New Approach to Lattice QCD Thermodynamics from Yang-Mills Gradient Flow

FlowQCD Collaboration

M. Asakawa (Osaka), T. Hatsuda (RIKEN), E. Itou (KEK), M. Kitazawa (Osaka), H. Suzuki (Kyushu)

arXiv:1312.7492 [hep-lat]

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1. Energy-momentum tensor : why should we care ?
2. Yang-Mills gradient flow : 4D world from (4+1)D bulk
3. Yang-Mills thermodynamics : power of plateau
4. Future

Energy-Momentum Tensor : why should we care?

$T_{\mu\nu}$ Generator of Poincare group (Translation + Lorentz)

Conservation law

$$\partial^\mu T_{\mu\nu} = 0$$

YM Trace anomaly

$$T_\mu{}^\mu = \frac{\beta}{2g} G_{\mu\nu} G^{\mu\nu}$$

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Yang-Mills theory at finite T

$$\langle T_{\mu\nu} \rangle \Longleftrightarrow \varepsilon(T), P(T), s(T)$$

Bulk thermodynamics

$$\langle (T_{\mu\nu})^n \rangle \Longleftrightarrow C_V(T) \text{ etc}$$

Fluctuations

$$\langle T_{\mu\nu}(x) T_{\lambda\rho}(0) \rangle \Longleftrightarrow \eta(T), \zeta(T)$$

Transports

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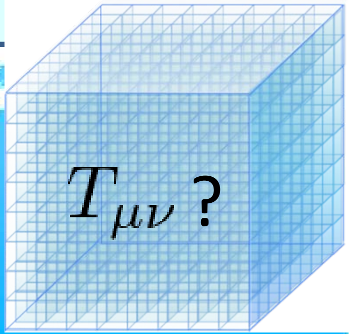
Transports

However,

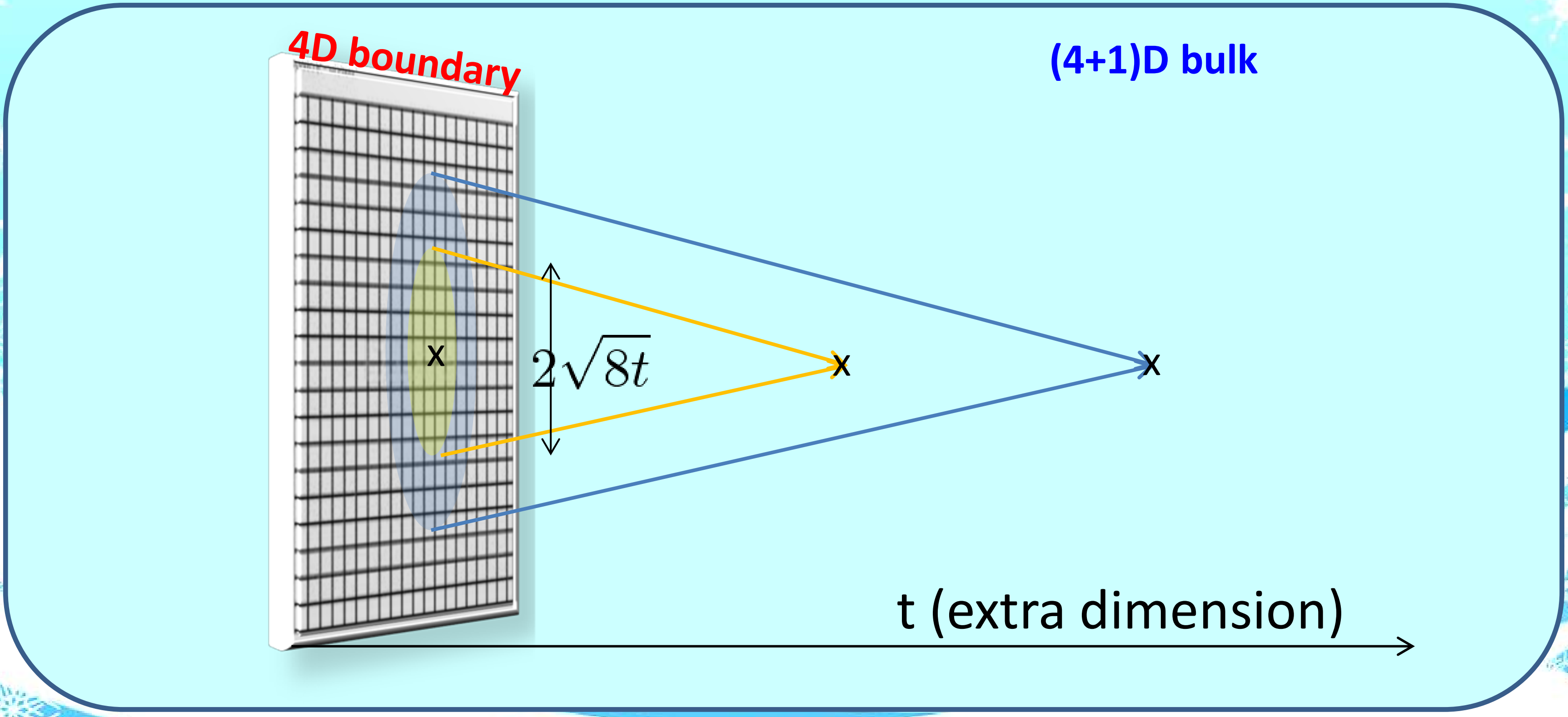
apparent conflict with Lattice discretization



