A scaling relation between pA and AA collisions

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GB, D.Teaney arXiv:1312.6770
Motivation and introduction

The two particle correlations show a striking similarity between high multiplicity pA and AA collisions at fixed multiplicity.

(CMS, arxiv:1305.0609)
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\[ \text{red triangles: } (v_2 \{2\})_{\text{Pb}} \]

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**Question:** Do they originate from the same physics?

**Idea:** They both emerge from a *collective response* to the geometry dictated by \( \frac{l_{mfp}}{L} = f(dN/dy) \).

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“Conformal dynamics” (as an elliptical cow)

- The initial state is described by $N_{\text{clust}}$ independently distributed clusters such that the multiplicity $N \propto N_{\text{clust}}$

- There is a single dimensionful parameter that controls the equilibration dynamics:

$$l_{mp} \propto 1/T_i \quad \text{(e.g. saturation inspired model: } \propto Q_s)$$

- The equilibration dynamics is conformal:

$$\frac{l_{mp}}{L} \propto \frac{1}{T_i L} = f \left( \frac{dN}{dy} \right)$$

- Flow emerges as a collective response to the geometry:

$$v_{2,3} = k_{2,3} \frac{l_{mp}}{L} \propto \sqrt{\frac{dN}{dy}}$$

\text{(e.g. saturation inspired model: } N_{\text{clust}} = \pi Q_s^2 L^2 \Rightarrow \frac{l_{mp}}{L} \propto \frac{1}{Q_s L} \propto \frac{1}{\sqrt{dN/dy}})$$
Independent cluster model and flow in AA

- $N_{clust}$ point like, independent clusters distributed as

$$n(x) = \bar{n}(x) + \delta n(x), \quad \langle \delta n(x) \delta n(y) \rangle = \bar{n}(x) \delta^{(2)}(x - y)$$

- Flow is sourced both by:
  - average geometry $\bar{n}(x)$
  - fluctuations $\delta n(x)$

- Eccentricity in AA:

$$\langle \epsilon_2^2 \rangle_{AA} = \underbrace{\epsilon_s^2}_{\text{average}} + \underbrace{\langle \delta \epsilon_2^2 \rangle}_{\text{fluctuations}}, \quad \langle \delta \epsilon_2^2 \rangle = \frac{\langle r^4 \rangle}{N_{clust} \langle r^2 \rangle^2}$$
Eccentricity in pA and scaling relation

- Eccentricity in AA: \( \langle \epsilon_2^2 \rangle_{AA} = \langle \epsilon_s^2 \rangle + \langle \delta \epsilon_2^2 \rangle \) average fluctuations

- Eccentricity in pA: \( \langle \epsilon_2^2 \rangle_{pA} = \langle \delta \epsilon_2^2 \rangle \) fluctuations

⇒ To compare the \( v_2 \)s, scale out the average geometry from AA:

\[
(v_2 \{2\})_{PbPb,rscl} \equiv \sqrt{1 - \frac{\epsilon_s^2}{\langle \epsilon_2^2 \rangle_{PbPb}}} (v_2 \{2\})_{PbPb}
\]

- If the conformal scaling relation \( (k_{2AA} = k_{2pA}) \) holds expect

\[
(v_2 \{2\})_{PbPb,rscl} \equiv k_2 \sqrt{\langle \delta \epsilon_2^2 \rangle_{PbPb}} \sim k_2 \sqrt{\langle \delta \epsilon_2^2 \rangle_{pPb}} \equiv (v_2 \{2\})_{pPb}
\]
Remarkable agreement between the fluctuation driven part \((v_2^{(2)})_{\text{PbPb}}\) and \((v_2^{(2)})_{\text{pPb}}\)!

The scaling factor \(\sqrt{1 - \epsilon^2} \langle \epsilon^2 \rangle_{\text{PbPb}}\) is a nontrivial function of multiplicity and is calculated by Glauber model (not a fit!).

A scaling relation between pA and AA collisions.
Remarkable agreement between the fluctuation driven part of \((v_2(2))_{\text{PbPb}}\) and \((v_2(2))_{\text{pPb}}\)!

