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QCD Phase Diagram from the Lattice at Strong Coupling

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Strong Coupling QCD - Motivation and Setup

Why Strong Coupling QCD?

- With conventional lattice simulations based on Hybrid Monte Carlo: due to the **sign problem**, the full QCD phase diagram at finite density is out of reach (all methods limited to $\mu/T < 1$).
- Limit where the sign problem can be made mild: strong coupling $g\rightarrow\infty \quad\Rightarrow\quad \beta=\frac{2N_c}{g^2}\rightarrow 0$.
- In this limit, the gauge action $S_G[U]$ is absent, only the fermionic action $S_F[U,\bar\psi,\psi]$ persists.
- With simulations based on SC-LQCD, phase diagram and nuclear phase transition can be studied!

General Strategy for SC-LQCD

- SC-limit allows to integrate out gauge fields completely since integration factorizes!
- After further integrating out Grassmann variables, new degrees of freedom are [1]:
  - **Monomers** n_x correspond to mesons, $M(x)=\bar\psi(x)\psi(x)$,
  - **Dimers** k_b correspond to (non-oriented) meson hoppings $(M(x)M(x+\hat\mu))^{k_b}$,
  - **Baryons** which form oriented loops ℓ with segements $\bar B(x)B(x+\hat\mu)$ and $B(x)=\frac{1}{N_c}\epsilon_{i_1\dots i_{N_c}}\psi_{i_1}(x)\dots\psi_{i_{N_c}}(x)$.
- SC-LQCD exhibits **confinement** and **chiral symmetry breaking**.
- Drawback: lattice remains coarse, as SC-limit is opposite of continuum limit.
- Results depend on choice of fermion discretization: staggered or Wilson?
- Range of validity differs; only in continuum limit $g\rightarrow 0$, results become universal.
- Staggered and Wilson fermions in SC-limit can be studied via **Worm algorithm**.

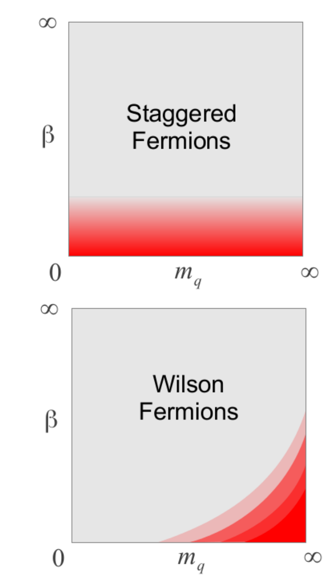

Fig. 1: Range of validity for stagggered fermions and Wilson fermions.

SC-LQCD with Staggered Fermions:

- Advantage: valid for all quark masses, easy to get to **chiral limit** and to study chiral dynamics.
- Disadvantage: fermions have no spin (staggered phases result in Grassmann constraint).
- Strong coupling partition function (exact rewriting of S_F):

$$Z_{\mathrm{SC}}(m_q,\mu)=\sum_{\{k,n,\ell\}}\underbrace{\prod_{b=(x,\mu)}\frac{(N_c-k_b)!}{N_c!k_b!}\gamma^{2k_b\delta_{\hat\mu 0}}}_{\text{meson hoppings } M_xM_y}\underbrace{\prod_x\frac{N_c!}{n_x!}(2am_q)^{n_x}}_{\text{chiral condensate } M_x}\underbrace{\prod_\ell w(\ell,\mu)}_{\text{baryon hoppings } \bar B_x B_y}\qquad\underbrace{\sum_{\hat\mu}k_{\hat\mu}+n_x=N_c}_{\text{Grassmann constraint}}$$

SC-LQCD with Wilson Fermions:

- Advantage: spins also present at strong coupling; gauge corrections simpler to obtain.
- Disadvantage: Partition function can only be written in a hopping parameter expansion in $\kappa(m_q)$ $\Rightarrow$ Restricted to heavy quarks, no chiral dynamics.
- Strong coupling partition function (here the static limit, leading order in κ, with $C\equiv(2\kappa e^{a\mu})^{N_\tau}$):

$$Z_{\mathrm{SC}}(C)=\prod_{\vec x}\left\{\sum_{k=0}^{2N_c}\binom{3+\min(k,2N_c-k)}{3}(\bar C_{\vec x}C_{\vec x})^k+\sum_{k=0}^{N_c}(1+k)(1+N_c-k)(\bar C_{\vec x}C_{\vec x})^k(\bar C_{\vec x}^{N_c}+C_{\vec x}^{N_c})+\bar C_{\vec x}^{2N_c}+C_{\vec x}^{2N_c}\right\}$$

Recent Results for Staggered SC-LQCD

The Phase Diagram in the Strong Coupling Limit & Chiral Limit:

- The **chiral and nuclear transition coincide**, at large densities, baryonic crystall forms (saturation).
- The 1st order line terminates at tricritical point $(aT_T, a\mu_T)$, which becomes the critical point at finite am_q.
- The nuclear interaction is an **entropic force** (order effects in pion gas [3]), no meson exchange between baryons at $\beta=0$.
- Behavior at low $a\mu$ is qualitatively the same, but first order transition strongly N_τ-dependent. [4]
- N_τ-dependence of phase boundary due to anisotropy γ, no re-entrance in continuous time ($N_\tau\rightarrow\infty$).

