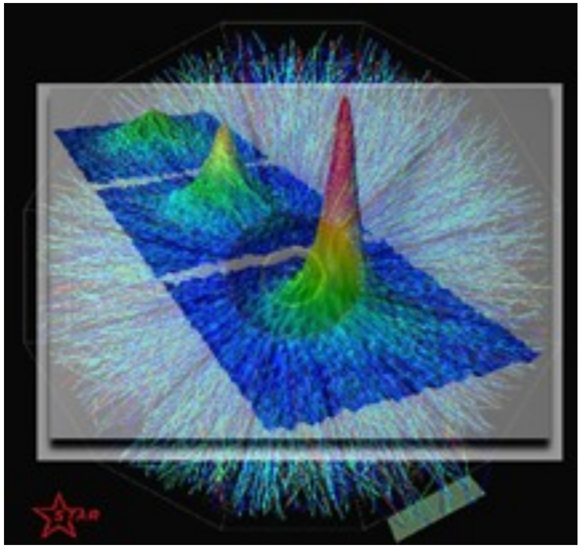
QM2014 @ Darmstadt, May 19, 2014

Gluon Transport Equation in the Small Angle Approximation and the Onset of Bose-Einstein Condensation



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Research Supported by NSF

Outline

- Overpopulated Glasma in Heavy Ion Collisions
- Kinetic Equation under Small Angle Approximation
- The Onset of Dynamical (Out-of-equilibrium) BEC
- The Robustness of the Onset Dynamics
- Summary

Work done in collaboration with: Jean-Paul Plaizot, Larry McLerran

References:

Blaizot, JL, McLerran, Nucl. Phys. A920, 58(2013);

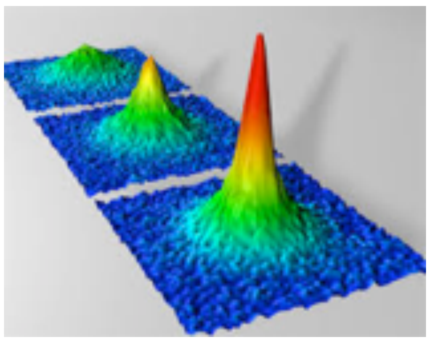
Blaizot, Jiang, & JL, in preparation;

Huang & JL, arXiv: 1303.7214; arXiv:1402.5578[invited review for IJMPE];

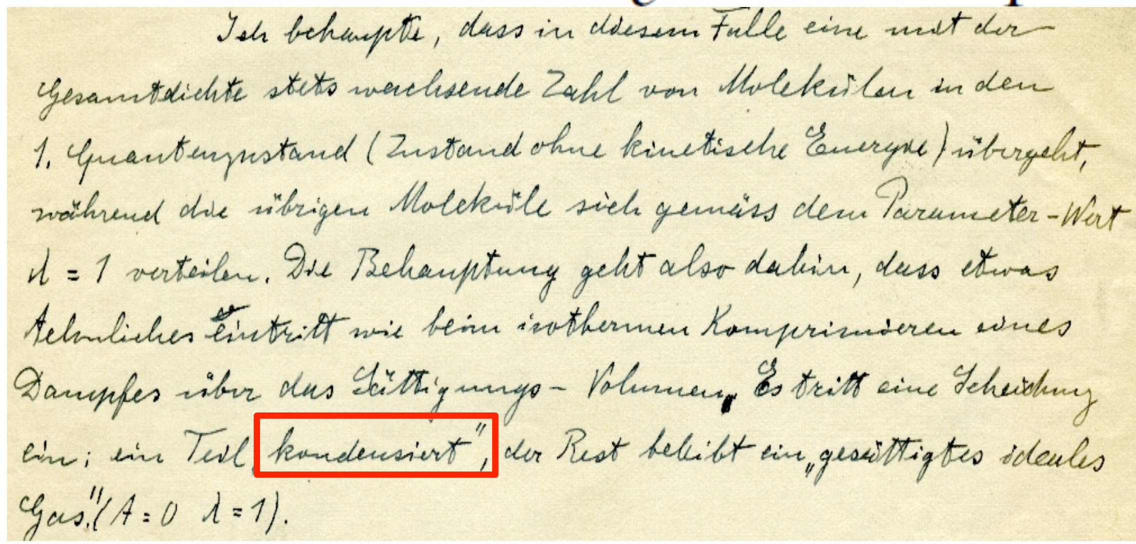
Blaizot, Gelis, JL, McLerran, Venugopalan, Nucl. Phys. A873, 68 (2012);

