

New results in the formalism and application of relativistic hydrodynamics

Rajeev S. Bhalerao, Amaresh Jaiswal, Subrata Pal and V. Sreekanth
Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, India

Introduction

- In heavy-ion collisions, after initial thermalisation, relativistic hydrodynamics is used to model the evolution of the expanding matter
- Formulation of second-order hydrodynamics is not unique and is a topic of considerable research
- We derive relativistic viscous hydrodynamic equations for various forms of the non-equilibrium single-particle phase-space distribution function $f(x, p)$, and apply these results to relativistic heavy-ion collisions

Relativistic Hydrodynamics

- Energy-momentum tensor of the fluid element is $T^{\mu\nu} = \varepsilon u^\mu u^\nu - P \Delta^{\mu\nu} + \Pi^{\mu\nu}$, with viscous contribution $\Pi^{\mu\nu} = \pi^{\mu\nu} - \Delta^{\mu\nu} \Pi$, where $\pi^{\mu\nu}$ is the shear pressure and Π is the bulk pressure.
- Structure of $\Pi^{\mu\nu}$ determines the order of the hydrodynamics formalism.
- First-order (in gradients of u^μ) theory is *acausal* and prone to instabilities.
- There are different formulations of second-order dissipative hydrodynamics
- In *hydrodynamic formalism based on the generalised second law of thermodynamics*, we write $f(x, p)$ close to equilibrium as $f = f_0 + \delta f \equiv f_0(1 + \phi)$, $f_0 = \exp(-\beta u \cdot p)$. Entropy current and its divergence are $S^\mu(x) = - \int dp p^\mu f (\ln f - 1)$, $\partial_\mu S^\mu = - \int dp p^\mu [\phi(1 + \phi/2)(\partial_\mu f_0) + \phi(\partial_\mu \phi) f_0]$. To proceed further, we need to specify $\delta f \equiv f_0 \phi$.
- We propose the new form

$$\text{Case 1 : } \phi_1 = \frac{\Pi}{P} + \frac{p^\mu p^\nu \pi_{\mu\nu}}{2(\varepsilon + P)T^2},$$

which is consistent with Grad's 14-moment approximation in orthogonal basis. Consider also the well-known form

$$\text{Case 2 : } \phi_2 = \frac{p^\mu p^\nu}{2(\varepsilon + P)T^2} \left(\pi_{\mu\nu} + \frac{2}{5} \Pi \Delta_{\mu\nu} \right),$$

with corrections which are only quadratic in momenta.

- Now, calculating $\partial_\mu S^\mu$ and imposing the second law of thermodynamics, $\partial_\mu S^\mu \geq 0$, gives *dynamical evolution equations* for $\pi^{\mu\nu}$ and Π

$$\pi^{\mu\nu} = 2\eta \left[\sigma^{\mu\nu} - \beta_2 \dot{\pi}^{\langle\mu\nu\rangle} - \frac{4}{3} \beta_2 \theta \pi^{\mu\nu} \right],$$

$$\Pi = -\zeta \left[\theta + \beta_0 \dot{\Pi} + \frac{4}{3} \beta_0 \theta \Pi \right].$$

The coefficients β_0 and β_2 are related to the relaxation times as $\tau_\Pi = \zeta \beta_0$, $\tau_\pi = 2\eta \beta_2$. For Case 1: $\beta_2^{(1)} = 3/(\varepsilon + P) + m^2 \beta^2 P / [2(\varepsilon + P)^2]$ and $\beta_0^{(1)} = 1/P$, and Case 2: $\beta_2^{(2)} = \beta_2^{(1)}$ and $\beta_0^{(2)} = \frac{18}{5(\varepsilon + P)} + \frac{3m^2 \beta^2 P}{5(\varepsilon + P)^2}$.

