Towards the Test of Saturation Physics Beyond Leading Logarithm

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Saturation

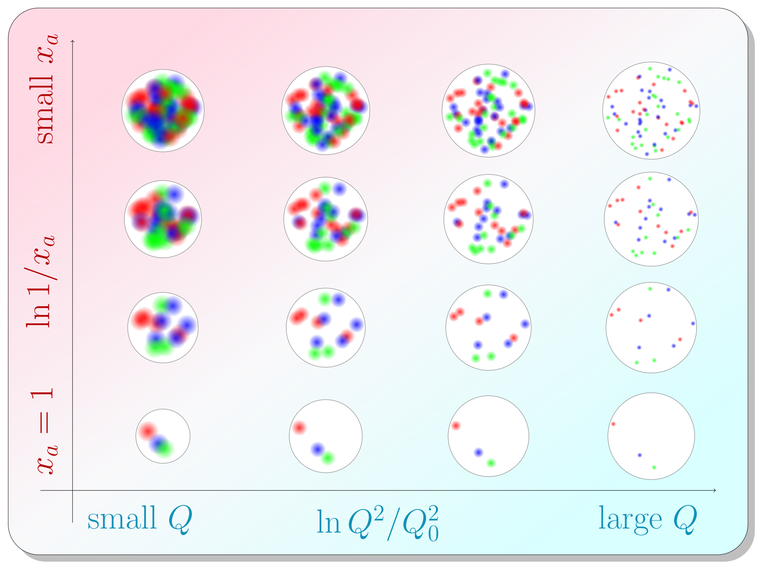
Over half a century, collider experiments have resulted in a set of parton distributions that describe the effective structures of protons and atomic nuclei: both integrated parton distributions $f_i(x, Q^2)$, which represent parton number density per unit momentum, and the unintegrated parton distributions, the Weizsäcker-Williams distribution and the dipole gluon distribution $S_Y^{(2)}(r)$, the latter of which relates to hadron production in pA collisions.

The dipole distribution is expected to obey the BK equation which governs its evolution in $Y = \ln \frac{1}{x_a}$:

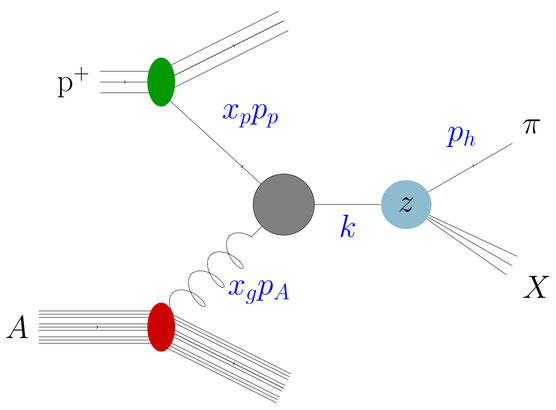
$$\frac{\partial S_Y^{(2)}(r_\perp)}{\partial Y} = -\frac{N_c \alpha_s}{\pi} \int \frac{\mathrm{d}^2\mathbf{b}_\perp}{2\pi} \left[\frac{r_\perp^2}{s_\perp^2 t_\perp^2} + \alpha_s(\ldots) \right] \left[S_Y^{(2)}(r_\perp) - S_Y^{(2)}(s_\perp) S_Y^{(2)}(t_\perp) \right] \tag{1}$$

At small values of x_a and $r_\perp$, this equation predicts $S_Y^{(2)} \to 1$, a phenomenon known as *saturation*. Saturation is theoretically necessary to avoid unbounded exponential growth of the gluon distribution, which would lead to violations of unitarity.

Despite solid theoretical evidence for its existence, saturation and the underlying physical models have not been definitively observed in experiments. Experiments at the LHC and RHIC are just starting to reach the necessary kinematic threshold.



Cross section in the hybrid formalism



The hybrid factorization scheme, also called dilute-dense factorization, expresses a cross section in terms of an integrated parton distribution, representing the structure of the projectile proton, and an unintegrated dipole distribution, representing the structure of the target nucleus.