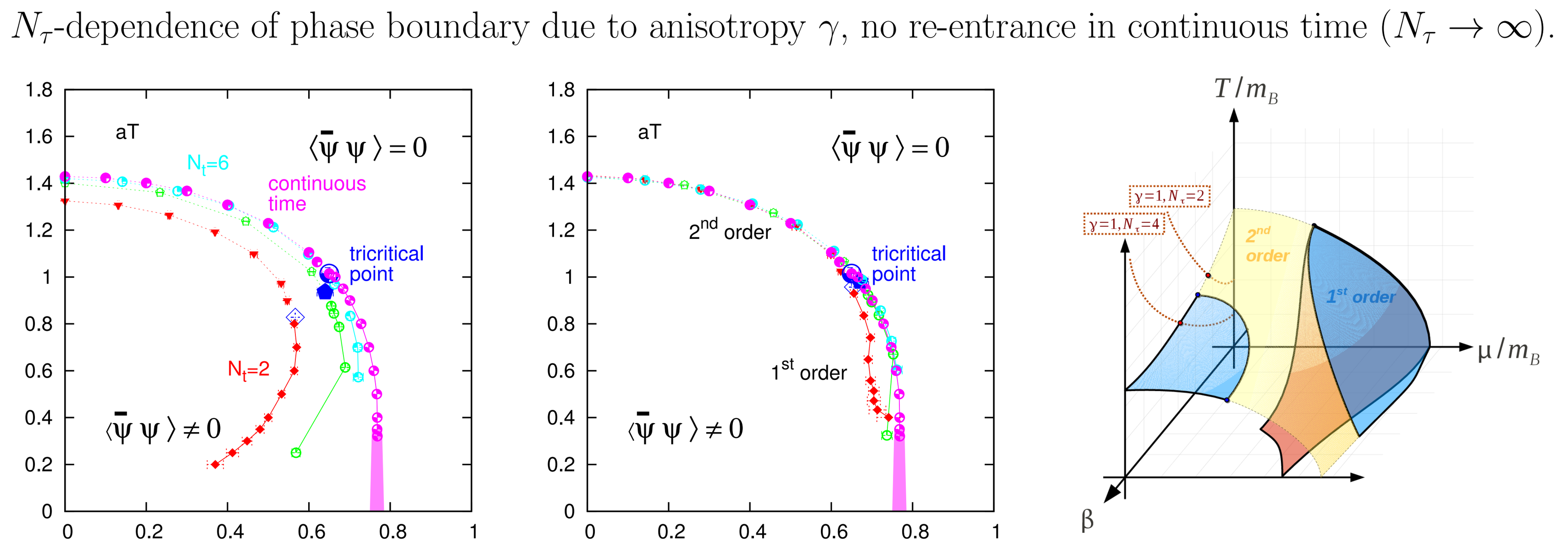

Fig. 2: SC phase diagram measured with Worm algorithm [2,3,4]. *Left:* with identifications: $aT=\frac{\gamma^2}{N_\tau}$, $a\mu=\gamma^2 a_\tau\mu$. *Center:* corrected anisotropy $\frac{a}{a_\tau}=f(\gamma,N_\tau)$. *Right:* One of several possible scenarios on how the SC phase diagram evolves into the $N_f=4$ continuum phase diagram.

Questions we want to address by making β finite (towards continuum limit):

- does the nature of nuclear interactions change qualitatively? (meson exchange now possible)
- do the **nuclear and chiral transition split?**
- does the **tricritical point** move to smaller or larger μ as β is increased?
  $\Rightarrow$ relevant for existence of chiral critical endpoint in continuum limit!

Gauge Corrections to the Strong Couling Limit

- Full partition function including gauge action linearized in β to obtain corrections to SC-limit:

$$Z=\int d\psi d\bar\psi dU e^{S_G+S_F}\approx\int d\psi d\bar\psi Z_F\left(1-\beta\left\langle S_G\right\rangle_U\right),\qquad Z_F=\int dU e^{-S_F}.$$

- Plaquette expectation value at strong coupling [5]:

$$\left\langle \mathrm{tr}[U_P+U_P^\dagger]\right\rangle_{Z_F}=\left(\prod_{l\in P}z_l\right)^{-1}\sum_{s=1}^{19}F_P^s(M,B,\bar B)$$

- $\mathcal{O}(\beta)$ partition function obtained by introducing discrete variables q_p for **excited plaquettes**:
  - modified link weights and site weights,
  - modified Grassman constraint $N_c\rightarrow N_c+q_p(x)$.

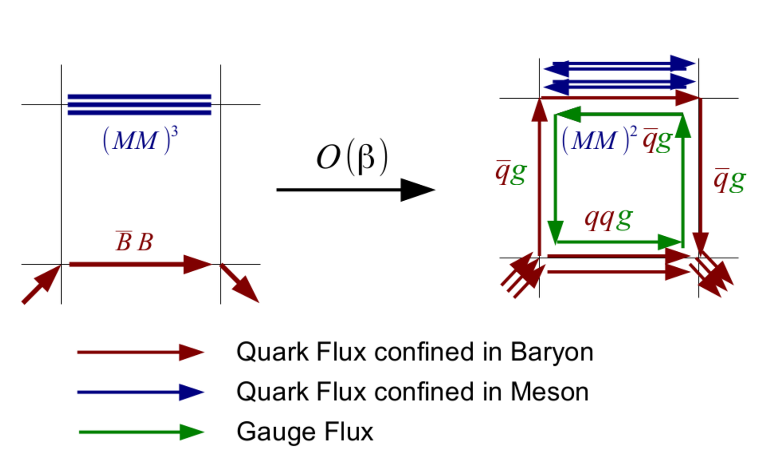

Fig. 3: *Left:* Graphical representation of a one of 19 diagrams at $\mathcal{O}(\beta)$.

Gauge Corrections to the SC-Phase Diagram

Gauge Observables at non-zero Density

- Polyakov loop $\langle L\rangle$ and spatial/temporal plaquettes $\langle P_s\rangle$, $\langle P_t\rangle$ measured via **reweighting**:
- $\langle L\rangle=\frac{\int d\bar\chi d\chi\langle L\rangle_U Z_F}{\int d\bar\chi d\chi Z_F}$ and $\langle P_t\rangle$ are sensitive to the chiral transition.
- Scan at **finite density** in polar coordinates $(aT,a\mu)\mapsto(\rho,\phi)$ across the phase boundary.