Chiu, Hemmick, Khachatryan, Leonidov, JL, McLerran, Nucl. Phys. A900, 16(2013).

BEC: Quantum Coherence <=> Overpopulation



$$f_{\mathrm{eq}}(\boldsymbol{k}) = n_c \delta(\boldsymbol{k}) + \frac{1}{e^{\beta(\omega_{\boldsymbol{k}} - m_0)} - 1}$$



Ich behaupte, dass in diesem Falle eine mit der Gesamtdichte stets wachsende Zahl von Molekülen in den 1. Quantenzustand (Zustand ohne kinetische Energie) übergeht, während die übrigen Moleküle sich gemäss dem Parameter-Wert $\lambda = 1$ verteilen. Die Behauptung geht also dahin, dass etwas Aehnliches eintritt wie beim isothermen Komprimieren eines Dampfes über das Sättigungs-Volumen. Es tritt eine Scheidung ein; ein Teil „kondensiert", der Rest bleibt ein „gesättigtes ideales Gas" ($A = 0$ $\lambda = 1$).

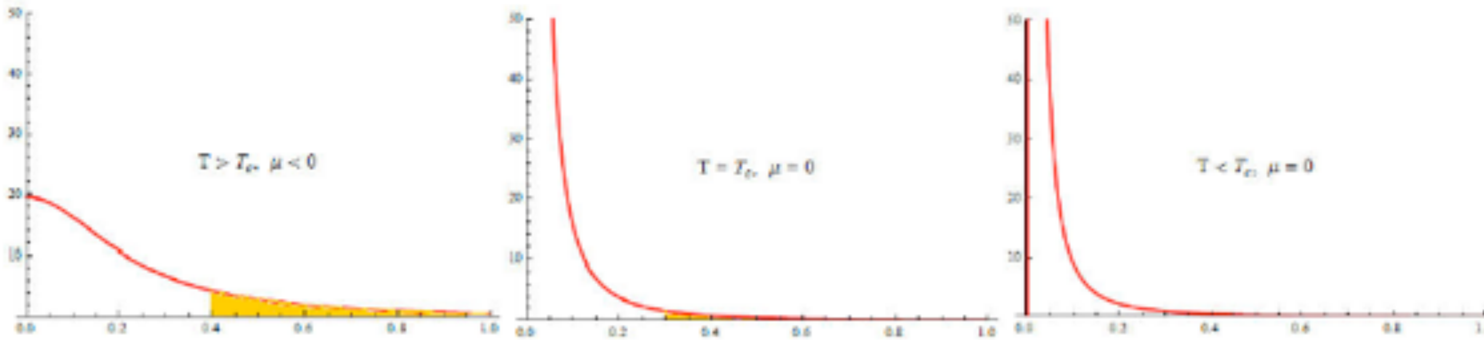
Einstein: new phase emerges with condensate, when quantum wave scale overlaps with inter-particle scale (--- the 1st application of de Broglie wavelength idea)

Quantum Coherence implies **OVERPOPULATION**:

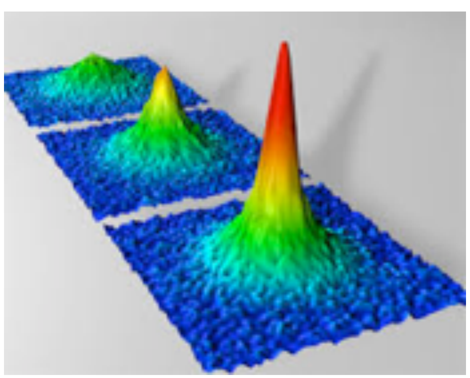
$$\boxed{\frac{\lambda_{dB}}{d} \sim \left(n\epsilon^{-3/4}\right)^{\alpha} \sim \hat{O}(1)}$$

BEC in The Very Cold

Brilliant evaporative cooling: precisely to achieve
OVERPOPULATION



Cooling procedure: kick out fast atoms (truncating UV tail);
then let system relax toward new equilibrium;
relaxation via IR particle cascade & UV energy cascade.