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}Y\,\mathrm{d}^2\mathbf{p}_\perp} = \sum_{i,j\in q,g} \sum_{k\in\{0,1\}} \int_\tau^1 \frac{\mathrm{d}z}{z^2} \int_{\tau/z}^1 \mathrm{d}\xi x f_i(x, Q^2) D_{h/j}(z, Q^2) \int \mathrm{d}^2\mathbf{x}_\perp \mathrm{d}^2\mathbf{y}_\perp \, \mathcal{H}_{ij}^{(k)}(z, \xi, \mathbf{x}_\perp, \mathbf{y}_\perp) S(z, \mathbf{x}_\perp, \mathbf{y}_\perp) \tag{2}$$

For numerical evaluation we perform partial integration to remove singularities:

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}Y\,\mathrm{d}^2\mathbf{p}_\perp} = \sum_{i,j\in q,g} \sum_{k\in\{0,1\}} \int_\tau^1 \mathrm{d}z \int_{\tau/z}^1 \mathrm{d}\xi \left[\frac{F_s(z,\xi)}{(1-\xi)_+} + F_n(z,\xi) + F_d(z,\xi)\delta(1-\xi) \right] \tag{3}$$

$$= \sum_{i,j\in q,g} \sum_{k\in\{0,1\}} \left\{ \int_\tau^1 \mathrm{d}z \int_{\tau/z}^1 \mathrm{d}\xi \left[\frac{F_s(z,\xi) - F_s(z,1)}{1-\xi} + F_n(z,\xi) \right] + \int_\tau^1 \mathrm{d}z \left[F_s(z,1) \ln\left(1 - \frac{\tau}{z}\right) + F_d(z,1) \right] \right\} \tag{4}$$

When the NLO corrections are incorporated, results turn out negative for large $p_\perp$. (see plots at the bottom) Does this mean the hybrid formalism is invalid?

Chirilli, Xiao, and Yuan 2012; Stasto, Xiao, and Zaslavsky 2014.

Resummation

One option to fix the negativity is resummation of higher-order diagrams. This resummation scheme incorporates diagrams with multiple gluon loops.
The cross section in large $p_\perp$ limit, with $x_0 = \frac{\tau}{z}$, takes the form

$$\frac{\mathrm{d}^3\sigma_{qq}}{\mathrm{d}Y\,\mathrm{d}^2p_\perp} \to \frac{\alpha_s N_c S_\perp}{4\pi^2} \int_\tau^1 \frac{\mathrm{d}z}{z^2} D_q(z) \frac{Q_s^2}{k_\perp^4} \int_{x_0}^1 \mathrm{d}\xi \, x q(x) \mathcal{J}_q(\xi) \tag{5}$$

When $p_\perp$ is large, τ is large and $\xi > \tau \approx 1$. So set $\xi = 1$, or equivalently $x = x_0$, in the parton distribution, and:

$$\int_{x_0}^1 \mathrm{d}\xi \, x q(x) \mathcal{J}_q(\xi) \to x_0 q(x_0) \int_{x_0}^1 \mathrm{d}\xi \, \mathcal{J}_q(\xi) = x_0 q(x_0) \mathcal{I}_q(x_0) \tag{6}$$

Now use exponential resummation on the divergent factor $\mathcal{I}_q(x_0) \sim 4\ln(1 - x_0)$

$$S_\perp \int_\tau^1 \frac{\mathrm{d}z}{z^2} D_q(z) \frac{Q_s^2}{k_\perp^4} \frac{x_0 q(x_0)}{c} \left[\exp\left(c \frac{\alpha_s N_c}{4\pi^2} \mathcal{I}_q(x_0) \right) - c \frac{\alpha_s N_c}{4\pi^2} \mathcal{I}_q(x_0) - 1 \right] \tag{7}$$

Exact kinematics

Another option is exact kinematics: we do not take the limit $\sqrt{s} \to \infty$. This results in the addition of the red term to the gluon momentum fraction x_a

$$x_a = \frac{k_\perp}{\sqrt{s}} e^{-y_h} \left(1 + \frac{(\mathbf{k}_{g\perp} - \mathbf{k}_\perp)^2}{k_\perp^2} \frac{\xi}{1-\xi} \right) \tag{8}$$

When $\mathbf{k}_\perp \neq \mathbf{k}_{g\perp}$, requiring $x_a \leq 1$ puts an upper limit on ξ which is less than 1,

