Quark-Gluon Plasma connected to Finite Heat Bath

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What is the meaning of 'q' in an entropy formula?

Open questions related to this:

- *What is responsible for the power-law tail for high p_T?*
- *Can we assume thermal equilibrium at high p_T?*
- *What is the origin of the 'collectivity' (quarks or hadrons)?*
- *What is the statistical origin of the hadron formation?*

Nature provides several power-law-tailed distributions especially for high-energy physics. In a specific matter, in the quark-gluon plasma phase – created in high-energy nuclear collisions – an exponential (thermal) distribution is also observed at the low-momentum regime. Recently, due to new developments in thermodynamics [1-5] this Janus-face behavior might be understood via generalized entropy formula which connects the two above distribution in one global description.

Mathematical suggestions for entropy formulas, different from the well-known logarithmic formula due to Boltzmann–Gibbs statistics are numerous. Two most cited stems from Alfred Rényi (1961) and Constantino Tsallis (1988). Both contain a parameter denoted by *q*. We elaborated a new chain of thoughts connecting the q-entropy formula with fundamental thermodynamical principles and derived that $q=1-1/C+\Delta T^2/T^2$ is related to the heat capacity of the heat reservoir. This correspondence is exact for ideal systems, and can be viewed as an approximation for the general case. The Universal Thermostat Independence (UTI) principle presented and used in Refs. [1,2,5] guides in the construction of the optimal finite-size corrected entropy formula characterized by the heat capacity, C of the reservoir and fluctuation of the temperature, *ΔT*.

Quark-scaling in hadron spectra

Hadron spectra measured in high-energy nuclear collisions follow Boltzmann-Gibbs statistics at low transverse momenta, p_T. At high transverse momenta the statistical distribution approaches a power law. Applying a simple blast-wave picture in AuAu collisions, the identified hadron spectra present two bunches for mesons and baryon states as seen on Fig 1.

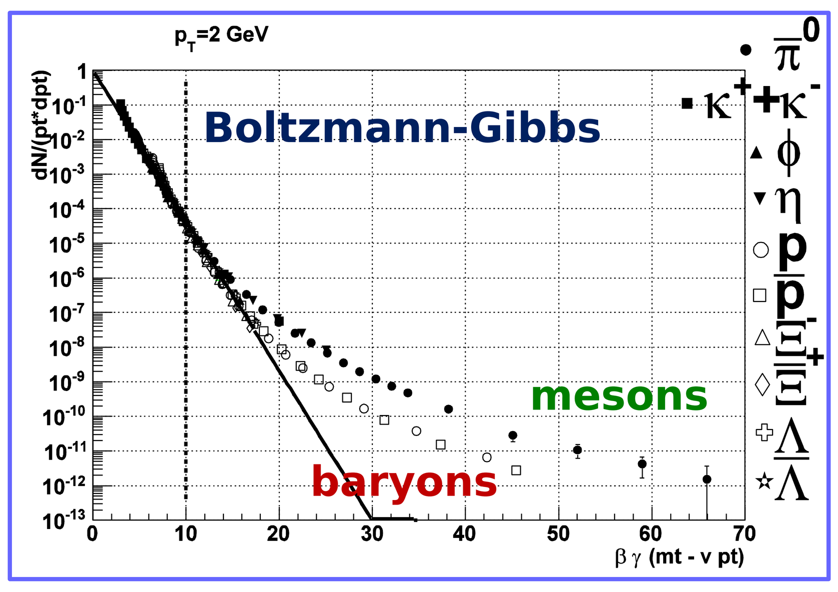

Fig 1: Measured identified hadron spectra as a function of $\beta\gamma\,(m_T - v\,p_T)$