It took ~70 years to achieve
OVERPOPULATION,
thus BEC in ***ultra-cold*** bose gases.

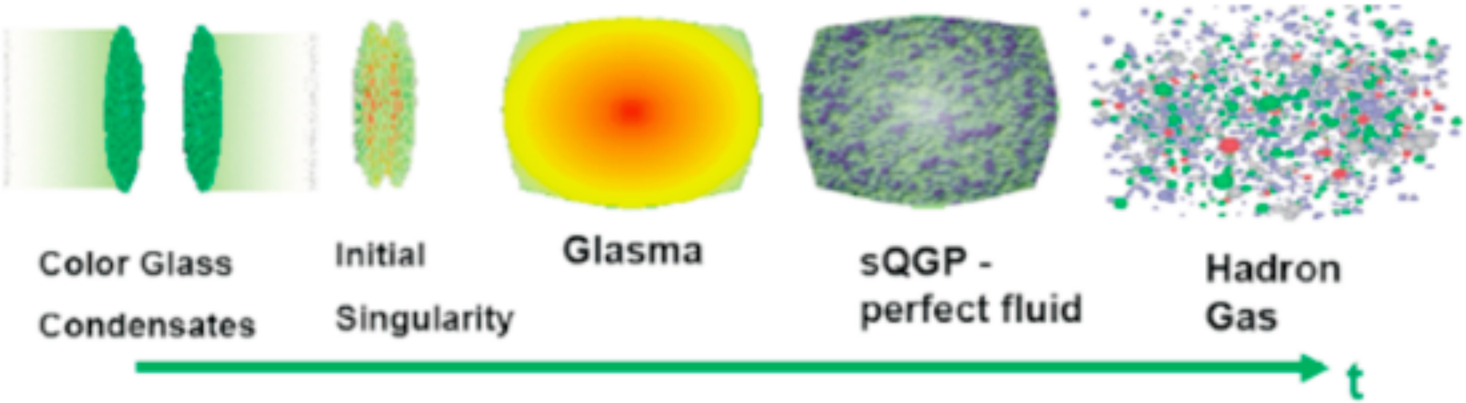
$$n \cdot \epsilon^{-3/4} > \hat{O}(1)\ threshold$$

BEC in the Very Hot!

