Color path-integral Monte-Carlo simulations of strongly coupled of quark-gluon plasma

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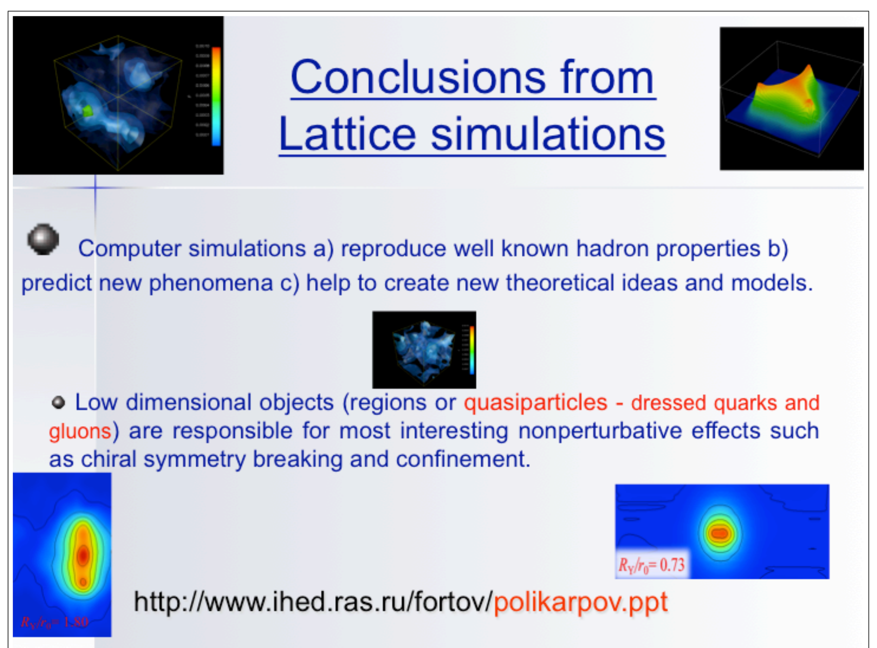
Abstract

The most fundamental way to compute properties of the strongly coupled quark-gluon plasma (QGP) is provided by the lattice QCD. Interpretation of these complicated computations requires application of various QCD motivated models simulating various aspects of the full theory. More over these models are needed in cases when the lattice QCD fails, e.g. at large baryon chemical potentials and out of equilibrium. A quasi-particle model has been recently introduced in literature. It is expected that it allows to treat soft processes in the QGP which are not accessible by the perturbative means and the main features of non-Abelian plasmas can be understood without the difficulties inherent to quantum field theory.
For quasi-particle QGP model we propose stochastic simulation of thermo-dynamics and kinetic properties *in the wide region of temperature, density and quasi-particles masses. We extend previous classical simulations based on a color Coulomb interaction to the relativistic quantum regime. In grand canonical ensemble for finite and zero baryon chemical potential we use the direct quantum path integral Monte Carlo method (PIMC) within Feynman formulation of quantum mechanics. For the strongly correlated QGP we have done calculations of equation of state, spatial and color pair distribution functions, diffusion coefficients and shear viscosity.*

Conclusions from Lattice simulations



Computer simulations a) reproduce well known hadron properties b) predict new phenomena c) help to create new theoretical ideas and models.

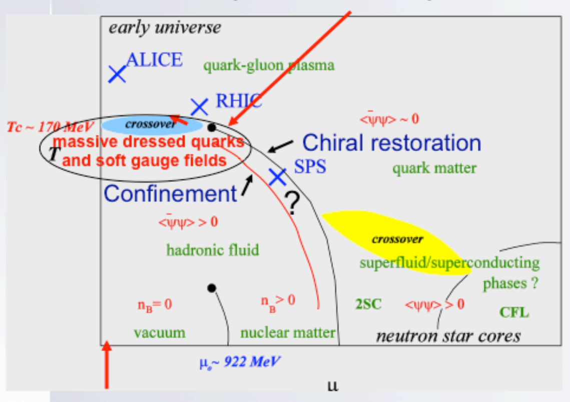
Low dimensional objects (regions or quasiparticles - dressed quarks and gluons) are responsible for most interesting nonperturbative effects such as chiral symmetry breaking and confinement.