Hydrodynamical Evolution & Particle Production

In the 1D Bjorken model, equations governing the longitudinal expansion of the medium become,

$$\frac{d\varepsilon}{d\tau} = -\frac{1}{\tau}(\varepsilon + P + \Pi - \Phi),$$

$$\tau_\pi \frac{d\Phi}{d\tau} = \frac{4\eta}{3\tau} - \Phi - \frac{4\pi_\pi \Phi}{3\tau},$$

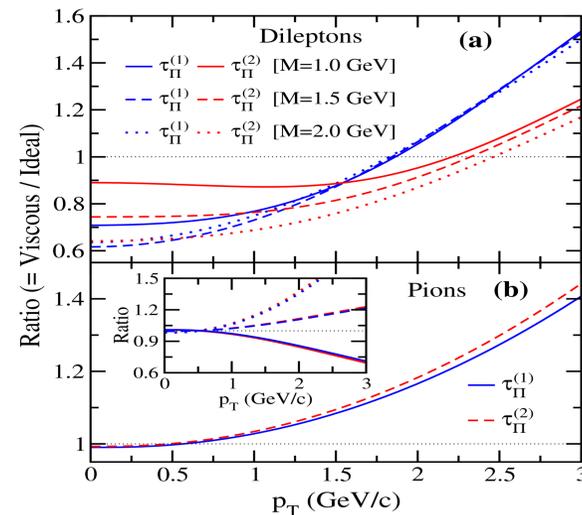
$$\tau_\Pi \frac{d\Pi}{d\tau} = -\frac{\zeta}{\tau} - \Pi - \frac{4\eta\Pi}{3\tau},$$

where $\tau_\Pi^{(1)} = \frac{\varepsilon + P}{PT} \left(\frac{\zeta}{s} \right)$, $\tau_\Pi^{(2)} = \frac{18}{5T} \left(\frac{\zeta}{s} \right)$, and $\tau_\pi = \frac{6}{T} \left(\frac{\eta}{s} \right)$.

- We use the lattice QCD EoS of A. Bazavov *et al.* (2009), with $T_c = 170$ MeV. We use the lattice QCD result of Meyer (2008) for the bulk viscosity to entropy density ratio ζ/s , which indicates the existence of a peak in ζ/s near T_c . We use the KSS bound of the shear viscosity to entropy density ratio $\eta/s = 1/4\pi$.

Hydrodynamical equations are solved numerically to get evolution of $T(\tau)$, $P(\tau)$, and viscous stresses $\Phi(\tau)$ and $\Pi(\tau)$. Initial conditions (RHIC): $\tau_0 = 0.5$ fm/c, $T_0 = 310$ MeV, $\Phi(\tau_0) = \Pi(\tau_0) = 0$, and freezeout temperature $T(\tau_f) = 160$ MeV.

Particle Production rates are modified depending upon the nonequilibrium $f(x, p)$ used. Thermal dilepton spectrum is obtained by integrating the total rate over the space-time history of the collision and hadron spectra are calculated using the Cooper-Frye prescription.



- Derived hydrodynamical equations using two different forms of $f(x, p)$, with ϕ_1 and ϕ_2 .
- Found that $\tau_\pi^{(1)} = \tau_\pi^{(2)}$, but $\tau_\Pi^{(1)} \neq \tau_\Pi^{(2)}$.
- $f(x, p)$ used in the derivation of hydro equations should be the same as $f(x, p)$ used in the freezeout prescription.
- Otherwise particle production affected significantly.

Chapman-Enskog-like method

- We solve the Boltzmann equation, in the relaxation-time approximation, iteratively

$$p^\mu \partial_\mu f = - (u \cdot p) \frac{f - f_0}{\tau_R}$$

$$f = f_0 + \delta f, \quad \delta f = \delta f^{(1)} + \delta f^{(2)} + \dots$$

$$f_1 = f_0 - \frac{\tau_R}{u \cdot p} p^\mu \partial_\mu f_0, \quad f_2 = f_0 - \frac{\tau_R}{u \cdot p} p^\mu \partial_\mu f_1.$$