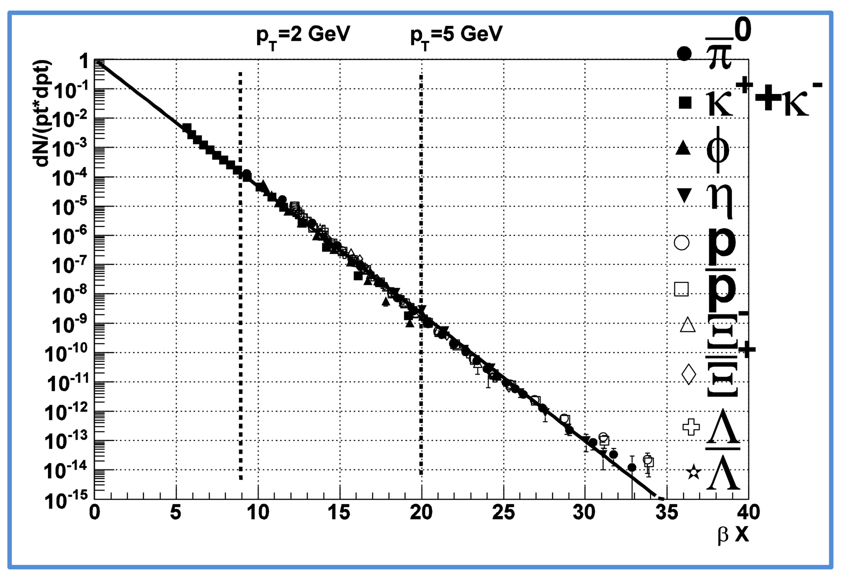

Fig 2: The scaling variable βX ~ nC ln(1+β E/nC) unites the meson and baryon branches with corresponding n=2 and 3 quark compositions.

The steepness for mesons and baryons appear in proportion 2 : 3, suggesting a quark coalescence-like hadronization picture. Fig. 2 shows the result of quark scaling of the hadron spectra. Setting *K=2* for mesons and *K=3* for baryons the $T_{slope}(E_i)$ of the hadron spectra is

$$T^{\rm hadron}_{\rm slope}(E) = T^{\rm quark}_{\rm slope}(E/K)$$

Application: a simple thermal model

The slope parameter, *T(E)* can be fitted to the experimental data. It contains the information on the thermodynamics, thus fit of $T_{slope}(E_i)$ is:

$$T_{\rm slope}(E_i) = \left(-\frac{\mathrm{d}}{\mathrm{d}\,E_i}\ln P_i\right)^{-1} = T_0 + E_i/C,$$

$$T_0 = Te^{-S/C}Z^{1/C}(1-1/C)$$

Here T_0 is the Tsallis slope parameter and *C* represents the heat capacity of the finite reservoir. Thus, connection between the temperature of the subsystem, *T* (set of the measured hadrons), and the Tsallis parameter T_1 is given by

$$T_1 = 1/\beta_1 = Te^{-S/C}$$

Applying this for a bag model with the Stefan–Boltzmann formula, we get:

$$C = \frac{\mathrm{d}\,E}{\mathrm{d}\,T} = 4\sigma VT^3 + (\sigma T^4 + B)\,\frac{\mathrm{d}\,V}{\mathrm{d}\,T}$$