http://www.ihed.ras.ru/fortov/polikarpov.ppt

Phase diagram

Quark-gluon plasma is considered as system of dressed quarks, anti-quarks and gluons presented by color Coulomb quasiparticles with T-dependent dispersion curves and width at around μ=0 or above T_d and below T_c.

Litim, Manuel, Stoecker, Bleicher, Feinberg, Richardson, Bonasera, Maruyama, Hatsuda, Shuryak, Fukushima,....



Phase diagram (F.Karsch)

Basic assumptions of the quantum quasiparticle model of quark – gluon plasma

is based on resummation technique and lattice simulations allowing for consideration of quark-gluon plasma as system of dressed quarks, antiquarks and gluons presented by color Coulomb quasiparticles with T-dependent dispersion curves and width.
(Shuryak , Phys.Lett.B478,161(2000), Phys. Rev. C, 74, 044909, (2006))

- We consider relativistic color quasiparticles representing gluons and the most stable quarks and anti-quarks of three flavors (up, down and strange).
- Up, down and strange quasiparticles have the same masses
- Interparticle interaction is domonated by a color Coulomb potential with distance dependent coupling constant.
- The color operators are substituted by their average values – classical color vectors in SU(3) (8D vectors with 2 Casimirs condit.).

The model input requires :

- The temperature dependence of the quasiparticle mass.
- The temperature dependence of the coupling constant.

All input quantities should be deduced from lattice QCD calculations or experimental data and substituted in quantum Hamiltonian.

Thermodynamics of quark - gluon plasma in grand canonical ensemble within Feynman formulation of quantum mechanics

$$H_\beta = K_\beta + U_C = \sum_a \sqrt{p_a^2 + m_a^2(\beta)} + U_C =$$
$$= \sum_a \sqrt{p_a^2 + m_a^2(\beta)} + \sum_{a,b} \frac{g^2(|r_a - r_b|,\beta) C_{ab} < \vec{Q}_a | \vec{Q}_b >}{4\pi |r_a - r_b|}$$

Grand canonical partition function

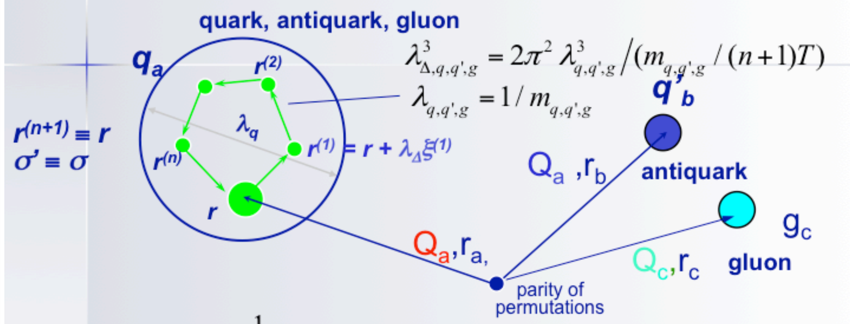
$$\Omega(\mu, \mu_s = 0, V, \beta) = \sum_{N_q, N_{\bar q}, N_g} \exp(\beta\mu(N_q - N_{\bar q})) Q(N_q, N_{\bar q}, N_g, \beta)/N_q! N_{\bar q}! N_g!$$
$$Q(N_q, N_{\bar q}, N_g, \beta) = \sum_\sigma \int_V dr dQ \rho(r, Q, \sigma; \beta)$$
$$\rho = \exp(-\beta H(\beta)) = \exp(-\Delta\beta H(\beta)) \times K \times \exp(-\Delta\beta H(\beta))$$

