Frame independent formulation of energy loss in evolving bulk medium †

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Abstract

In an earlier work[1] we found that realistic transverse expansion strongly suppresses elliptic flow with GLV energy loss. On the other hand, Betz and Gyulassy simultaneously reproduce[2] R_{AA} and v_2 with simple power-law dE/dL formulas. To resolve the apparent contradiction we study different bulk medium evolution models and implementations of Djordjevic-Gyulassy-Levai-Vitev (DGLV) radiative energy loss for light partons and heavy quarks. A key new ingredient relative to earlier dE/dL and (D)GLV studies is a frame independent formulation of energy loss[3]. The covariant formulation accentuates the interplay between bulk medium evolution and parton energy loss, which affects light hadron and heavy flavor anisotropic flow.

Covariant DGLV energy loss

In the Djordjevic-Gyulassy-Levai-Vitev (DGLV) formalism[4, 5] of parton energy loss, to first order in opacity χ, the medium-induced radiated gluon spectrum off a fast parton of mass M that scatters at distance L is

$$x\frac{dN^{(1)}(L)}{dx\,d\mathbf{k}} = \frac{C_R\alpha_s}{\pi^2}\chi\int\frac{d\mathbf{q}}{\pi}\frac{\mu^2}{\mathbf{q}^2(\mathbf{q}^2+\mu^2)}\frac{2\mathbf{Q}}{\mathbf{Q}^2+X}\left(\frac{\mathbf{Q}}{\mathbf{Q}^2+X}-\frac{\mathbf{k}}{\mathbf{k}^2+X}\right)(1-\cos\omega L)\,. \qquad (1)$$

Here E is the jet parton energy, μ is the local Debye screening mass, $X = x^2M^2 + \frac{(1-x)\mu^2}{2}$, $\omega \equiv \frac{\mathbf{Q}^2+X}{2Ex}$, and $\mathbf{Q} = \mathbf{k}+\mathbf{q}$. As in [1] we average the energy loss along the jet path

$$\langle \Delta E^{(1)}\rangle = E\int_0^\infty dL\,\rho(\vec{x}_0+\hat{v}L, L/v)\,\sigma_{gg}(L)\int dx\,d^2\mathbf{k}\,x\frac{dN^{(1)}(L)}{dx\,d\mathbf{k}}\,, \qquad (2)$$

using $\sigma_{gg} = 9\pi\alpha_s^2/2\mu^2$ for the scattering cross section in the medium for gluons, and observing finite energy and kinematic bounds ($|k| \lesssim xE$, $|q| \lesssim \sqrt{6ET}$, $xE \gtrsim \mu$). We set $\mu \approx 2T$, and approximate $X \approx x^2M^2$ so that in the massless limit we recover ordinary GLV energy loss (except for one unscreened "dynamical" $1/q^2$ term).

To define a **Lorentz covariant** approach, we calculate energy loss in the LR frame where the fluid is **at rest** at the point of scattering. This implies[3] a replacement $dL\,\rho\,\sigma_{gg} \to dL\,\rho\,\sigma_{gg}(1-\vec{v}\vec{v}_F)$, where $\vec{v}_F$ is the local three-velocity of fluid flow. Kinematic bounds are also evaluated in the LR frame. Covariance thus leads to an interplay between jet motion and medium flow, similarly to the approach in Ref. [6].

Light hadron results

Figure 1 shows the influence of covariant energy loss on **neutral pion** observables in $Au+Au$ at $\sqrt{s_{NN}} = 200$ GeV with impact parameters $b = 7.5$ fm. The calculation is analogous to Ref. [1] (LO pQCD jets with binary collision profile), except for the bulk medium evolution we take 2+1D boost-invariant viscous hydrodynamic solutions[7] publicly available from the TECHQM Collaboration [8] (shear viscosity $\eta/s = 0.08$, realistic lattice QCD equation of state, Glauber initial profile). Three cases are contrasted: i) transversely frozen dynamics with longitudinal expansion only ("1D"); ii) realistic transverse expansion included ("3D"); or iii) transverse expansion and frame independent energy loss ("covariant 3D"). All three give the same R_{AA} once α_s is set to $R_{AA} \sim 0.4$ at high p_T. Transverse expansion suppresses elliptic flow, as was shown in [1]. However, this effect is compensated by jet-medium flow coupling from covariant treatment of energy loss as seen in the right plot.

