Quark-Hadron Phase Transition in the PNJL Model with Hadronic Excitations

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K. Yamazaki and T. Matsui, Nucl. Phys. A913 (2013) 19.

K. Yamazaki and T. Matsui, Nucl. Phys. A922 (2014) 237.

1. Motivation

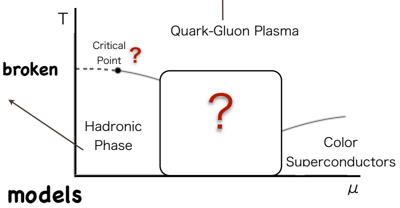
★ QCD phase diagram

Quark-Gluon Plasma

- Chiral Phase Transition
- De-confining Phase Transition

Hadronic Phase

- Chiral symmetry is broken
- Color is confined



★ Study by effective models

We need the following model ;

- Containing order-parameters of the chiral transition and the de-confining transition
- Hadrons in low temperature phase

Nambu-Jona-Lasinio model with Polyakov-loop K. Fukushima, 2004

Mean field approximation (MFA) + hadronic excitations

2. PNJL model with mesonic excitations (μ=0)

★ Partition Function

$$Z(T, A_4) = \int [d\bar{q}][dq] \exp\Big[\int_0^\beta d\tau \int d^3x\, \mathcal{L}(q, \bar{q}, A_4)\Big]$$

2 flavor

$$\mathcal{L}(q, \bar{q}, A_4) = \bar{q}(i\gamma^\mu D_\mu - m_0)q + G\Big[(\bar{q}q)^2 + (\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\Big] \qquad D_\mu = \partial_\mu + gA_4\delta_{\mu,0}$$

4th component of gauge field four fermi interaction

3 flavor

$$\mathcal{L}(q, \bar{q}, A_4) = \bar{q}(i\gamma^\mu D_\mu - \hat{m}_0)q + G\Big[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2\Big] + \mathcal{L}_6$$

★ Derivation of equation of state

- Introducing bosonic fields as auxiliary fields
- Calculating effective action up to the second order of fluctuation
- Gaussian integral over boson fields

$$\Omega(T, A_4) = T\Big(I_0 + \frac{1}{2}\mathrm{Tr}_M \ln \frac{\delta^2 I}{\delta\phi_i \delta\phi_j}\Big)$$

Contribution from mean field Contribution from mesonic excitations

- Taking statistical average over gauge field

3. Equation of state

★ MFA

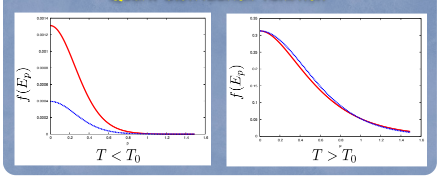
$$p_{MF}(T, \Phi) = p^0_{MF}(M_0) - \Delta p_{\mathrm{vac}} + 4\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{3E_p} f_\Phi(E_p) - \mathcal{U}(T, \Phi)$$

zero point motion vacuum pressure Effective potential of Φ

$$L(\boldsymbol{r}) = \mathcal{P}\exp\Big[ig\int_0^\beta d\tau A_4(\tau, \boldsymbol{r})\Big] \rightarrow e^{\beta A_4}$$

$$\Phi = \frac{1}{3}\langle \mathrm{tr}_c L\rangle$$

Quark distribution function



Red line : $f(E_p)$ by NJL model.

Blue line : $f(E_p)$ by PNJL model (containing Polyakov loop).

Single quark excitations are suppressed by Polyakov loop. H. Hansen, et al. 2007

★ Mesonic excitations

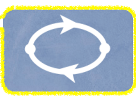
$$p_M(T, \Phi) = -\frac{T}{2}\sum_n \int \frac{d^3q}{(2\pi)^3}\sum_\alpha \ln \mathcal{M}_\alpha(\omega_n, q)$$

2 flavor : α = π, σ

3 flavor : α = π, K, η, η', σ, κ, a0, f0

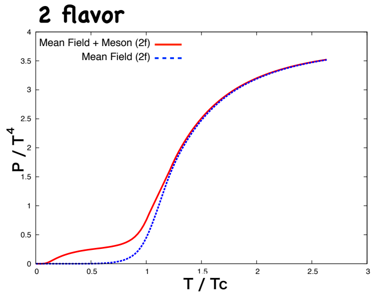
Ishida, 1999

$$\mathcal{M}_\alpha(\omega_n, q) = 1 - 2G\Pi_\alpha(\omega_n, q)$$



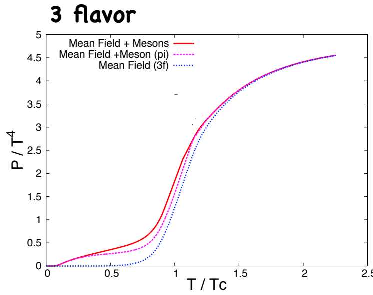
4. Numerical results of pressure

2 flavor



π and σ are taken into this calculation.