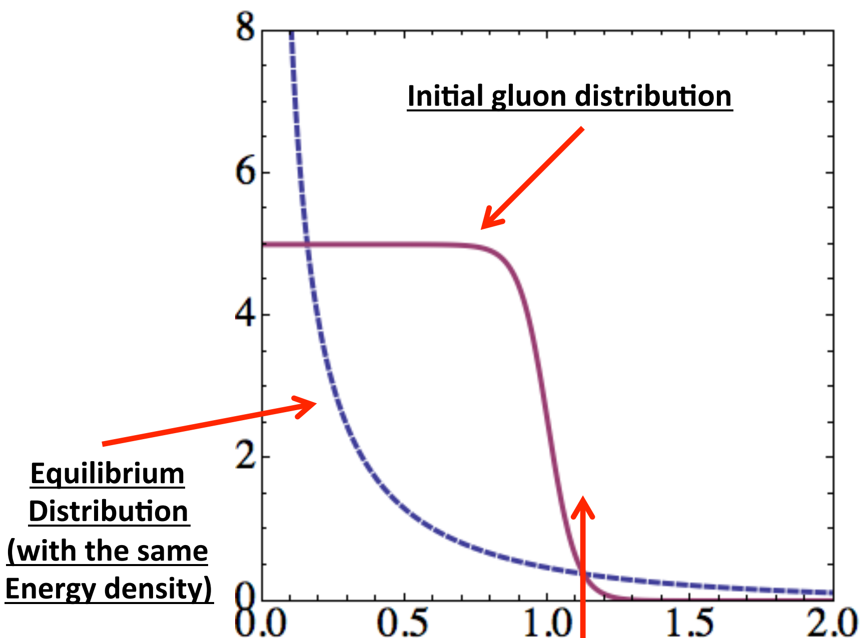
Temperature

$10^{-8}K$	$10^{0}K$	$10^{1}K$	$10^{2}K$	~~ $10^{12}K$
cold atomic gas	liquid helium; cosmic axion?	magnon	cavity photon; magnon	overpopulated glasma!

Overpopulated Glasma



The precursor of a thermal quark-gluon plasma, known as glasma, is born as a gluon matter with **HIGH OVERPOPULATION**:



Saturation Scale Qs ~ 1 GeV or larger, weakly coupled

Very large occupation number

$$f \sim \frac{1}{\alpha_s}$$

Saturation fixes initial scale

$$\epsilon_0 \sim \frac{Q_s^4}{\alpha_s}$$

$$n_0 \sim \frac{Q_s^3}{\alpha_s}$$

$$\epsilon_0 / n_0 \sim Q_s$$

Overpopulation: Thermodynamic Consideration

Our initial gluon system is highly OVERPOPULATED:

$$f(p) = f_0\, \theta(1 - p/Q_s),$$

$$\epsilon_0 = f_0\, \frac{Q_s^4}{8\pi^2}, \qquad n_0 = f_0\, \frac{Q_s^3}{6\pi^2}, \qquad n_0\, \epsilon_0^{-3/4} = f_0^{1/4}\, \frac{2^{5/4}}{3\, \pi^{1/2}},$$

This is to be compared with the thermal BE case:

$$n\, \epsilon^{-3/4}|_{SB} = \frac{30^{3/4}\, \zeta(3)}{\pi^{7/2}} \approx 0.28$$

Overpopulation occurs when: $\boxed{f_0 > f_0^c \approx 0.154}$

Identifying f_0 -> 1/alpha_s, even for alpha_s =0.3,
the system is highly overpopulated!!

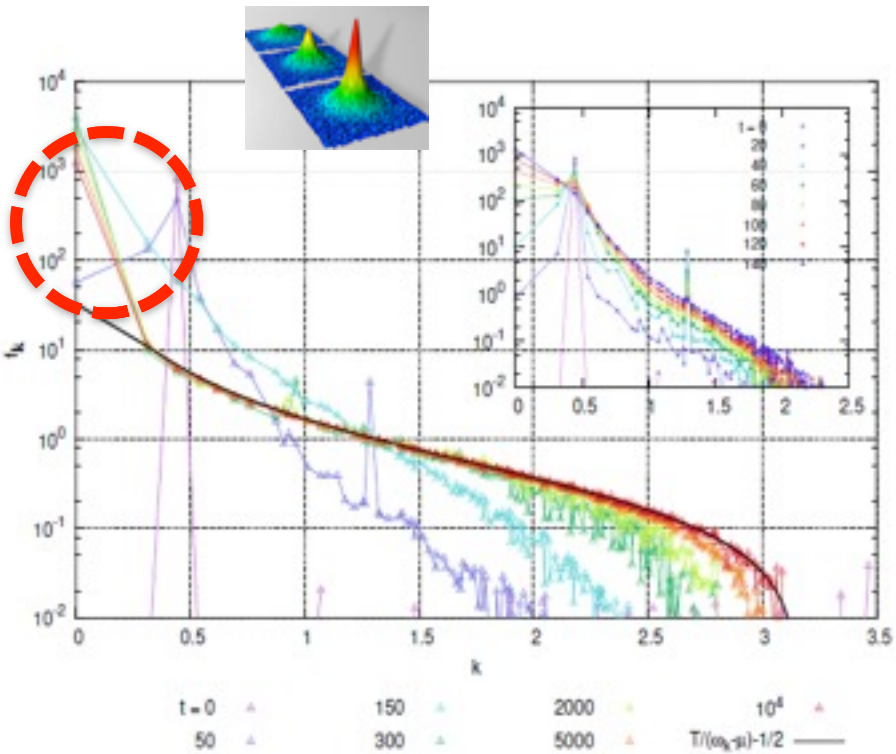
Will the system accommodate the excessive particles by forming a Bose-Einstein Condensate (BEC) ? AND HOW???