Figure 2 shows results with "fKLN" initial profile instead of Glauber. Elliptic flow is $\sim$ 15% higher with "fKLN" but otherwise the results are much the same as for the Glauber profile.

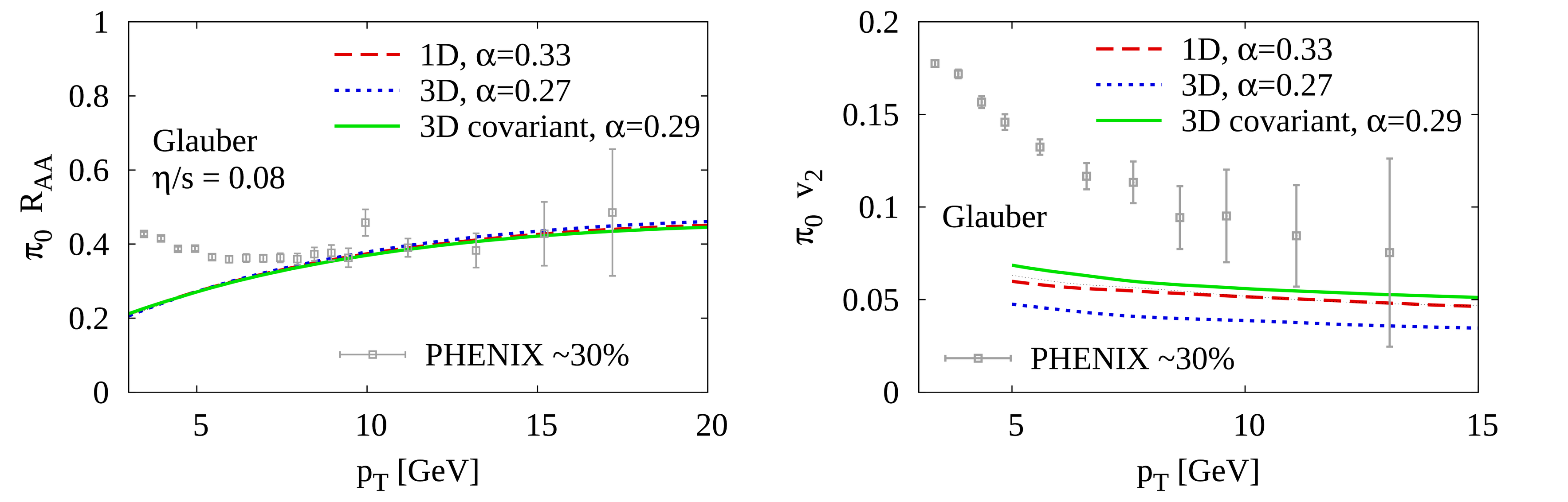

FIGURE 1: Influence of covariant GLV energy loss on neutral pion R_{AA} and v_2.