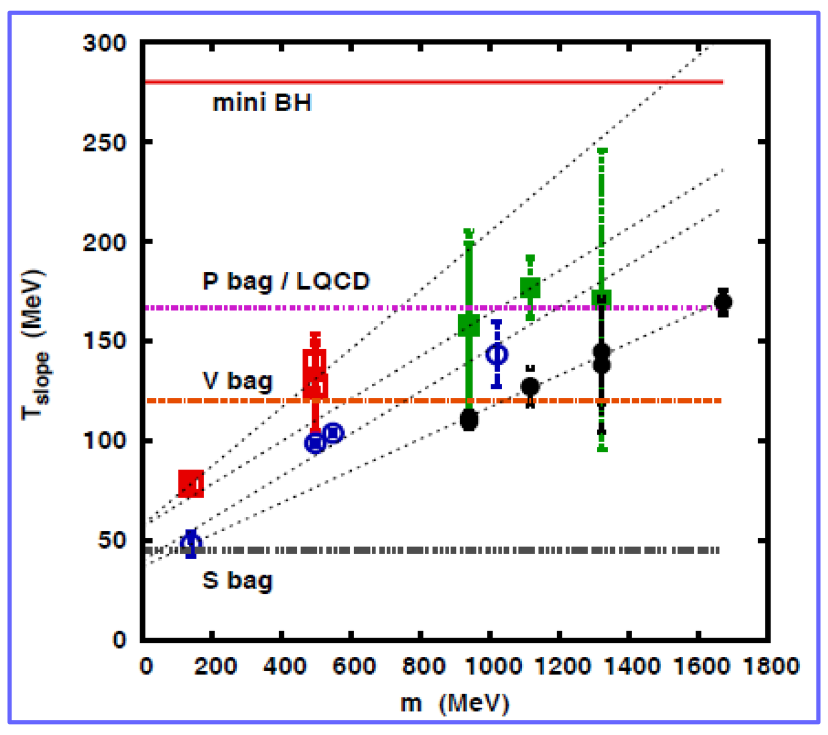

Fig 3: Extrapolated inverse slopes from RHIC AuAu data at 200 GeV (circles) and LHC pp data at 900 GeV.

Fig. 3 presents model result parameters of the Tsallis distribution of valence quarks .

Reservoir's C	Subsystem's T
$C_V = 4\sigma VT^3 = 3S$	$T_{1V} = Te^{-1/3} \approx 120$ MeV
$C_p = \infty$	$T_{1P} = T = 167$ MeV
$C_S = 3S(1-T_*^4/T^4)/4$	$T_{1S} \leq Te^{-4/3} \approx 45$ MeV
$C = -2S$	$T_1 = T\mathrm{e}^{1/2}$

Results: Finiteness vs. fluctuations

Tsallis parameter *q* is derived. There are cometing contributions from the finite heat capacity and from temperature fluctuation to the Tsallis parameter *q* [1,5,6]. We have found two parameters/effects are playing against to each other in order to determine the value of the *q* parameter:

$$q = 1 + \frac{\Delta T^2}{T^2} - \frac{1}{C}$$

Note, that the Boltzmann-Gibbs exponential is recovered for *q=1*. This is the case among others for Gaussian fluctuations:

$$\Delta T/T = 1/\sqrt{|C|}$$

For a general fluctuation pattern of particle number, *n* the Tsallis–Pareto form is only an approximation, but it goes beyond the traditional exponential, Boltzmann–Gibbs. In the general case, these parameters are given like:

$$T = \frac{E}{\langle n\rangle} \quad \text{and} \quad q = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2}$$

We have derived the exact Tsallis–Pareto distribution for fixed total energy, *E* and fluctuating particle number, *n*. For an negative binomial distribution, (NBD) (bosonic reservoir) *q=1+1/(1+k) >1*, for a binomial (BD) one (fermionic reservoir), *q=1-1/k < 1*.

The use of Tsallis–Pareto distribution is supported by experimental findings (Fig 4):

- NBD was observed for *n*-fluctuations,
- better spectra fit by 'soft+hard' Tsallis,
- radial flow is included in the framework

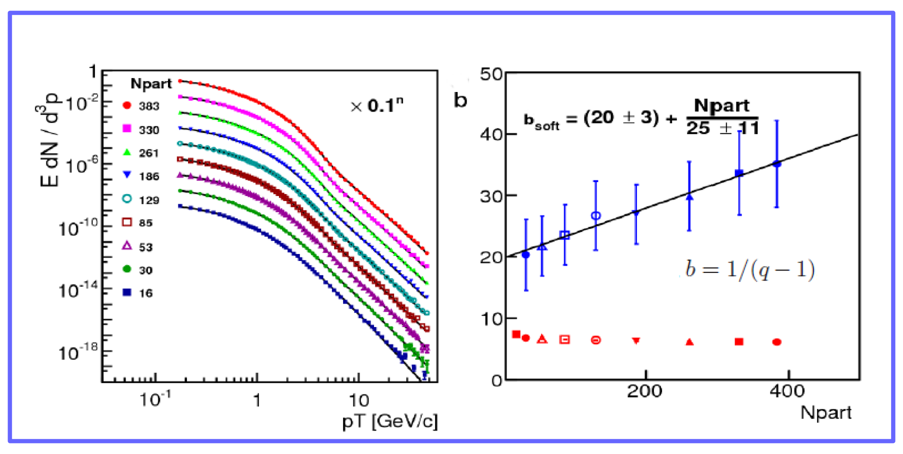

Fig 4: Two-component, "soft+hard" Tsallis fit for b=1/(1-q) parameter on hadron spectra measured by ALICE [5] at 2.76 TeV LHC energy.

References

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