Strong Evidence of BEC from Scalar Field Theory Simulations

Bose–Einstein condensation and thermalization of the quark–gluon plasma

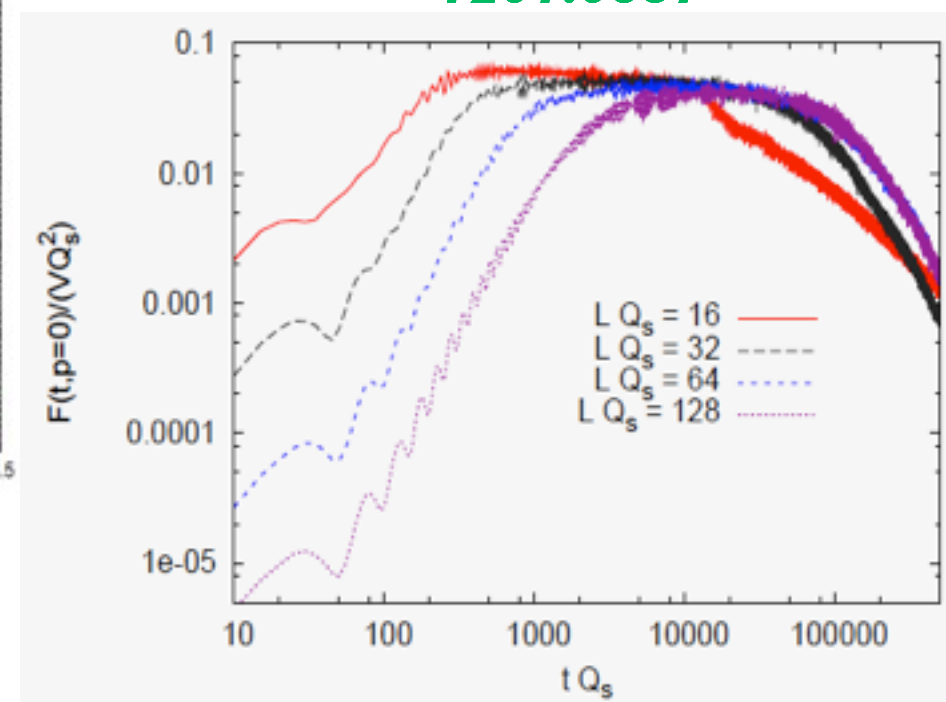
Jean-Paul Blaizot [a], François Gelis [a], Jinfeng Liao [b,*], Larry McLerran [b,c], Raju Venugopalan [b]

Absolutely true for pure elastic scatterings;
True, in transient sense, for systems with inelastic processes