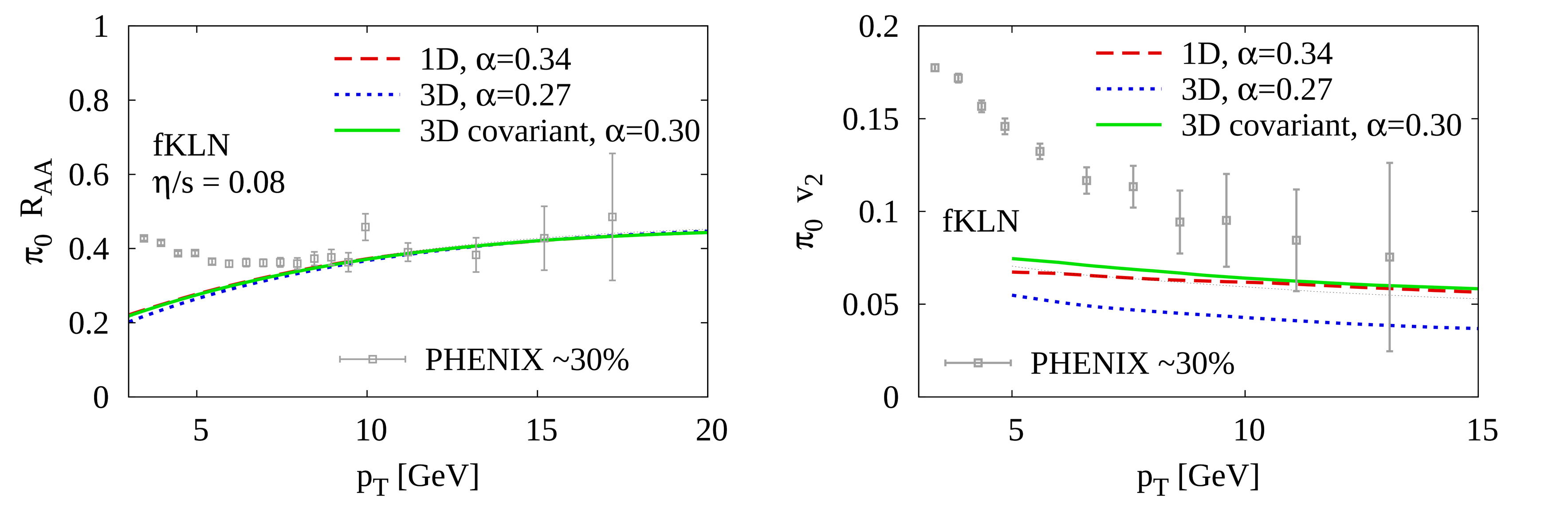

FIGURE 2: Neutral pion R_{AA} and v_2. from the calculation as in Fig. 1, except with fKLN initial profile.

Heavy quark results

With energy loss parameters fixed to pion R_{AA}, we turn to heavy quark suppression and elliptic flow. For **charm quarks** in $Au+Au$ at RHIC ($b \approx 7.5$ fm), Figure 3 shows practically no difference in R_{AA} between the "1D", "3D", and "covariant 3D" calculations. However, very similar cancellation is seen in charm elliptic flow between transverse expansion (suppression) and covariant energy loss treatment (enhancement) to the cancellation for pions earlier. For **bottom quarks** (Fig. 4), elliptic flow behaves qualitatively similarly to charm quark elliptic flow, and there is also a modest variation in low-p_T R_{AA} between the three scenarios.