Yang-Mills Gradient Flow (1): 4D world from (4+1)D bulk

Diffusion equation towards extra dimension

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \quad B_\mu(0,x) = A_\mu^{\mathrm{bare}}(x)$$



Luscher & Weisz, arXiv: 1101.0963[hep-lat], JHEP 1102 (2011) 051

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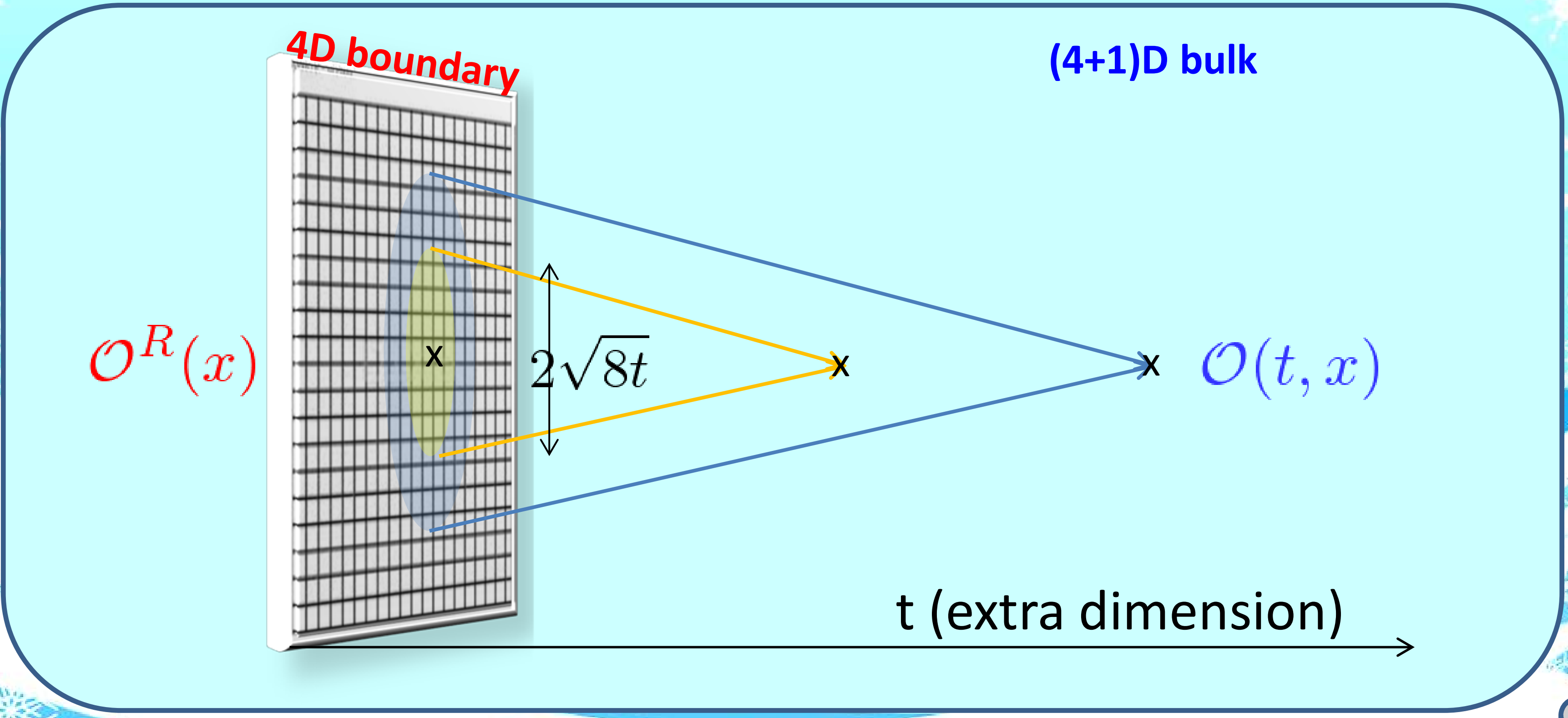
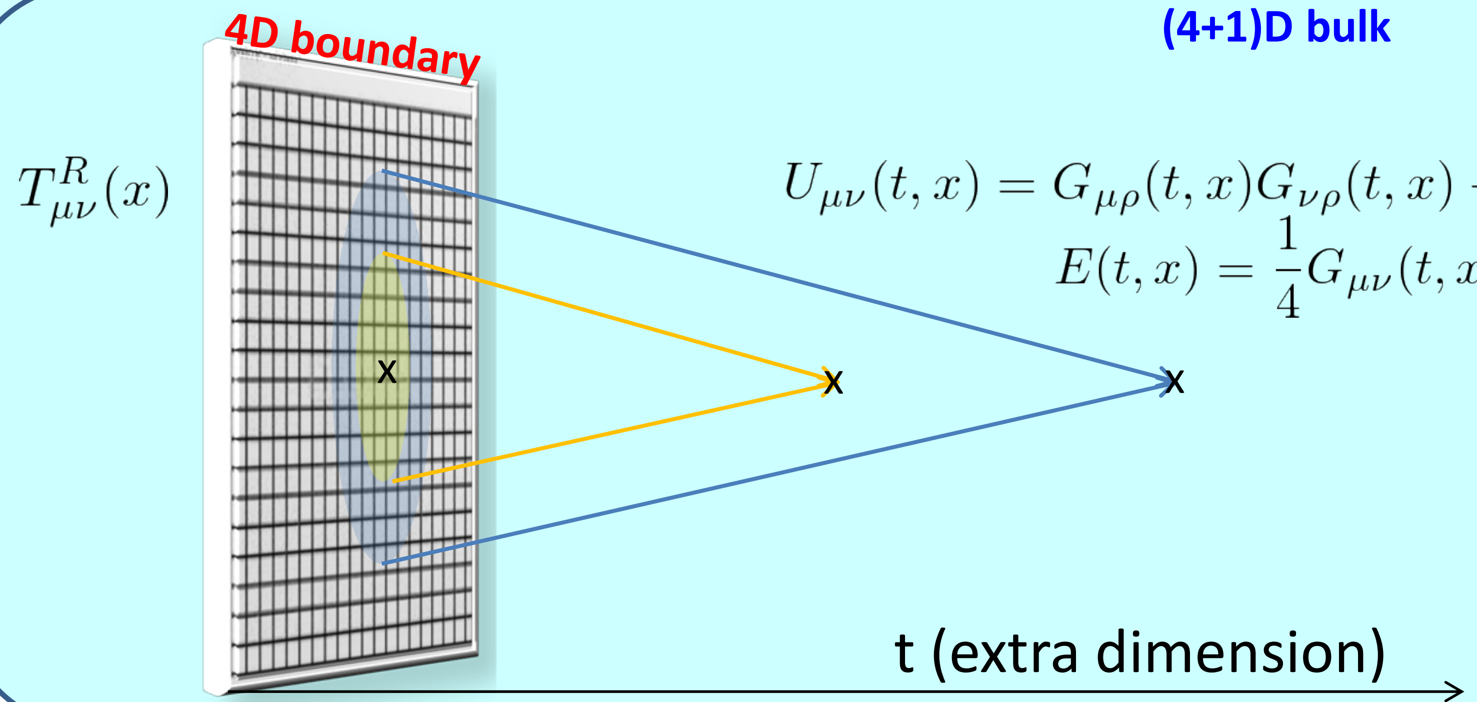