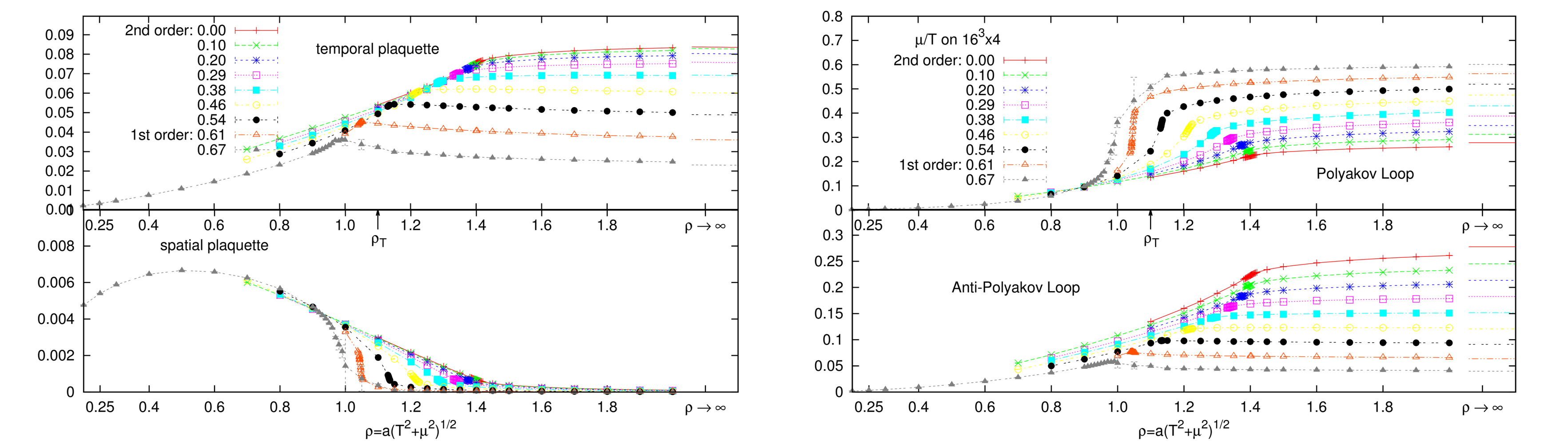

Fig. 4: *Left:* Temperature dependence of $\langle P_s\rangle$, $\langle P_t\rangle$. *Right:* Temperature dependence of Polyakov $\langle L\rangle$ and Antipolyakov loop $\langle L^*\rangle$

Fermionic Observables at non-zero Density

- Chiral susceptibility (and also baryon density) at finite β determined by $\mathcal{O}(\beta)$ **Taylor coefficient**:

$$\chi(\beta)=\chi_0+\beta c_\chi^{(1)}+\mathcal{O}\left(\beta^2\right),\qquad c_\chi^{(1)}=\frac{d}{d\beta}\frac{Z_2(\beta)}{Z(\beta)}\bigg|_{\beta=0}=\langle(\bar\psi\psi)^2P\rangle-\langle(\bar\psi\psi)^2\rangle\langle P\rangle$$

- From 2nd order scaling of $\chi(\beta)$, we obtain for the **slope**: $\frac{d}{d\beta}aT_c(\beta)\simeq-0.446(7)$ at $\mu=0$, which decreases with increasing μ and vanishes at the tricritical point and along the first order line. [5]
- Reweighting of baryon density allows to determine how the tricritical point $(aT_T,a\mu_T)$ moves along the first order line as a function of β.