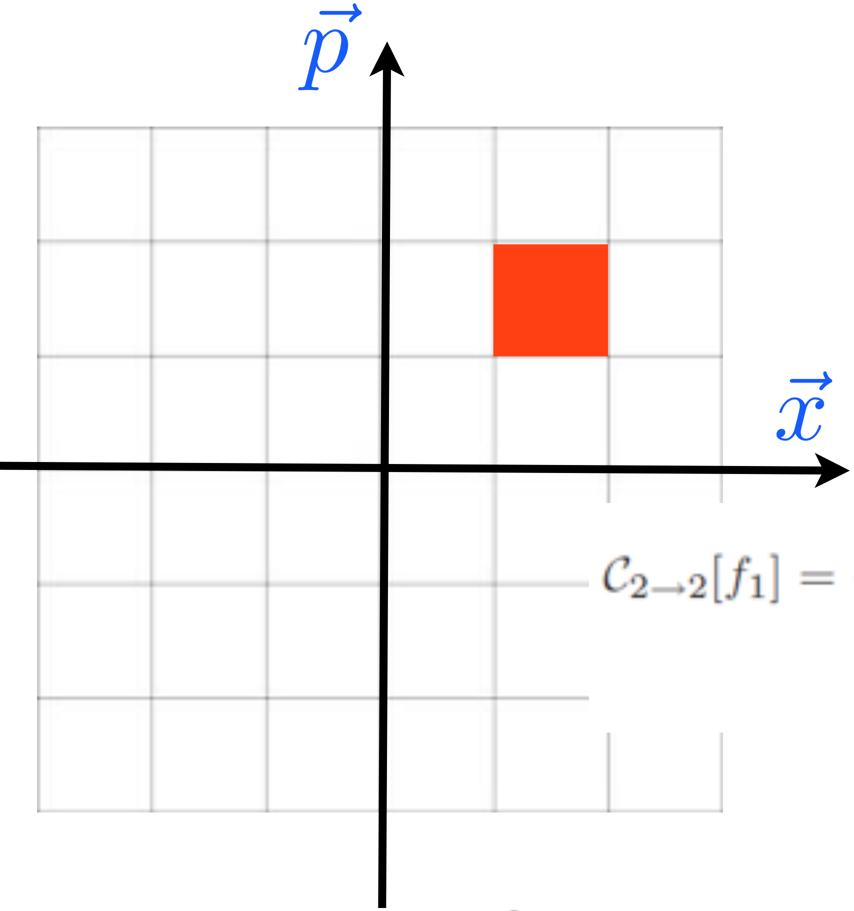


From: Epelbaum & Gelis 1107.0668

From: Berges & Sexty 1201.0687



Kinetic Evolution of A Dense Gluon System



$$f(\boldsymbol{x}, \boldsymbol{p}) = \frac{(2\pi)^3}{2(N_c^2 - 1)} \frac{dN}{d^3\boldsymbol{x}\, d^3\boldsymbol{p}}$$

phase space density

Transport equation:

$$(\partial_t + \mathbf{v}_p \cdot \nabla_{\mathbf{x}}) f(t, \mathbf{x}, \mathbf{p}) = \mathcal{C}[f]$$

Example of 2--2 scattering

$$|\, M_{12\to 34}\,|^2 = 72 g^4 \left[3 - \frac{t\,u}{s^2} - \frac{s\,u}{t^2} - \frac{t\,s}{u^2}\right]$$

$$\mathcal{C}_{2\to 2}[f_1] = \frac{1}{2}\int_{234} \frac{1}{2E_1} |M_{12\to 34}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4)$$
$$\times [(1 + f_1)(1 + f_2) f_3 f_4 - f_1 f_2 (1 + f_3)(1 + f_4)].$$

Input:

* cross-section

* initial condition

Destination:

* fixed point solution

Kinetic Equations with Long Range Interactions

For describing kinetic evolution of a system with long range interactions, the small angle approximation is a very useful approach.

* Landau ~1950 for NR QED plasma (in Boltzmann limit), known as Landau collision integral.

* A. Mueller, 1999, generalized to relativistic gluon plasma with QCD interactions (also in Boltzmann limit).

* Blaizot-Liao-McLerran, 2011, gluon plasma with quantum statistics.