$$\mathcal{O}(t,x) \xrightarrow[t\to 0]{} \sum_k c_k(t)\mathcal{O}_k^R(x) + (\text{powers in } t)$$

Luscher & Weisz, arXiv: 1101.0963[hep-lat], JHEP 1102 (2011) 051

Yang-Mills Gradient Flow (2): 4D world from (4+1)D bulk

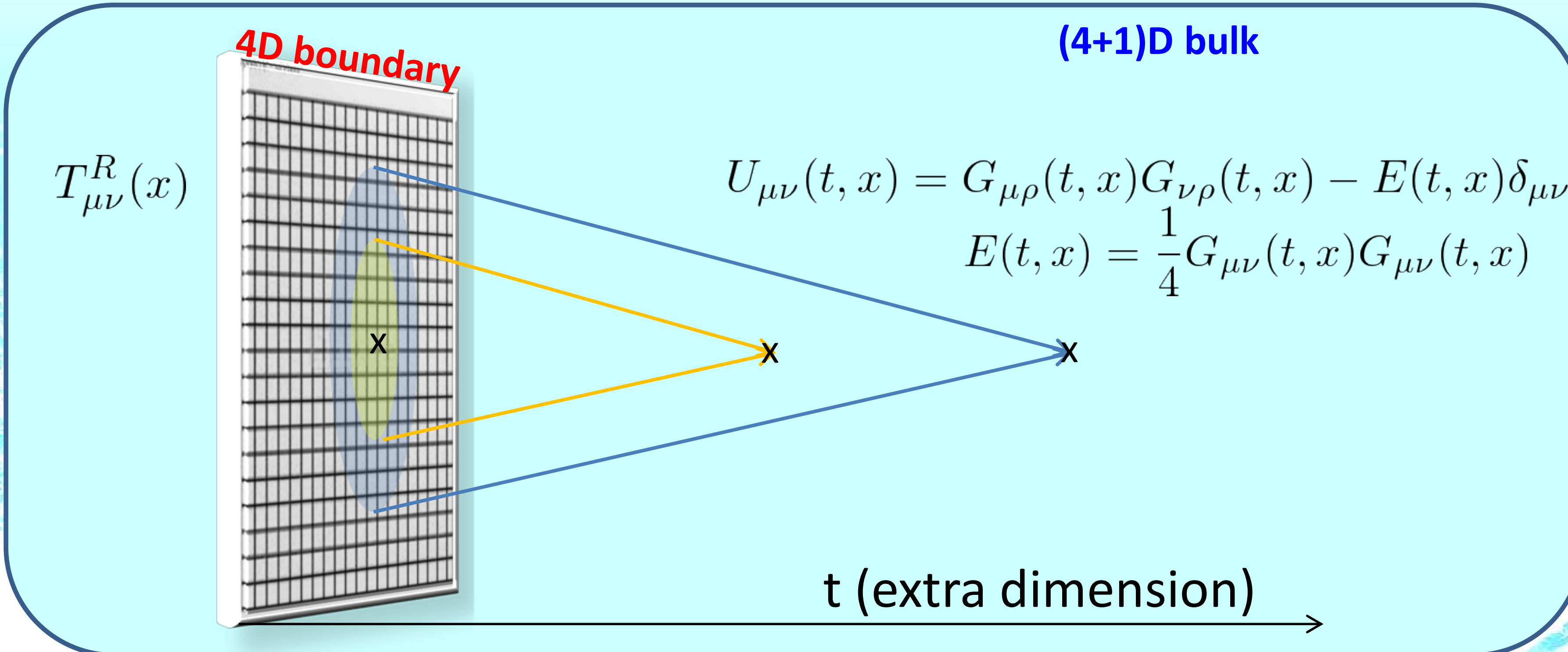


$$U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - E(t,x)\delta_{\mu\nu}$$