$n+1$, $\beta = 1/kT$, $\Delta\beta = \beta/(n+1)$

PATH INTEGRAL MONTE-CARLO METHOD

quark, antiquark, gluon

$$\lambda^3_{\Delta, q, \bar q, g} = 2\pi^2 \lambda^3_{q, \bar q, g} / (m_{q, \bar q, g}/(n+1)T)$$
$$\lambda_{q, \bar q, g} = 1/m_{q, \bar q, g}$$



$$\rho(r, Q, \sigma; \beta) = \frac{1}{\lambda_{\Delta}^{3N_q} \lambda_{\Delta}^{3N_{\bar q}} \lambda_{\Delta}^{3N_g}} \sum_{P = P_q P_{\bar q} P_g} (\pm 1)^{\kappa_P} \int_V dr^{(1)} \ldots dr^{(n)} dQ^{(1)} \ldots dQ^{(n)} \times$$
$$\rho(r, Q; r^{(1)}, Q^{(1)}; \Delta\beta) \ldots \rho(r^{(n)}, Q^{(n)}; Pr^{(n+1)}, PQ^{(n+1)}; \Delta\beta) S(\sigma, P\sigma')$$
$$\rho(r^{(l)}, Q^{(l)}; r^{(l+1)}, Q^{(l+1)}) \approx \delta(Q^{(l)} - Q^{(l+1)}) \rho(r^{(l)}, Q^{(l)}; r^{(l+1)}, Q^{(l)})$$

spin matrix

Density matrix

$$\sum_\sigma \rho(r, Q, \sigma; \beta) = \frac{1}{\lambda_\Delta^{3N_q} \lambda_\Delta^{3N_{\bar q}} \lambda_\Delta^{3N_g}} \sum_{s=0}^{N_q} \sum_{s'=0}^{N_{\bar q}} \sum_{s''=0}^{N_g} \rho_{s s' s''}([rQ], \beta)$$
$$\rho_{s s' s''}([rQ], \beta) = \frac{C_{N_q}^s}{2^{N_q}} \frac{C_{N_{\bar q}}^{s'}}{2^{N_{\bar q}}} \frac{C_{N_g}^{s''}}{2^{N_g}} \exp\{-\beta U([rQ], \beta)\} \times$$
$$\times \prod_{l=1}^{n} \prod_{p=1}^{N_q} \phi_{pp}^l \det|\psi_{ab}^{n,1}|_s \prod_{p=1}^{N_{\bar q}} \phi_{pp}^l \det|\psi_{ab}^{n,1}|_{s'} \prod_{p=1}^{N_g} \phi_{pp}^l \, per|\psi_{ab}^{n,1}|_{s''}$$
$$U([rQ], \beta) = \sum_{l=0}^{n} \frac{U_l^{q\bar q g}([r^{(l)} Q], \beta)}{n+1}$$

Pairwise sum of Kelbg potentials for each l=0,…,n

Exchange matrix
$$\|\psi_{ab}^{n,1}\|_s = \left\| K_2 \left\{ \sqrt{(m/((n+1)T))^2 + |(r_a - r_b)|^2 / \lambda^2_{q,\bar q,g}} \right\} \right\|_s$$

Color Kelbg potential

Richardson, Gelman, Shuryak, Zahed, Harmann, Donko, Leval, Kalman (r=0 ?)