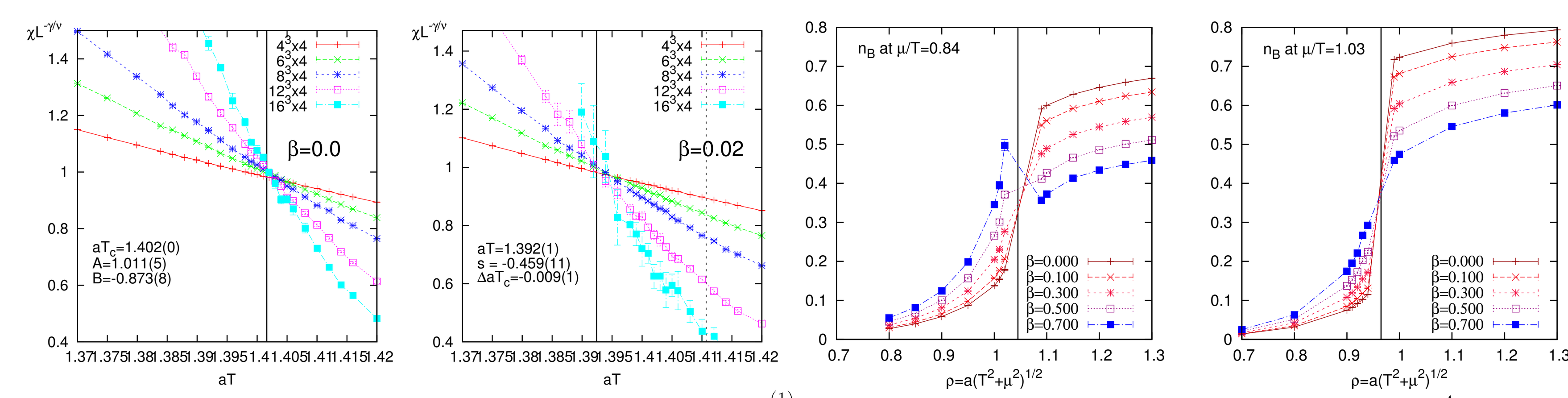

Fig. 5: *Left:* Shift in aT_c obtained from the Taylor coefficient $c_\chi^{(1)}$, related to scaling function parameters via $\Delta aT_c(\beta)\doteq-\beta aT_c\frac{A}{B}c_2$. *Right:* Baryon density n_B for various β and $\mu/T>\mu_T/T_T\approx0.62$. The nuclear transition weakens with increasing β and eventually turns into 2nd order.

- The ratio $\frac{T_c(\mu=0)}{3\mu_c(T=0)}\approx\frac{1.403}{0.57}=0.82$ at strong coupling is too large compared to the continuum estimate in the chiral limit: $\frac{T_c(\mu=0)}{3\mu_c(T=0)}\approx\frac{154\,\mathrm{MeV}}{0.93\,\mathrm{GeV}}=0.165$.
- But: $\frac{T_c(\mu=0)}{\mu_c(T=0)}\searrow\quad(\beta\nearrow)$

  as expected, since lattice spacing $a(\beta)$ decreases.
- Very good agreement with existing HMC data at $\mu=0$, qualitatively similar to mean field results [6].
- Phase boundary at finite density: slope vanishes at first order line, behaviour of tricritical point in agreement with scenario in Fig. 2 (right).