$$E(t,x) = \frac{1}{4}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x)$$

H. Suzuki, arXiv: 1304.0533[hep-lat], PTEP 2013 (2013) 083B03

Yang-Mills Gradient Flow (2): 4D world from (4+1)D bulk



$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left\{\alpha_U^{-1}(t)U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4}\alpha_E^{-1}(t)\left[E(t,x) - \langle E(t,x)\rangle_0\right]\right\}$$

$$\alpha_U(t) = \bar{g}(1/\sqrt{8t})^2\left[1 + 2b_0\bar{s}_1\bar{g}(1/\sqrt{8t})^2 + O(\bar{g}^4)\right], \quad \alpha_E(t) = \frac{1}{2b_0}\left[1 + 2b_0\bar{s}_2\bar{g}(1/\sqrt{8t})^2 + O(\bar{g}^4)\right]$$

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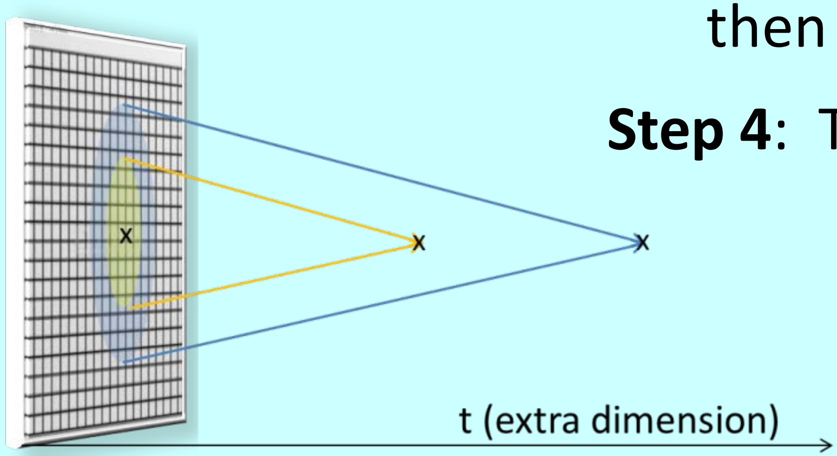
Yang-Mills thermodynamics (1): Four steps to go

Step 1: Generate gauge configuration at t=0 as usual

Step 2: Solve the gradient flow for each configuration

Step 3: Construct $U_{\mu\nu}(t,x)$ & $E(t,x)$, then calculate $\langle U_{\mu\nu}(t,x)\rangle$ & $\langle E(t,x)\rangle$

Step 4: Take the limit ($t\rightarrow 0$ after $a\rightarrow 0$) to obtain $\langle T_{\mu\nu}(x)\rangle$



$$2a \ll \sqrt{8t} \ll R_{\mathrm{UV}}$$

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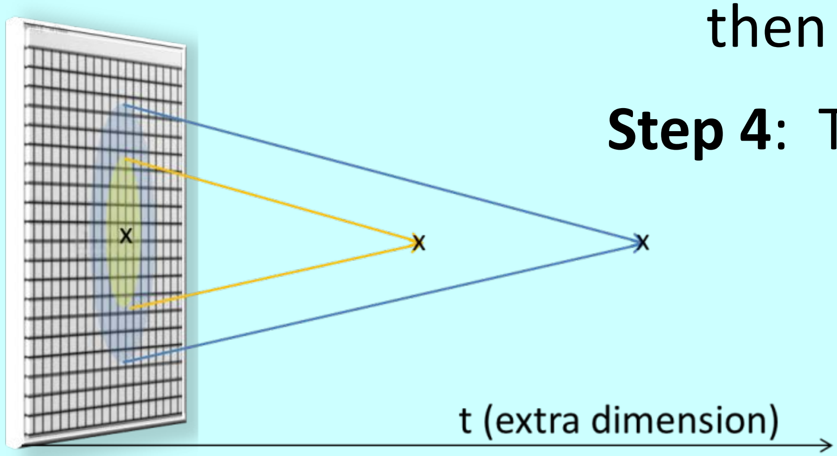
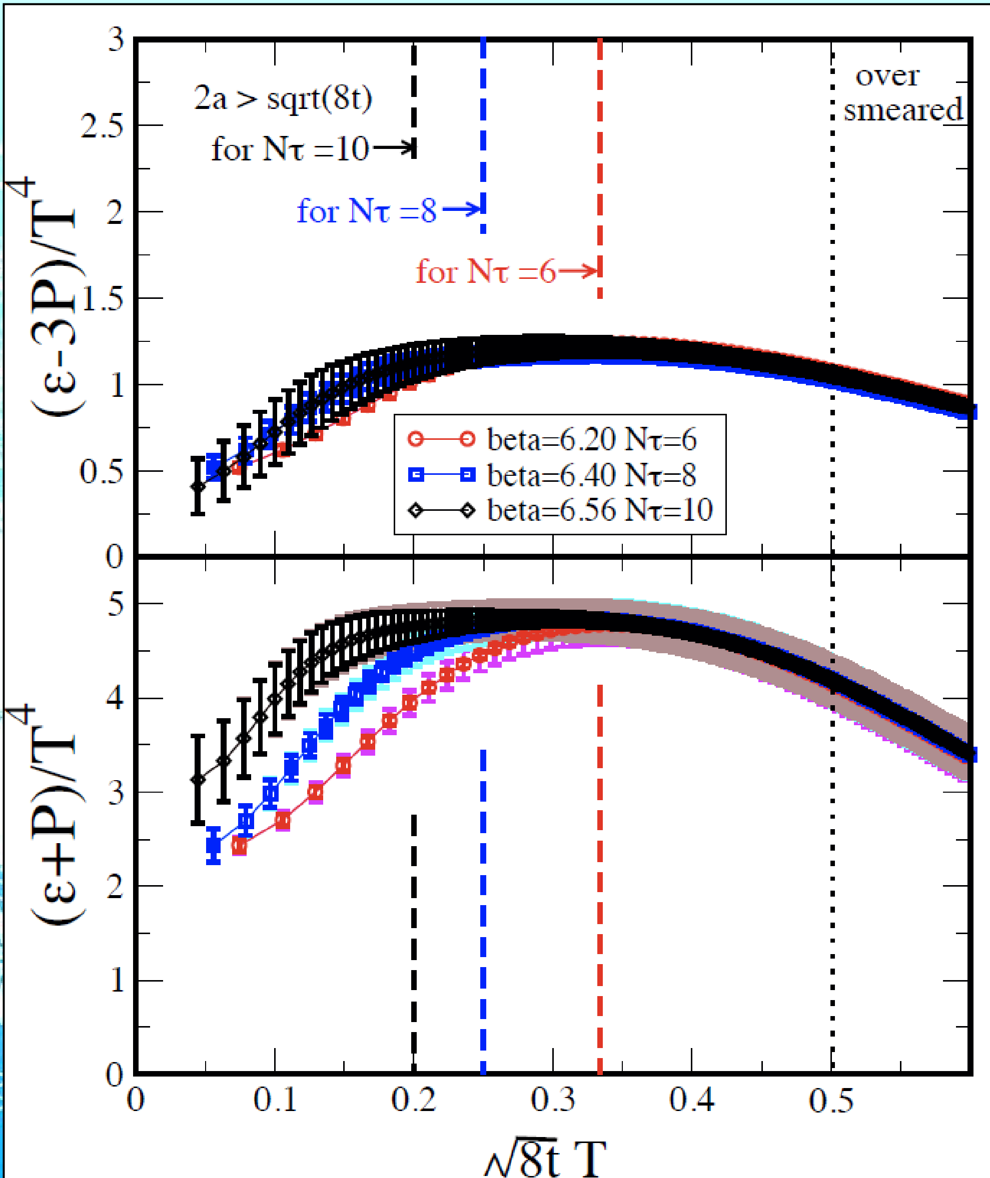