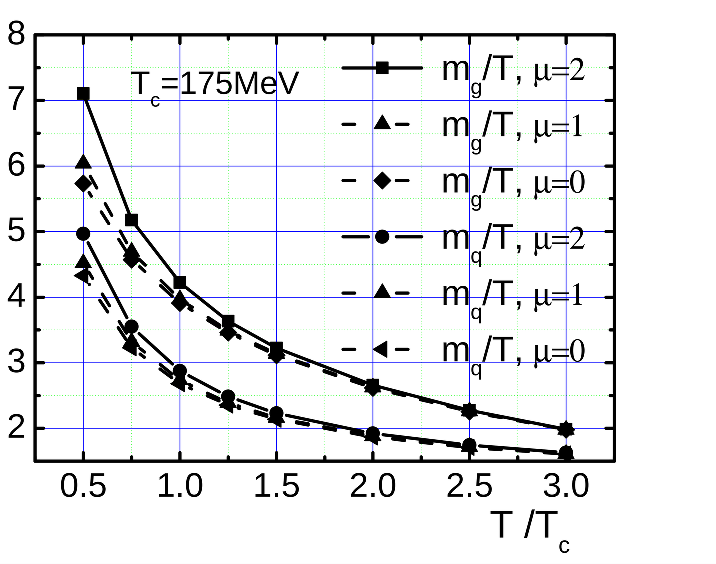
$$x_{ab} = |\mathbf{r}_{ab}| / \tilde\lambda_{ab}, \quad \tilde\lambda_{ab} = \hbar^2 \Delta\beta / 2\mu_{ab}$$
$$\Phi^{ab}(x_{ab}, \Delta\beta) = \frac{< \vec{Q}_a | \vec{Q}_b > g^2}{4\pi \tilde\lambda_{ab} x_{ab}} \left\{ 1 - e^{-x_{ab}^2} + \sqrt{\pi} x_{ab} \left[1 - erf(x_{ab}) \right] \right\}$$

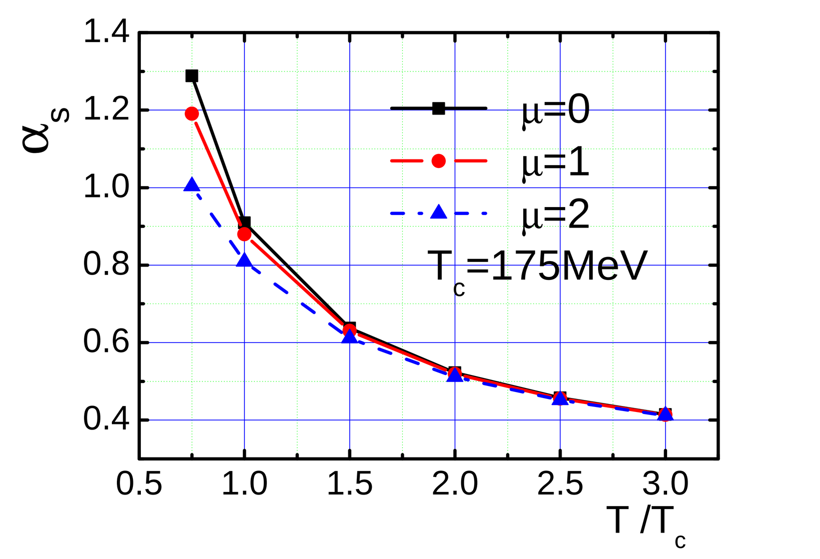
$|\mathbf{r}_{ab}| \to 0$: $\frac{< Q_a | Q_b > g^2 \sqrt{\pi}}{4\pi \tilde\lambda_{ab}}$

$|\mathbf{r}_{ab}| >> \tilde\lambda_{ab}$: $\frac{< Q_a | Q_b > g^2}{4\pi \lambda_{ab} |x_{ab}|}$