3 flavor



π, K and σ are taken into this calculation.

- At low T, pressure from quark excitation is suppressed by Polyakov loop.
- At high T, massless quark excitations dominate the pressure.

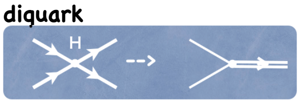
5. Extension to include baryons

$$\mathcal{L} = \bar{q}(i\not{D} - m)q + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq}$$

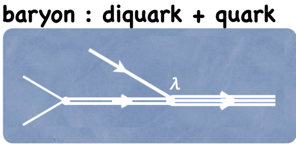
★ Introducing auxiliary fields

meson

diquark



baryon : diquark + quark



$$Z = \int [d\bar{q}][dq][d\phi][d\bar{\Delta}][d\Delta][d\bar{B}][dB]e^{-I(\bar{q},q,\phi,\bar{\Delta},\Delta,\bar{B},B)}$$

bilinear form of quark fields

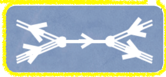
integral over quark fields --> fermion determinant

$$Z = \int [d\phi][d\bar{\Delta}][d\Delta][d\bar{B}][dB]e^{-I_{eff}(\phi,\bar{\Delta},\Delta,\bar{B},B)}$$

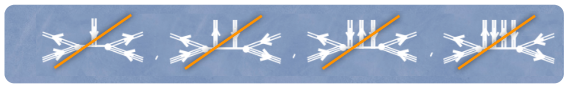
$$I_{eff} = \int d\tau \int d^3x\Big(\frac{1}{2G}\sigma^2 + \frac{1}{2H}\bar{\Delta}\Delta + \frac{1}{2\lambda}\bar{B}B\Big) - \frac{1}{2}\mathrm{Tr}\ln \mathbf{G}^{-1} - \frac{1}{2}\bar{\xi}\mathbf{G}\xi$$

$$I' = -\gamma\sum_n \int \frac{d^3p}{(2\pi)^3}\Big\{\ln\big[\beta^2(p_0^2 - E_\Delta^{+2})\big] + \ln\big[\beta^2(p_0^2 - E_\Delta^{-2})\big]\Big\}$$

$$E_\Delta^{\pm 2} = (E_p \pm \mu \mp iA_4)^2 + \Delta^2$$



neglect higher term of diquark fields



6. Effective action of mesons and baryons

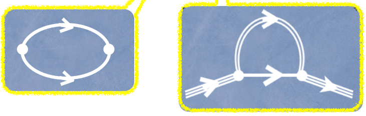
★ perform diquark integral first to get effective action for meson and baryon

- expand effective action up to 2nd order of diquark fields
- Gaussian integral over diquark fields

$$I_{eff}(\phi, \bar{B}, B) = I_0 + \frac{1}{2}\mathrm{Tr}\ln \frac{\delta^2 I}{\delta\bar{\Delta}\delta\Delta}\Big|_{\Delta_0} \qquad \frac{\delta^2 I}{\delta\bar{\Delta}\delta\Delta}\Big|_{\Delta_0} = \frac{1}{2H} - \Pi_{\Delta_0} - \bar{\mathbf{B}}\mathbf{G_0}\mathbf{B}$$

- Δ0, M0, Phi are determined by stationary conditions

$$\frac{\delta I}{\delta\phi}\Big|_{M_0} = 0\ , \quad \frac{\delta I}{\delta\bar{\Delta}}\Big|_{\Delta_0} = \frac{\delta I}{\delta\Delta}\Big|_{\Delta_0} = 0\ , \quad \frac{\delta I}{\delta\bar{\Phi}} = \frac{\delta I}{\delta\Phi} = 0$$



7. Summary and outlook

- We have got EOS at zero μ in 2- and 3-flavor PNJL model with mesonic excitations.
- To take baryons into the model is crucial, in order to study QCD phase diagram.
- We just write an effective action for hadrons.

- to solve gap equations --> obtain M0, Δ0, Phi
- to integrate over meson and baryon fields to get EOS
- to map the phase diagram