$$2a \ll \sqrt{8t} \ll R_{\rm UV}$$

$$\Delta = \varepsilon - 3P = -\langle T^R_{\mu\mu}(x)\rangle$$

$$sT = \varepsilon + P = -\langle T^R_{00}(x)\rangle + \frac{1}{3}\sum_{i=1,2,3}\langle T^R_{ii}(x)\rangle$$

Yang-Mills thermodynamics (2) : power of plateau



Flow QCD Coll., arXiv: 1312.7492[hep-lat]

- SU(3) YM theory
- Wilson action
- 32^3 x (6,8,10)
- β=5.89-6.56
- 100-300 config.

Emergent plateau !

$$2a \ll \sqrt{8t} \ll R_{\rm UV}$$

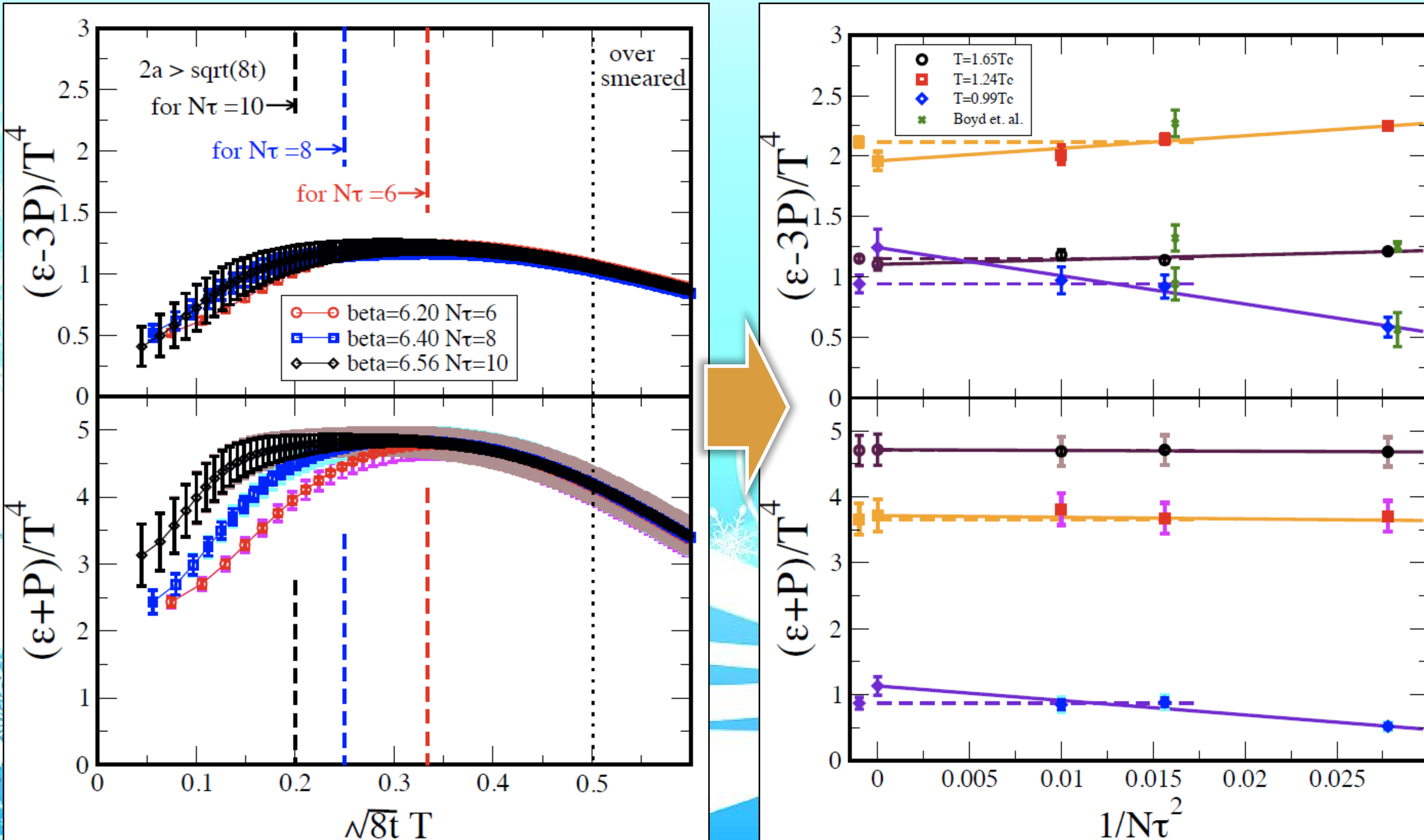