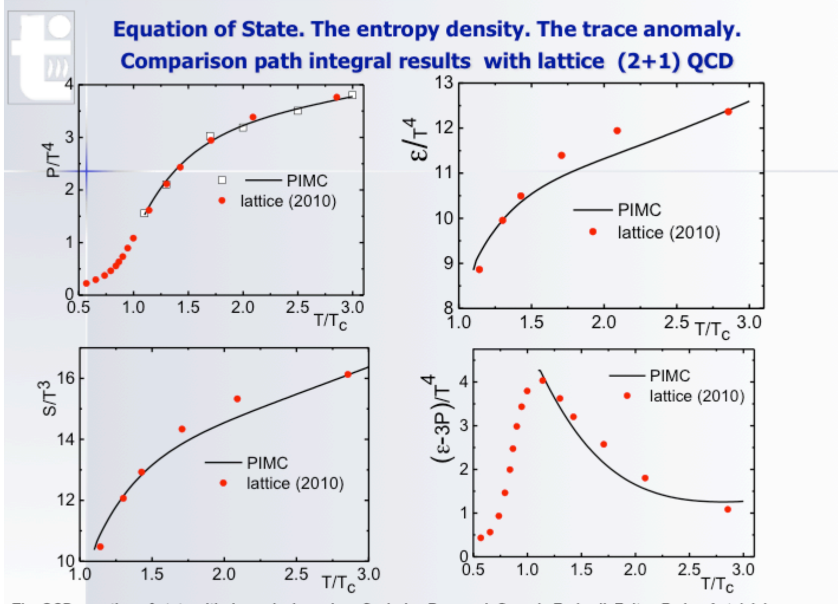
Objects Q are color coordinates of quarks and gluons
There is no divergence at small interparticle distances and it has a true asymptotics (T, x_{ab})

Ha -> $k_B T_c$, T_c =175 Mev, $T_c < T$, m_a~5$k_B T_c/c^2$, L_o ~ hc/$k_B T_c$, r_s= <r>/L_o<0.1, L_o~1.2 10^{-15} m, x_{ab} ~1

