Gamma_\sigma} + \frac{1}{t-m_\sigma^2} + \frac{1}{u-m_\sigma^2}\right]$$

$$M_{I=1} = g_\rho^2\left[\frac{2(t-u)}{s-m_\rho^2+im_\rho\Gamma_\rho(s)} + \frac{t-s}{u-m_\rho^2} - \frac{u-s}{t-m_\rho^2}\right] + g_\sigma^2 m_\sigma^2\left[\frac{1}{t-m_\sigma^2} - \frac{1}{u-m_\sigma^2}\right]$$

$$M_{I=2} = g_\rho^2\left[\frac{u-s}{t-m_\rho^2} + \frac{t-s}{u-m_\rho^2}\right] + g_\sigma^2 m_\sigma^2\left[\frac{1}{t-m_\sigma^2} + \frac{1}{u-m_\sigma^2}\right]$$

$$\overline{|M|^2} = \frac{1}{9}\sum(2I+1)\overline{|M_I|^2}$$

Introducing medium effect in the ππ cross section



The exact ρ propagator with π-meson loop diagrams from Dyson equation

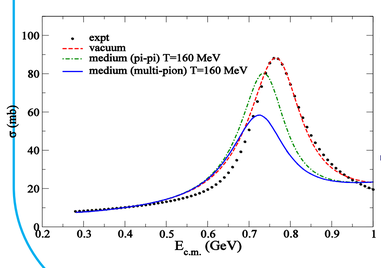
$$D_{\mu\nu} = D_{\mu\nu}^{(0)} + D_{\mu\rho}^{(0)}\Pi^{\rho\lambda}D_{\lambda\nu}$$

Vacuum propagator — **One loop self energy function**

The full ρ meson propagator in the medium

$$D_{\mu\nu}(q_0,\vec{q}) = \frac{-g_{\mu\nu} + q_\mu q_\nu / q^2}{q^2 - m_\rho^2 - \mathrm{Re}\,\Pi(q_0,\vec{q}) + i\,\mathrm{Im}\,\Pi(q_0,\vec{q})}$$

The in medium ρ propagator in terms of real and imaginary part of self energy



- The real part of the self energy function modifies the mass term in the propagator .
- The imaginary part of self energy is related to the decay width by the relation **$\mathrm{Im}\Pi(q_0,q) = -q_0\Gamma(q_0,q)$.**

Introduction of temperature dependent chemical potential - effect of early chemical freeze-out

Pions get out of chemical equilibrium early , at T~ **170 MeV.**

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Only elastic processes including the resonances dominate the dynamics of the system.

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At a lower temperature ~100 MeV momentum transfer ceases to give kinetic freeze out.

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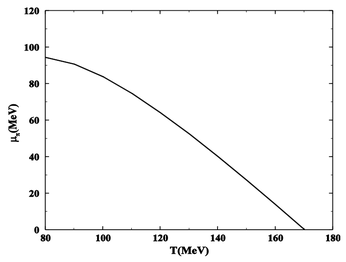
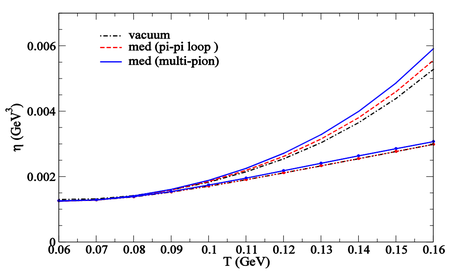
The chemical potential starts building up with decrease of temperature.



Ref: T. Hirano, and K. Tsuda Phys Rev. C 66, 054904 (2002)

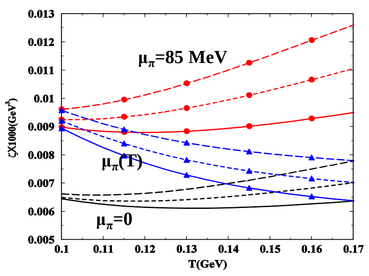
Shear viscosity as a function of temperature in Chapman-Enskog approximation



Ref: Sukanya Mitra , Sabyasachi Ghosh and Sourav Sarkar, Phys Rev. C 85, 064917 (2012)

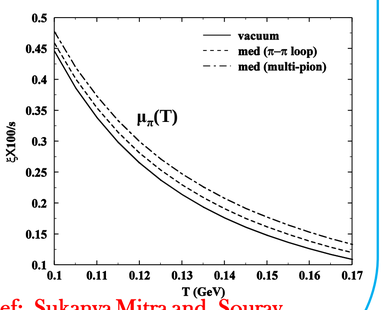
- **We can see a clear difference in the temperature dependence of shear viscosity with and without medium modification of the ρ propagator and it is more prominent from the heavy meson loops which considered as the multipion contribution to ρ self-energy compare to ππ loop.**
- **The upper set of curves uses the upper limit of integration over ψ~2 , which corresponds $E_{C.M.} = 2m_\pi\cosh\psi \sim 1$ GeV for ππ scattering while the lower set denotes actual upper limit , i.e. ∞, the difference between two sets of curve indicates the uncertainties of result due to insufficient information of cross section at higher energies.**

Bulk viscosity as a function of temperature in Chapman-Enskog approximation



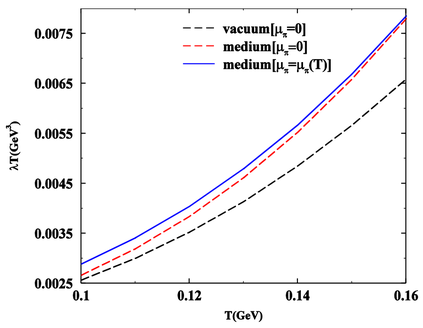
The three set of curves infer a large dependence on pion chemical potential, where in each set the effect of medium on ππ cross-section is clearly visible for both pion loop as well as heavy meson loop in ρ propagator.

The medium dependence is also observed for bulk viscosity to entropy density ratio as a function of temperature using a temperature dependent pion chemical potential .



Ref: Sukanya Mitra and Sourav Sarkar, Phys Rev. D 87, 094026 (2013)

Thermal conductivity as a function of temperature in Chapman-Enskog approximation



In case of thermal conductivity also the effect of medium is clearly seen through the self energy via π-π and π-meson loop. The zero and temperature dependent pion chemical potential gives small but distinctly different results.

Ref: Sukanya Mitra and Sourav Sarkar, Phys Rev. D 87, 094026 (2013)