T/T_c=1.65

a = 0.074 fm

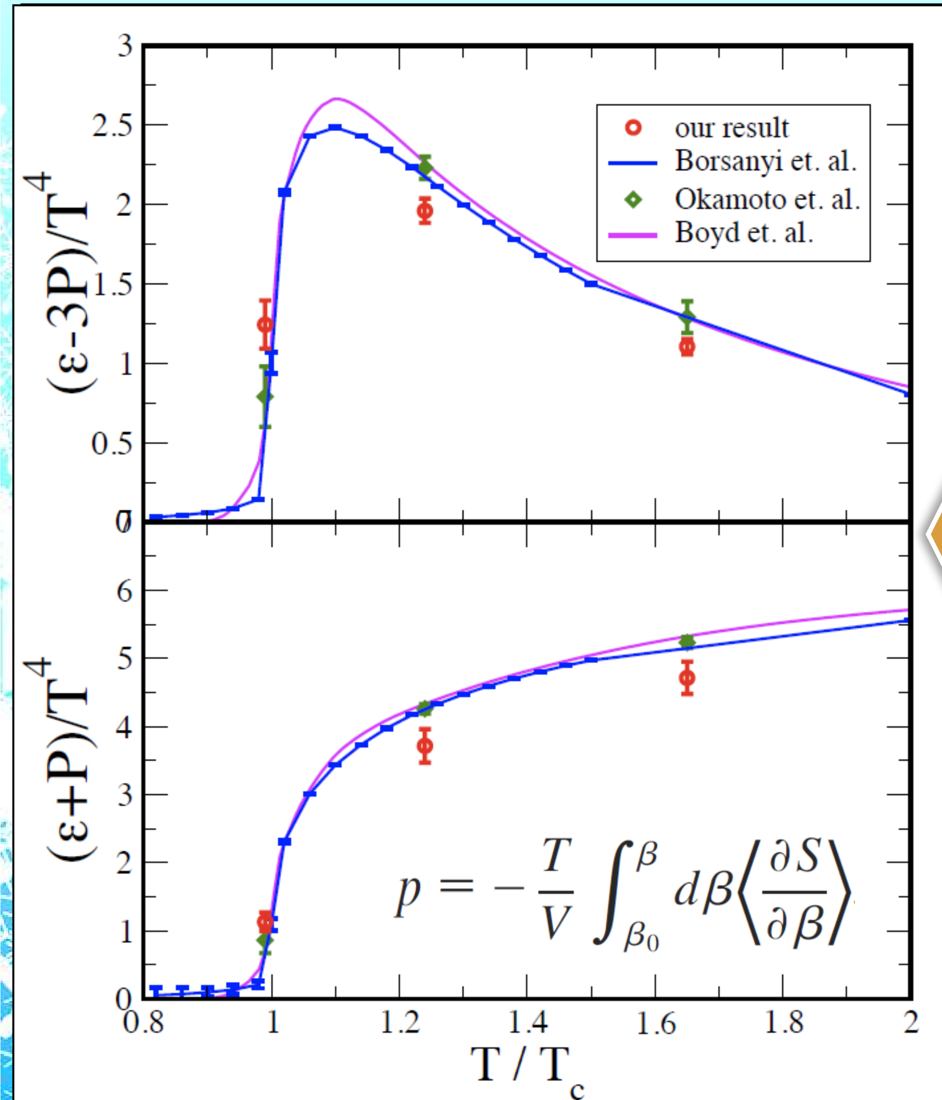
0.055 fm

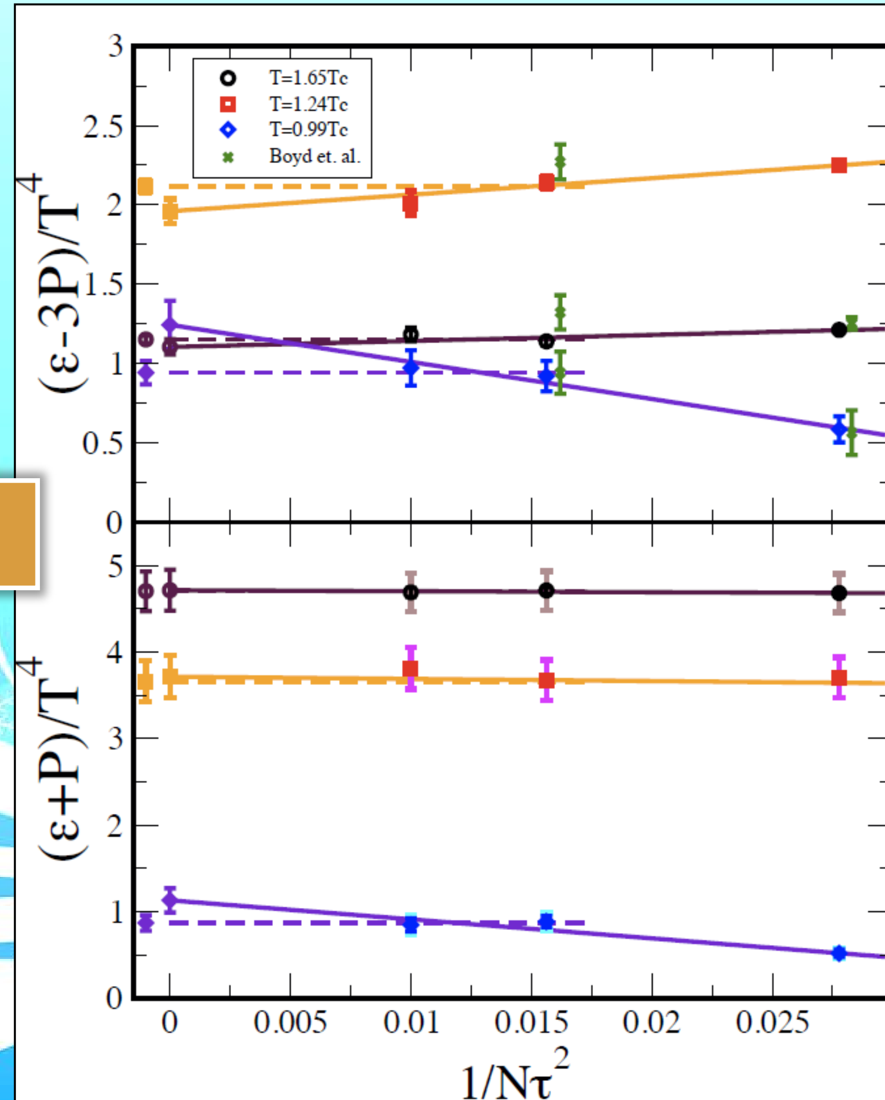
0.043 fm

Yang-Mills thermodynamics (3) : continuum extrapolation



Yang-Mills thermodynamics (3) : continuum extrapolation

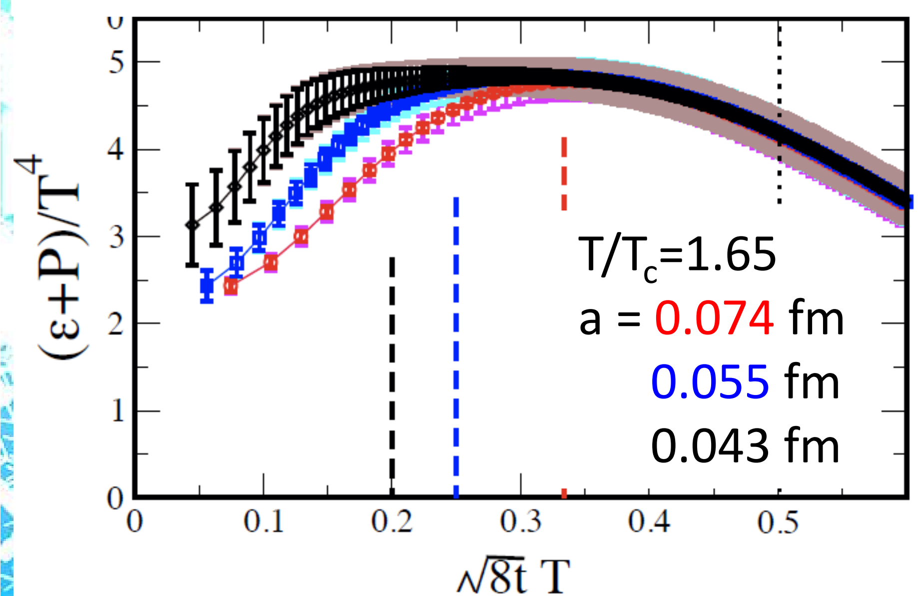


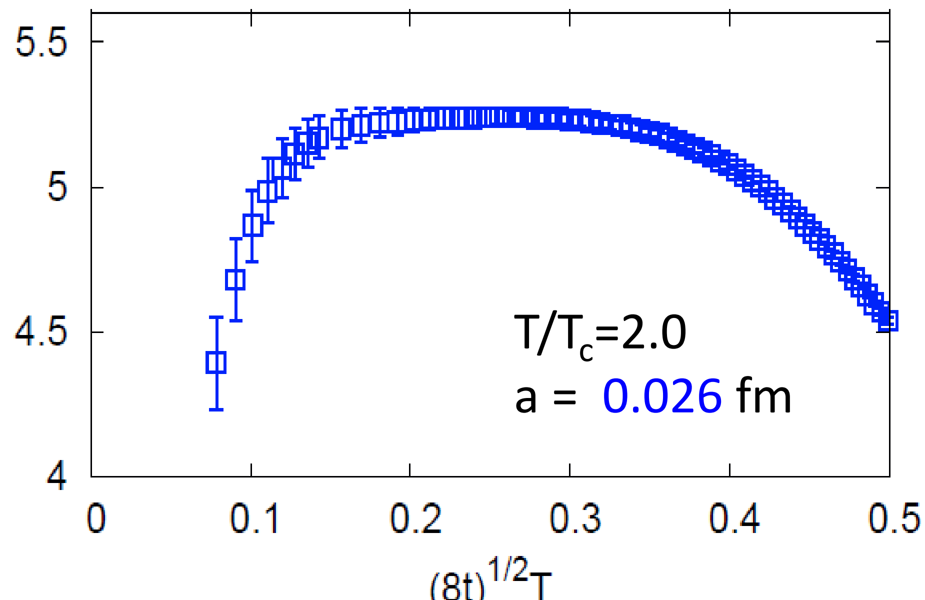


Yang-Mills thermodynamics (4) : larger & finer lattices

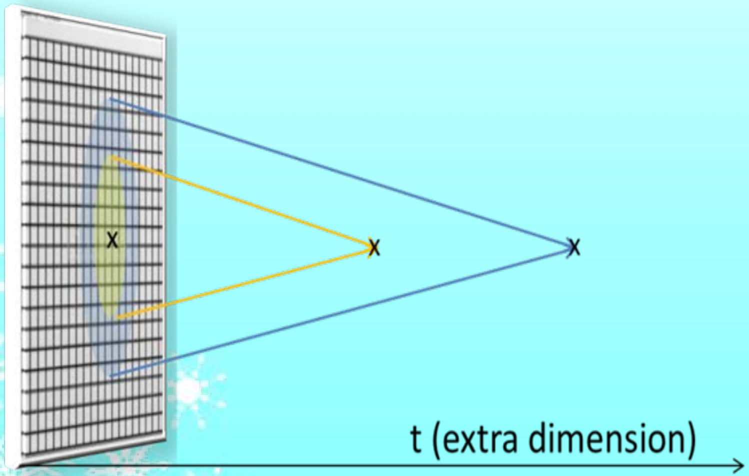
- SU(3) YM theory
- Wilson action
- 32^3 x (6,8,10)