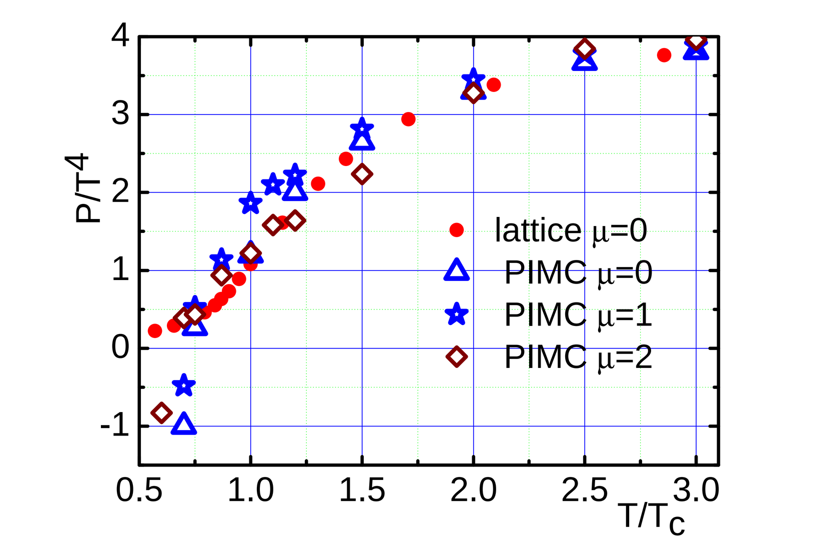


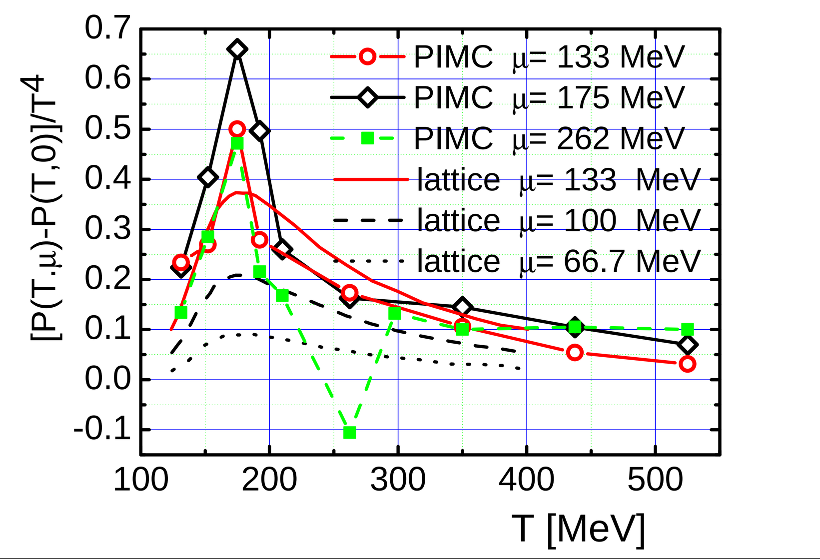


Equation of State. The entropy density. The trace anomaly. Comparison path integral results with lattice (2+1) QCD



The QCD equation of state with dynamical quarks Szabolcs Borsanyi, Gergely Endrodi, Zoltan Fodor, Antal Jakovac, Sandor D. Katz, Stefan Krieg, Claudia Ratti, Kalman K. Szabo, JHEP 11 (2010) 077





Kinetic properties of quark – gluon plasma in canonical ensemble

$$C_{FA}(t) = Z^{-1} Tr\{F \exp(i\frac{Ht_c}{h}) A \exp(-i\frac{Ht_c}{h})\};$$
$$H = K + V(qQ), t_c = t - i\frac{\beta h}{2}, \beta = \frac{1}{kT},$$
$$Z = Tr\{\exp(-\beta H)\}$$
$$C_{FA}(t) = \frac{1}{(2\pi h)^{2\nu}} \iint dQ_1 dp_1 dq_1 dp_2 dq_2 F(p_1, q_1) A(p_2, q_2) \times W(p_1, q_1, Q_1; p_2, q_2, Q_2; t; i\beta h),$$

In this model we use approximation $\delta(Q_1 - Q_2)\delta(Q_1 - Q_2)\delta(Q_1 - Q_2)$

$$A(p, q) = \iint d\xi \exp(-i\frac{p\xi}{h}) < q - \frac{\xi}{2} | A | q + \frac{\xi}{2} >$$

Weil symbols of operators

$$W(p_1, q_1, Q_1; p_2, q_2, Q_2; t; i\beta h) = Z^{-1} \iint d\xi_1 d\xi_2 \exp(i\frac{p_1\xi_1}{h}) \exp(i\frac{p_2\xi_2}{h}) \times$$
$$< q_1 + \frac{\xi_1}{2} | \exp(i\frac{Ht_c}{h}) | q_2 - \frac{\xi_2}{2} > < q_2 + \frac{\xi_2}{2} | \exp(-i\frac{Ht_c}{h}) | q_1 - \frac{\xi_1}{2} >$$