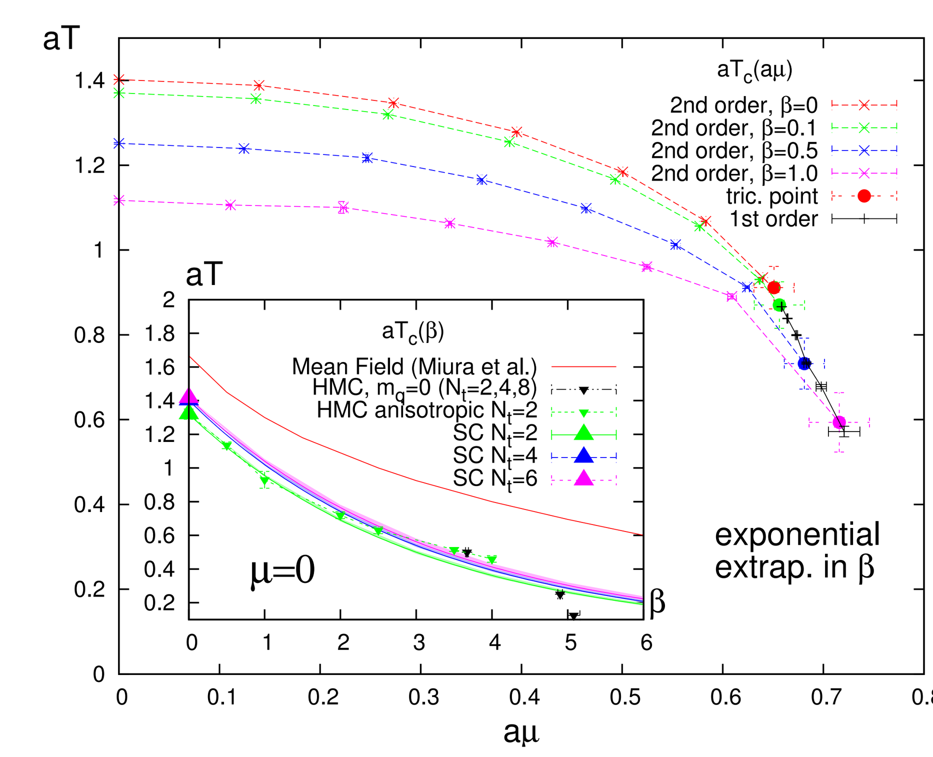

Fig. 6: Phase boundary in the strong coupling limit and exponentially extrapolated to finite β. *Inset:* $aT_c(\beta)$ for $\mu=0$, comparison with HMC and meanfield.

Prospects for SC-LQCD with Wilson Fermions

Wilson fermions were studied at finite density with a **3d-effective theory** based on Polyakov loops [7]:

- No backtracking, since $(1+\gamma_i)(1-\gamma_i)=0\quad\Rightarrow\quad$ mesons and baryons couple to Polyakov loops.
- Suitable to study finite temperature/density (no vacuum diagrams necessary).
- Joint expansion in $u(\beta)=\frac{\beta}{18}+\dots$ and κ, so far up to $\mathcal{O}(u^n,\kappa^m)$ with $n+m=4$ ($N_c=3$).
- Nuclear interaction is not entropic, meson exchange already at $\beta=0$.

Aim: also study Wilson fermions as **4d dimer system** to go to smaller quark masses:

- Mesons (dimers) and baryons (fluxes) carry spin, combinatorics governed by spin conservation.
- Valid configurations obtained by Wick contracting $4N_c$ monomers according to certain **vertex rules**.

Conclusion & Outlook

Achievements so far:

- Staggered fermions extensively studied in chiral limit, now the **phase diagram including gauge corrections** is charted, which seems to be valid up to $\beta\approx5$ ($a\approx0.3$ fm).
- All observables can be measured at finite density, the **slope** $\frac{d}{d\beta}aT_c(\mu)$ determined up to tricritical point.

Further Goals:

- $\mathcal{O}\left(\beta^2\right)$ **corrections** for staggered fermions feasible; extension to $N_f=2$ useful to compare to Wilson.
- 4d simulations with Wilson fermions in dimer representation necessary to go to smaller quark masses.

References

[1] P. Rossi and U. Wolff. *Nucl. Phys. B* **258** (1984) 105.
[2] D. H. Adams and S. Chandrasekharan. *Nucl. Phys. B* **662** (1989) 220-246
[3] P. de Forcrand and M. Fromm. *Phys. Rev. Lett.* **103** (2010) 112005
[4] W. Unger and Ph. de Forcrand. *PoS (Lattice 2011)* **218**, *hep-lat/1312.0589*
[5] Ph. de Forcrand, J. Langelage, O. Philipsen, W. Unger. *PoS (Lattice 2013)* **142**, hep-lat/1111.1434
[6] D. H. Miura, *et al.* *Phys. Rev. D* **80** (2009) 074034
[7] M. Fromm, J. Langelage, S. Lottini, M. Neuman and O. Philipsen. *Phys. Rev. Lett.* **110** (2013) 122001