- β=5.89-6.56
- 100-300 config.

- SU(3) YM theory
- Wilson action
- 64^3 x (8,10,12,14,16,18,64)
- β=6.4-7.4
- ～1000 config.





Summary + Future



Summary + Future

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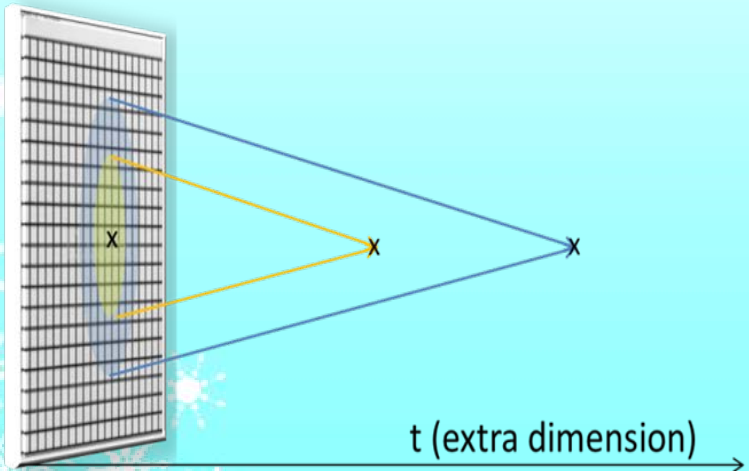

Summary

Energy-momentum tensor from Yang-Mills gradient flow provides a new way to study the hot plasma

- UV free definition
- small statistical error
- nice plateau in t-window
- no need of “integration”

Summary + Future