The scaling factor \(\sqrt{1 - \frac{\epsilon_s^2}{\langle \epsilon_2^2 \rangle_{\text{PbPb}}}}\) is a nontrivial function of multiplicity and is calculated by Glauber model (not a fit!).
Triangular flow

- Linear response: $v_3 = k_3 \sqrt{\langle \epsilon_3^2 \rangle}$

\[ \langle \epsilon_3^2 \rangle = \langle \delta\epsilon_3^2 \rangle = \frac{\langle r^6 \rangle}{N_{\text{clust}} \langle r^2 \rangle^3} \Leftrightarrow \text{fluctuations both in pA and AA} \]

- Expect $(v_3 \{2\})_{PbPb} \equiv k_3 \sqrt{\langle \delta\epsilon_3^2 \rangle_{PbPb}} \simeq k_3 \sqrt{\langle \delta\epsilon_3^2 \rangle_{pPb}} \equiv (v_3 \{2\})_{pPb}$
Triangular flow

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\[ \langle \epsilon_3^2 \rangle = \langle \delta \epsilon_3^2 \rangle = \frac{\langle r^6 \rangle}{N_{\text{clus}} \langle r^2 \rangle^3} \Leftrightarrow \text{fluctuations both in pA and AA} \]

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![Graph showing scaling relation between pA and AA collisions]

Blue circles: \((v_3\{2\})_{PbPb}\)

Red triangles: \((v_3\{2\})_{Pb}\)
Transverse momentum dependence of the flow

- **Scaling argument (dictated by “conformal dynamics”):**

\[ v_2(p_T) = \xi_2 \times \epsilon_2 \times f_2 \left( \frac{p_T}{\langle p_T \rangle} \right) \]

- **Response coefficient:** \( \xi_2 \)
- **Geometry:** \( \epsilon_2 \)
- **Universal function at fixed dN/dy**

- **Input:**

\[ \frac{\langle p_T \rangle_{pPb}}{\langle p_T \rangle_{PbPb}} \simeq 1.25 \] (ALICE, arXiv:1307.1094)

- **Expect:**

- \[ \frac{L_{PbPb}}{L_{pPb}} = \frac{T_{i \ pPb}}{T_{i \ PbPb}} \simeq 1.25 \] (pA is smaller and hotter)

- \[ [v_2 \{2\}(p_T)]_{pPb} = [v_2 \{2\} (\frac{p_T}{\kappa})]_{PbPb,rsc} \]
Scaling of $v_2(p_T)$

**PbPb**

<table>
<thead>
<tr>
<th>$120 \leq N_{\text{trk}}^{\text{offline}} &lt; 150$</th>
<th>$150 \leq N_{\text{trk}}^{\text{offline}} &lt; 185$</th>
<th>$185 \leq N_{\text{trk}}^{\text{offline}} &lt; 220$</th>
<th>$220 \leq N_{\text{trk}}^{\text{offline}} &lt; 260$</th>
</tr>
</thead>
</table>

**pPb**

Notice the slopes at small $p_T$!
Scaling of $v_3(p_T)$

Notice the slopes at small $p_T$!
The recent ALICE measurement reveals that $\frac{R_{PbPb}}{R_{pPb}} \approx 1.4$ at the highest multiplicity measured (ALICE, arXiv:1404.1194)

Compare with the conformal scaling result $\frac{L_{AA}}{L_{pA}} \approx 1.25$
Remarks: geometric fluctuations

- **pA and AA have different fluctuations in geometry:**
  \[
  \langle \delta \epsilon_2^2 \rangle = \frac{\langle r^4 \rangle}{N_{\text{clust}} \langle r^2 \rangle^2}, \quad \langle \delta \epsilon_3^2 \rangle = \frac{\langle r^6 \rangle}{N_{\text{clust}} \langle r^2 \rangle^3}
  \]

- Radial profile of pA? Does it matter? **Not so much!**
  \[
  \frac{(v_2)_{AA,rscl.}}{(v_2)_{pA}} = \sqrt{\frac{\langle \delta \epsilon_2^2 \rangle_{AA}}{\langle \delta \epsilon_2^2 \rangle_{pA}}}, \quad \sqrt{\frac{\langle \delta \epsilon_2^2 \rangle_{\text{hard-sphere}}}{\langle \delta \epsilon_2^2 \rangle_{\text{Gaussian}}}} \approx 0.85
  \]

- The difference comes as a square root of a double ratio!
- Gaussian profile does a good job for AA.
- Similar for $v_3$. 

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Conclusions

- The similarities as well as the differences between the high multiplicity pA and AA can be explained in a quantitative fashion by a simple conformal scaling framework.
- At fixed $dN/dy$, $l_{mfp}/L$ is the same for pA and AA (pA is smaller but hotter).
- No need to fine tune initial conditions for the scaling to work.
- It seems phenomenologically reasonable to conclude that $v_2$ and $v_3$ in pA and AA stem from the same collective physics (response to geometry).
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