To first- and second-orders in derivatives, we have

$$\delta f^{(1)} = -\frac{\tau_R}{u \cdot p} p^\mu \partial_\mu f_0, \quad \delta f^{(2)} = \frac{\tau_R}{u \cdot p} p^\mu p^\nu \partial_\mu \left(\frac{\tau_R}{u \cdot p} \partial_\nu f_0 \right).$$

With vanishing bulk viscosity and thermal conductivity, and using $\pi^{\mu\nu} = \Delta_{\alpha\beta}^{\mu\nu} \int dp p^\alpha p^\beta \delta f$, we can get hydrodynamical equations of first order: $\pi^{\mu\nu} = 2\tau_R \beta_\pi \sigma^{\mu\nu}$ and second order: $\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_R} = 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu} \omega^{\nu\rangle\gamma} - \frac{10}{7} \pi_\gamma^{\langle\mu} \sigma^{\nu\rangle\gamma} - \frac{4}{3} \pi^{\mu\nu} \theta$, where $\beta_\pi = 4P/5$ & $\tau_\pi = \eta/\beta_\pi$.

- We use $f(x, p)$ obtained in this method

$$\delta f_{1,CE} = \frac{f_0 \beta}{2\beta_\pi (u \cdot p)} p^\alpha p^\beta \pi_{\alpha\beta}$$

and in the well-known Grad's method

$$\delta f_G = \frac{f_0 \beta^2}{10\beta_\pi} p^\alpha p^\beta \pi_{\alpha\beta}$$

within the 1D Bjorken model.

- The longitudinal HBT radius, R_L , is calculated in terms of the transverse momentum, K_T , of the identical-particle pair

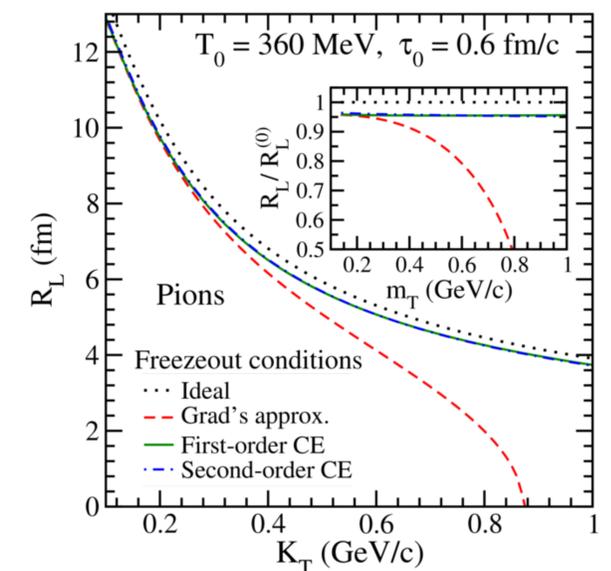
$$R_L^2(K_T) = \frac{\int K_\mu d\Sigma^\mu f(x, K) z^2}{\int K_\mu d\Sigma^\mu f(x, K)},$$

where K^μ is the pair four-momentum in the mid-rapidity region and $z = \tau_f \sinh(\eta_s)$. For large values of M_T/T :

$$f = f_0 : (R_L^2)^{(0)} = \tau^2 T / m_T,$$

$$f = f_0 + \delta f_1 : (\delta R_L^2)^{(1)} = -\frac{5\tau^2 T \Phi}{4\beta_\pi m_T},$$

$$f = f_0 + \delta f_G : (\delta R_L^2)^{(G)} = -\frac{\tau^2 T \Phi}{5\beta_\pi m_T} \left(3 + \frac{m_T}{T} \right).$$



- CE-like method, unlike Grad's, involves a small expansion parameter and is rapidly convergent up to second order.
- This method, unlike Grad's, preserves experimentally observed $1/\sqrt{m_T}$ scaling of R_L .
- At large p_T , this method yields smaller hadron multiplicities.
- References
R. S. Bhalerao, A. Jaiswal, S. Pal, and V. Sreekanth, *Phys. Rev. C* **88**, 044911 (2013);
Phys. Rev. C **89**, 054903 (2014)