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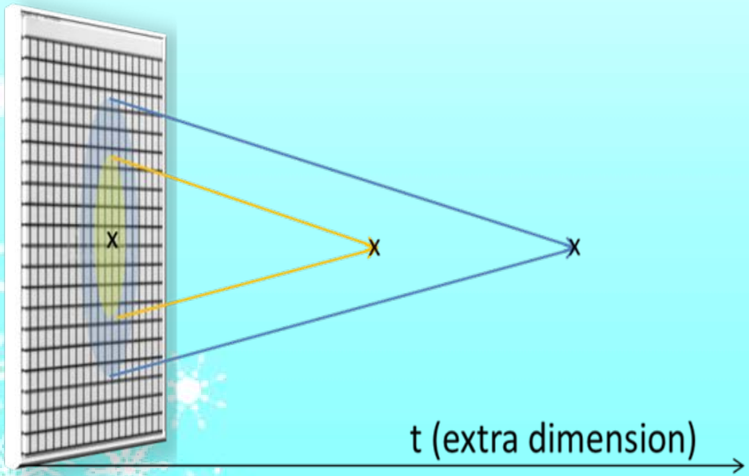

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Near Future

1. ε(T), P(T) and s(T) in YM theory with high statistics (under way)
2. Scale setting by $t^2 \langle E(t,x) \rangle$ (under way)
3. QCD with light quarks   H. Makino & H. Suzuki, arXiv: 1404.2758[hep-lat]
4. Heat capacity from $\langle (T_{00})^2 \rangle$, Viscosity from $\langle T_{\mu\nu}(x)\, T_{\lambda\rho}(0) \rangle$