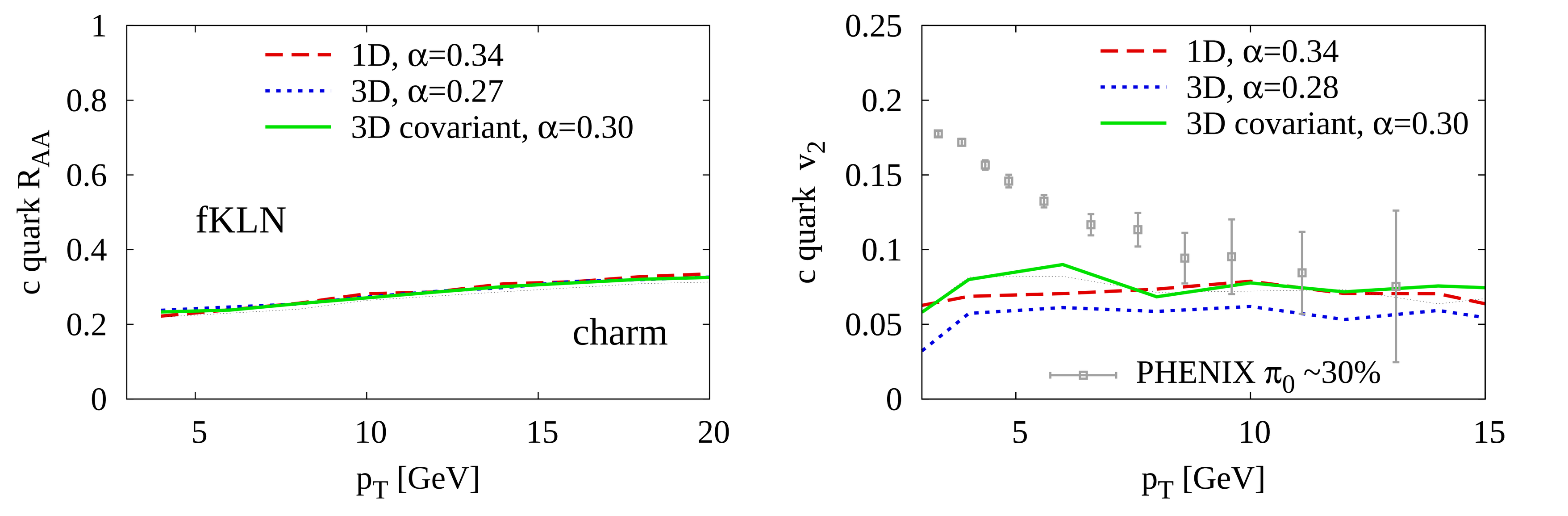

FIGURE 3: Charm quark R_{AA} and v_2 at RHIC, from the calculation in Fig. 2.

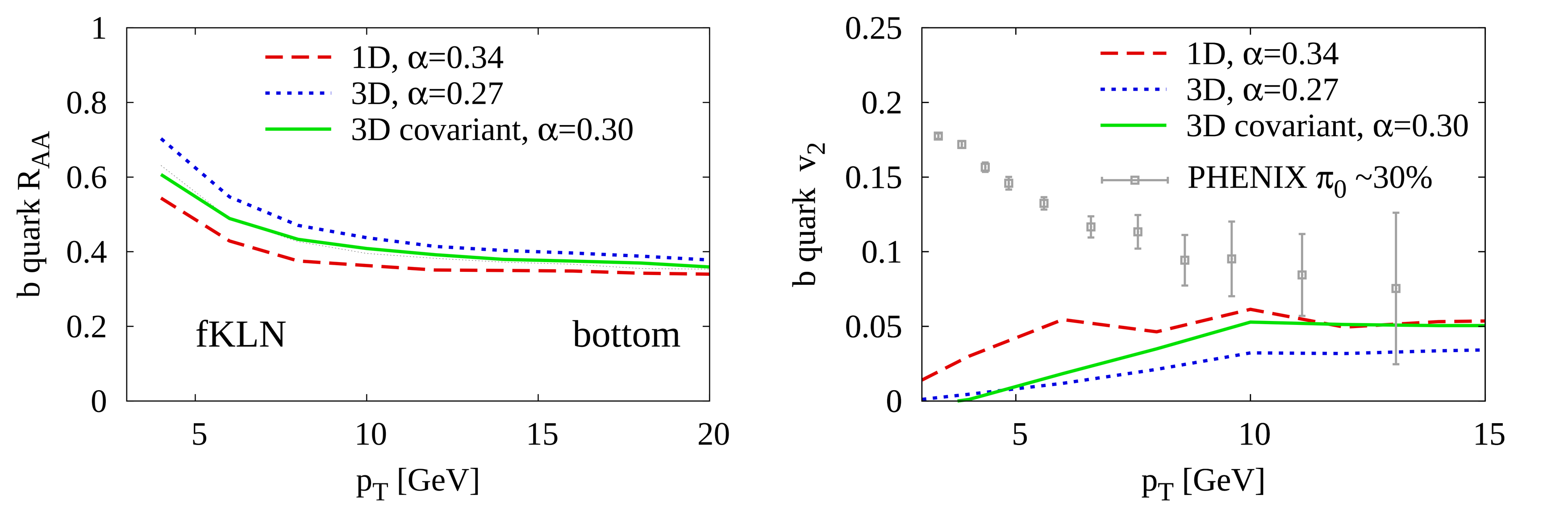

FIGURE 4: Bottom quark R_{AA} and v_2 at RHIC, from the calculation in Fig. 2.