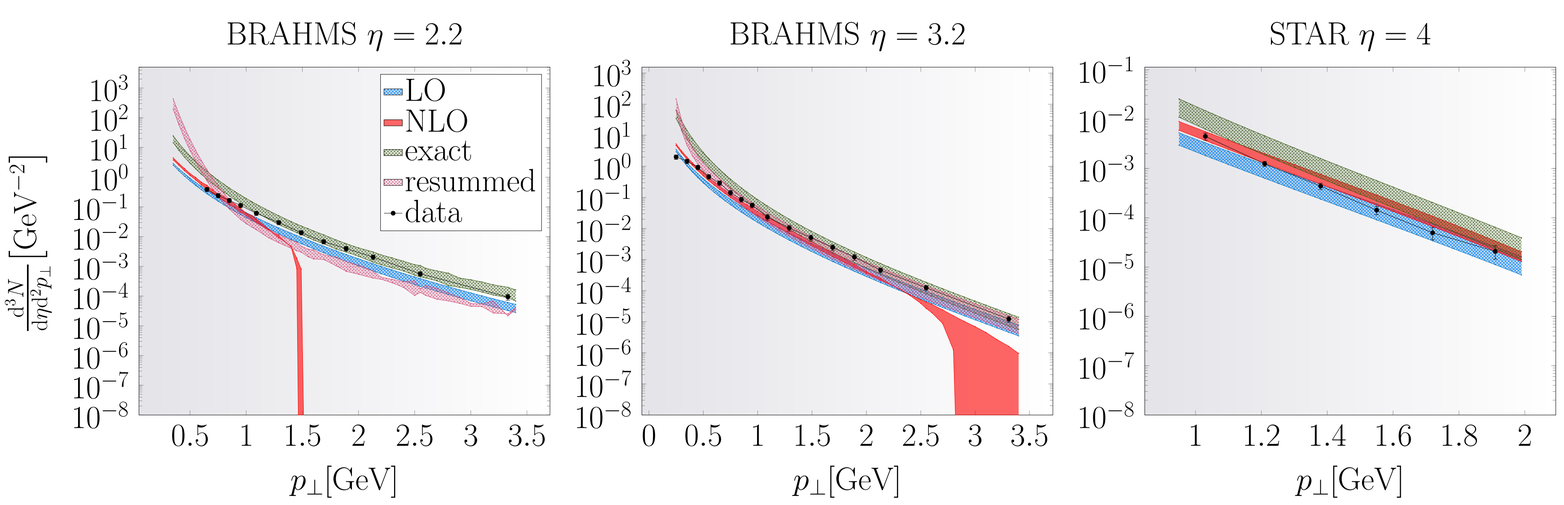
$$\xi \leq \frac{1 - k_\perp e^{-y_h}/\sqrt{s}}{1 - k_\perp e^{-y_h}/\sqrt{s} + (\mathbf{k}_{g\perp} - \mathbf{k}_\perp)^2 e^{-y_h}/(k_\perp \sqrt{s})} \tag{9}$$

which in turn removes the plus prescription subtraction, the source of the negativity.
Taking the high-$p_\perp$ limit results in simplified formulas, e.g. for the quark-quark channel

$$\frac{\alpha_s}{2\pi} \int_\tau^1 \frac{\mathrm{d}z}{z^2} D_{h/q}(z) \int_{\tau/z}^1 \mathrm{d}\xi \frac{1+\xi^2}{1-\xi} \frac{x q(x)}{k_\perp^4} \left(C_F (1-\xi)^2 + N_c \xi \right) \int \mathrm{d}^2\mathbf{k}_{g\perp} k_{g\perp}^2 \mathcal{F}(k_{g\perp}) \tag{10}$$

Requiring $x_a \leq 1$ also places a limit on the $\mathbf{k}_{g\perp}$ integral which would otherwise be divergent.

Results



Predictions at leading order, next-to-leading order, and using the resummation and exact kinematics compared to available data from RHIC. The band is indicative of the theoretical uncertainty due to the factorization scale. In each case the edges of the band are computed using $\mu^2 = 10\,\mathrm{GeV}^2$ and $50\,\mathrm{GeV}^2$.

- The NLO correction improves the fit at low $p_\perp$
- The LO+NLO result becomes negative at high $p_\perp$
- Adding NLO corrections reduces the theoretical uncertainty
- Using resummation produces a decent fit at high $p_\perp$
- Using exact kinematics produces a good fit at high $p_\perp$

Not shown in plots:

- The "crossover point" is approximately $1.2Q_s$
- Resummed result independent of c for large $c \gtrsim 5$

NLO prediction fits data at low $p_\perp \lesssim Q_s$
Resummation and exact kinematics fit at high $p_\perp$
Saturation model appears valid,
but LO+NLO terms are insufficient

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