Strong Coupling QCD - Motivation and Setup

Why Strong Coupling QCD?
- With conventional lattice simulations based on Hybrid Monte Carlo due to the sign problem, the full QCD phase diagram at finite density is out of reach (all methods limited to $T < 1$).
- Limit where the sign problem can be made mild: strong coupling, $g \to \infty \Rightarrow \beta = \frac{1}{g^2} \to 0$.
- In this limit, the gauge action $S[U,\bar{\psi},\psi]$ is absent, only the fermionic action $S[\bar{\psi},\psi]$ persists.
- With simulations based on SC-LQCD, phase diagram and nuclear phase transition can be studied!

General Strategy for SC-LQCD
- SC-Limit allows to integrate out gauge fields completely since integration factorizes!
- After further integrating out Grassmann variables, new degrees of freedom are [1]:
  - Monomers $n_\mu$ correspond to mesons, $M(x) = \bar{\psi}(x)(x)$.
  - Dimers $\Delta$ correspond to (non-oriented) monopole hopping ($M(x)M(x+\vec{p}))$.
  - Baryons which form oriented loops $f$ with segments $B(x)B(x+\vec{p})$ and $\bar{B}(\vec{p}) = \{\bar{\psi}_c(x) - \bar{\psi}_c(x+\vec{p})\} \propto \psi_c(x)$.
- SC-LQCD exhibits confinement and chiral symmetry breaking.
- Drawback: lattice remains coarse; as SC-limit is opposite of continuum limit.
- Results depend on choice of fermion discretization: staggered or Wilson?2
- Range of validity differs; only in continuum limit $g \to \infty$ results become universal.
- Staggered and Wilson fermions in SC-limit can be studied via Worm algorithm.

SC-LQCD with Staggered Fermions:
- Advantage: valid for all quark masses, easy to get to chiral limit and to study chiral dynamics.
- Disadvantage: fermions have no spin (staggered phases result in Grassmann constraint).
- Strong coupling partition function (exact rewriting of $S_p$):
  \[ Z_{S_p}(\gamma,\beta) = \prod_{(x,\mu)} \left[ \tau_{S_p}(x,\mu) \right] = \sum_{\{\bar{\psi}_c(x)\}} \prod_{(x,\mu)} \left[ \tau_{S_p}(x,\mu) \right] \approx \sum_{\{\bar{\psi}_c(x)\}} \prod_{(x,\mu)} \left[ \tau_{S_p}(x,\mu) \right] \]

SC-LQCD with Wilson Fermions:
- Advantage: spins also present at strong coupling, gauge corrections simpler to obtain.
- Disadvantage: Partition function can only be written in a hopping parameter expansion in $a(m_\mu^2)$.
- Restricted to heavy quarks; no chiral dynamics.
- Strong coupling partition function (here the static limit, leading order in $\gamma$ with $C \equiv (2\pi a)^{2N_c}$):
  \[ Z_{S_p}(\gamma,\beta) = \prod_{(x,\mu)} \left[ \left\{ \sum_{N_c} \left( 1 + \frac{1}{2a(m_\mu^2)N_c} \right) \right\} \prod_{(x,\mu)} \left[ \tau_{S_p}(x,\mu) \right] \right] \approx \sum_{\{\bar{\psi}_c(x)\}} \prod_{(x,\mu)} \left[ \tau_{S_p}(x,\mu) \right] \]

Recent Results for Staggered SC-LQCD

The Phase Diagram in the Strong Coupling Limit & Chiral Limit
- The chiral and nuclear transition coincide; at large densities, baryonic crystal forms (saturation).
- The 1st order line terminates at tricritical point $(\alpha_T,\eta_Q)$, which becomes the critical point at finite $\eta_Q$.
- The nuclear interaction is an entropic force (order effects in pion gas [3]), no meson exchange between baryons at $\beta = 0$.
- Behavior at low $\eta_Q$ is qualitatively the same, but first order transition strongly $N_c$-dependent [4].
- $N_c$-dependence of phase boundary due to anisotropy $\gamma$, no re-entrance in continuous time ($N_c \to \infty$).

Gauge Corrections to the SC-Phase Diagram

Gauge Observables at non-zero Density
- Polyakov loop ($L$) and spatial/temporal plaquettes ($P_{\vec{q}}$, $P_{\vec{q}}$) measured via reweighting.
- $\langle L \rangle = \int d\bar{U}dU \langle L \rangle$ and $\langle P \rangle$ are sensitive to the chiral transition.
- Scan at finite density in polar coordinates $(\rho_T,\eta_Q) \to (\rho,\eta)$ across the phase boundary.

Fermionic Observables at non-zero Density
- Chiral susceptibility (and also baryon density) at finite $\beta$ determined by $\langle \bar{\psi} \psi \rangle$ Taylor coefficient.
- From 2nd order scaling of $\langle \bar{\psi} \psi \rangle$, we obtain for the slope $d\langle \bar{\psi} \psi \rangle /d\beta = -\frac{4}{3} \rho_T^2$ at $\rho_T = 0$, which decreases with increasing $\rho_T$ and vanishes at the tricritical point and along the first order line. [3]
- Reweighting of baryon density allows to determine how the tricritical point $(\alpha_T,\eta_Q)$ moves along the first order line as a function of $\beta$.

Prospects for SC-LQCD with Wilson Fermions

Wilson fermions were studied at finite density with a 3d-effective theory based on Polyakov loops [7]:
- No backtracking, since $(1 + \gamma)(1 - \gamma) = 0$ movable and baryons couple to Polyakov loops.
- Suitable to study finite temperature/density (no vacuum diagrams necessary).
- Joint expansion in $\rho_T$ and $\eta_Q$ to $\rho_T^2,\eta_Q^2$; similar to Poincaré coordinates.
- Nuclear interaction is not entropic, meson exchange already at $\beta = 0$.
- Also study Wilson fermions as 4d Wilson system to go to smaller quark masses.
- Mesons (dimes) and baryons (flaxes) carry spin, combinations governed by spin conservation.
- Valid configurations obtained via Wick contracting 4Nc monomers according to certain vertex rules.

Conclusion & Outlook

Advancements so far
- Staggered fermions extensively studied in chiral limit, now the phase diagram including gauge corrections is charted, which seems to be valid up to $3 \approx 5$ ($\mu = 0.3$ fm).
- All observables can be measured at finite density, the slope $d\langle \bar{\psi} \psi \rangle /d\beta$ determined up to tricritical point.
- Further Goals:
  - $O(3\beta)$ corrections for staggered fermions feasible; extension to $N_c = 2$ useful to compare to Wilson.
  - 4d simulations with Wilson fermions in dimer representation necessary to go to smaller quark masses.

References