Comparison to $dE/dL = \kappa E^a L^b T^c$

Now compare to recent results[2] by Betz and Gyulassy, obtained with a non-covariant, power-law energy loss parameterization ($a = 1/3$, $b = 1$, $c = 8/3$). As shown in Fig. 5, elliptic flow is suppressed with this dE/dL formula as well for both Glauber and fKLN initial profiles. At high p_T, $v_2 \sim 4-5\%$ in agreement with our findings in [1] for bulk evolution using the parton transport MPC[9]. On the other hand, the hydrodynamic solution used in [2] (dotted, magenta) gives much higher elliptic flow. The key difference [3] is in the equation of state – the solutions we use are for realistic lattice QCD EoS, while the one in [2] is for a "bag-model" parameterization with a first-order phase transition. Covariant formulation of dE/dL would enhance the v_2 results in the right plot just like for GLV energy loss.

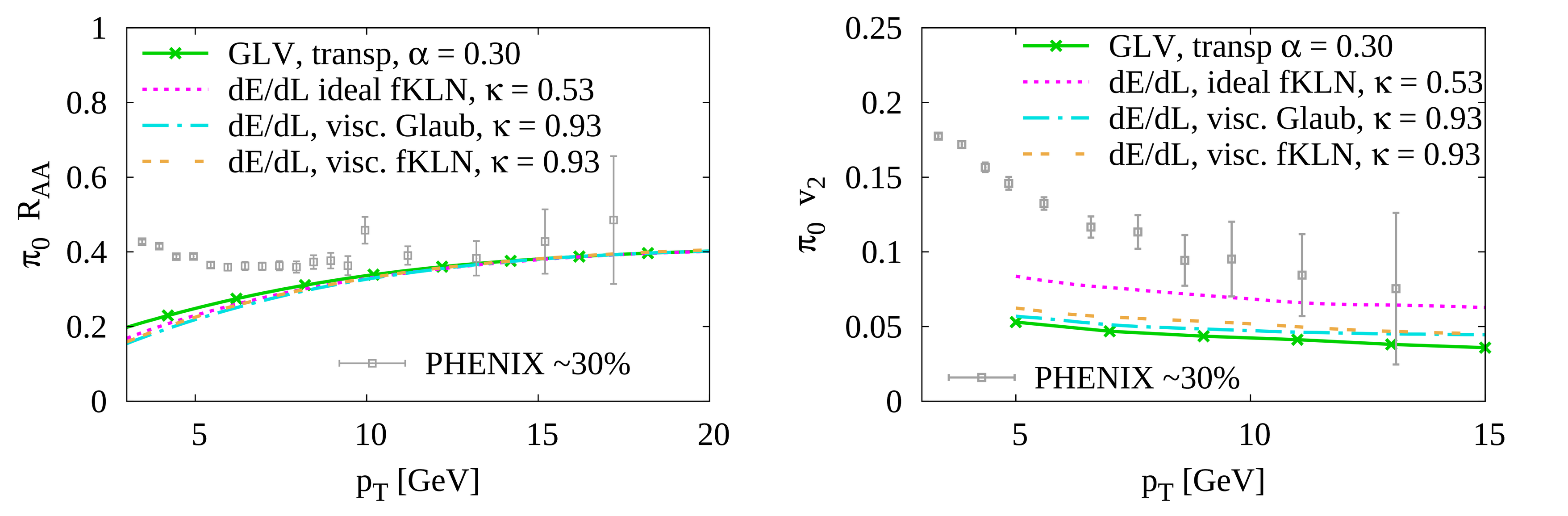

FIGURE 5: Bulk medium evolution dependence of power-law dE/dL energy loss.

Conclusions:

- With (D)GLV energy loss, transverse expansion reduces v_2. This is natural because energy loss in GLV is biased towards later collisions, at which time the medium is more symmetric azimuthally.
- **Covariant** (D)GLV formulation, on the other hand, gives smaller energy loss for jets moving in the same direction as the medium flow. The reduction is larger the higher the flow velocity, which leads to an enhancement of the in-plane vs out-of-plane difference in energy loss ⇒ larger elliptic flow.

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