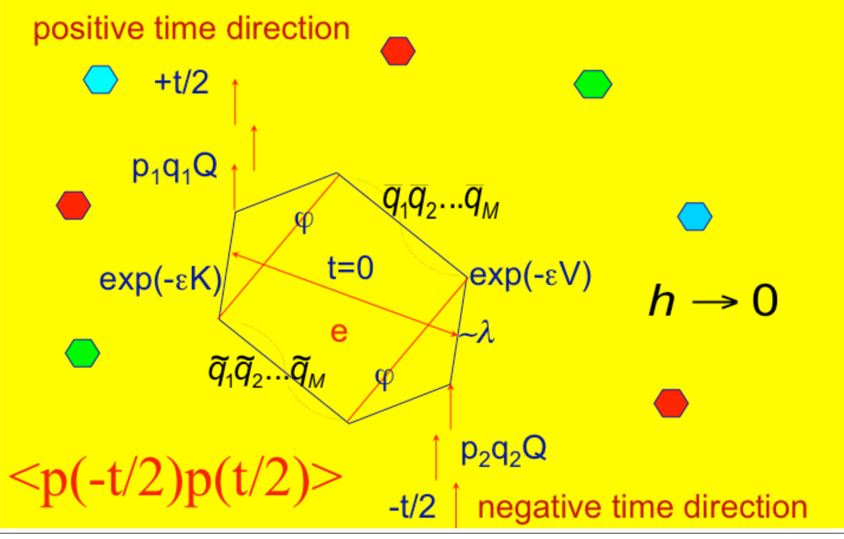
Integral equation

$$W(p_1, q_1, Q_1; p_2, q_2, Q_2; t; i\beta h) = \bar{W}(p_1^0, q_1^0, Q_1^0; p_2^0, q_2^0, Q_2^0; 0; i\beta h) +$$
$$+ \int_0^t d\tau \int ds \iint d\eta W(p_1^\tau - s, q_1^\tau, Q_1^\tau; p_2^\tau - \eta, q_2^\tau, Q_2^\tau; \tau; i\beta h) \gamma(s, q_1^\tau, Q_1^\tau; \eta, q_2^\tau, Q_2^\tau),$$
$$\gamma(s, q_1^\tau, Q_1^\tau; \eta, q_2^\tau, Q_2^\tau) = \frac{1}{2}[\omega(s, q_1^\tau, Q_1^\tau)\delta(\eta) - \omega(\eta, q_2^\tau, Q_2^\tau)\delta(s)], F(q, Q) = -\nabla_q V(q, Q)$$
$$\omega(s, q, Q) = \frac{4}{(2\pi h)^\nu h} \int dq' V(q - q', Q) Sin(\frac{2sq'}{h}) + F(q, Q)\frac{d\delta(s)}{ds}$$

Positive time direction
Color dynamics in SU(2) or SU(3)
Initial conditions
Hamiltonian eqations
Negative time direction

$$\frac{dq_1^\tau}{d\tau} = \frac{1}{2m} p_1^\tau; \quad \frac{dp_1^\tau}{d\tau} = \frac{1}{2} F(q_1^\tau, Q_1^\tau),$$
$$\frac{dQ_1^{a,\tau}}{d\tau} = \frac{1}{2} \sum_{b,c} f^{abc} Q_1^b \nabla_{Q_1^c} V(q_1^\tau, Q_1^\tau),$$
$$p_1^\tau(t; p_1, q_1, Q_1) = p_1, q_1^\tau(t; p_1, q_1, Q_1) = q_1, Q_1^\tau(t; p_1, q_1, Q_1) = Q_1$$
$$\frac{dq_2^\tau}{d\tau} = -\frac{1}{2m} p_2^\tau; \quad \frac{dp_2^\tau}{d\tau} = -\frac{1}{2} F(q_2^\tau, Q_2^\tau),$$
$$\frac{dQ_2^{a,\tau}}{d\tau} = -\frac{1}{2} \sum_{b,c} f^{abc} Q_2^b \nabla_{Q_2^c} V(q_2^\tau, Q_2^\tau),$$
$$p_2^\tau(t; p_2, q_2, Q_2) = p_2, q_2^\tau(t; p_2, q_2, Q_2) = q_2, Q_2^\tau(t; p_2, q_2, Q_2) = Q_2$$

Schematic snapshot for color phase space dynamics



Time autocorrelation function of the stress energy tensor and shear viscosity of quark –gluon plasma

$$\eta(\tau) = \frac{n}{3k_B T} \left\langle \sum_{x \neq y} \sigma_{xy}(\tau/2) \sigma_{xy}(-\tau/2) \right\rangle$$
$$\sigma_{xy}(\tau) = \frac{1}{N} \left(\sum_{i=1}^N m_i v_{x_i} v_{y_i} + \frac{1}{2} \sum_{i \neq j} r_{x_{ij}} F_{y_{ij}} \right)$$
$$\eta = \lim_{t \to \infty} \eta(t) = \lim_{t \to \infty} \int_0^t d\tau \eta(\tau)$$