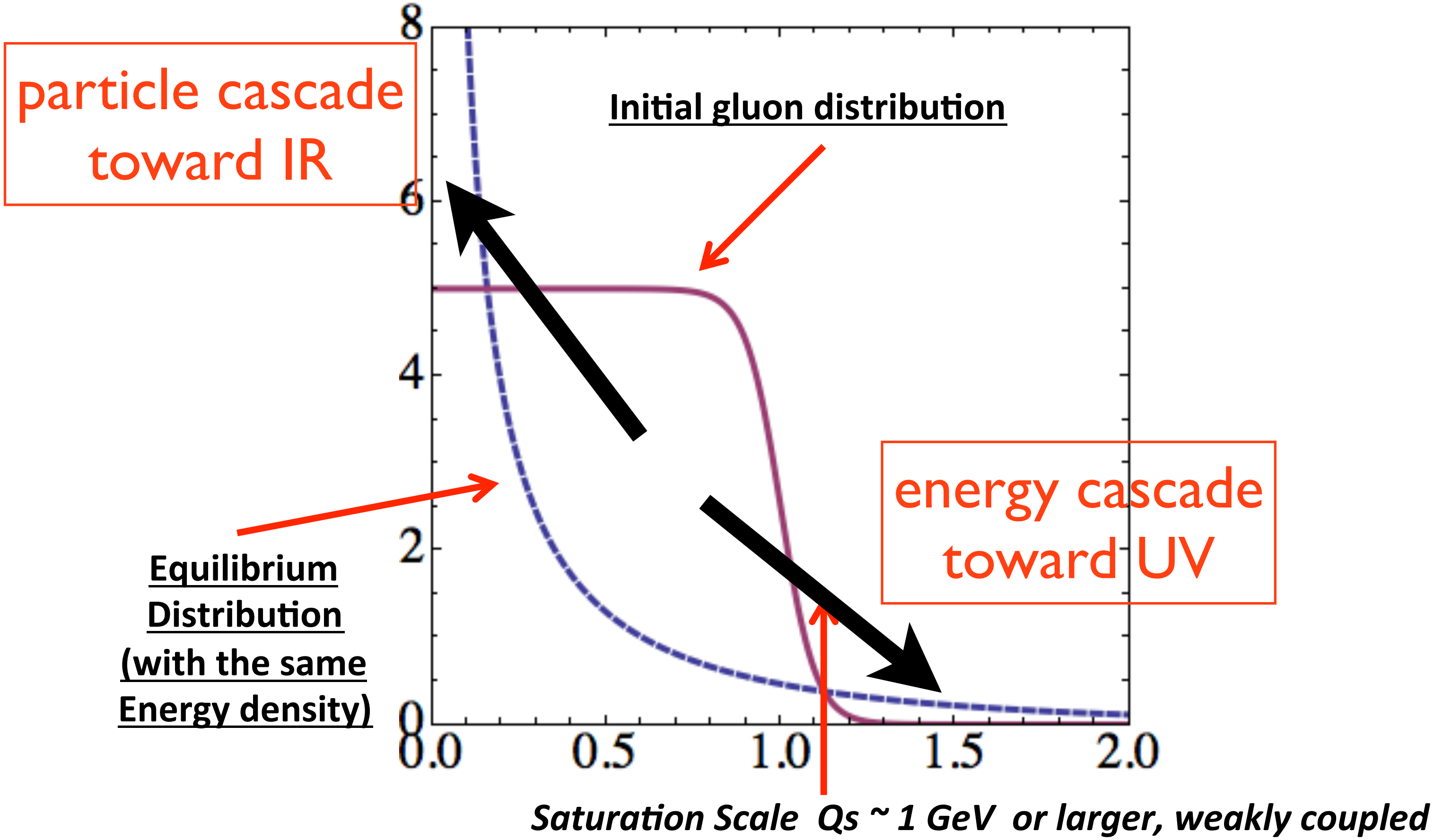
$$\boxed{\mathcal{D}_t f(\vec{p}) = \xi \left(\Lambda_s^2 \Lambda\right) \vec{\nabla} \cdot \left[\vec{\nabla} f(\vec{p}) + \frac{\vec{p}}{p} \left(\frac{\alpha_S}{\Lambda_s}\right) f(\vec{p})[1 + f(\vec{p})]\right]}$$

$$\Lambda \left(\frac{\Lambda_s}{\alpha_S}\right)^2 \equiv (2\pi^2) \int \frac{d^3p}{(2\pi)^3} f(\vec{p}) [1 + f(\vec{p})]$$

$$\Lambda \frac{\Lambda_s}{\alpha_S} \equiv (2\pi^2)\, 2 \int \frac{d^3p}{(2\pi)^3} \frac{f(\vec{p})}{p}$$

Two important scales:
hard scale Lambda
soft scale Lambda_s

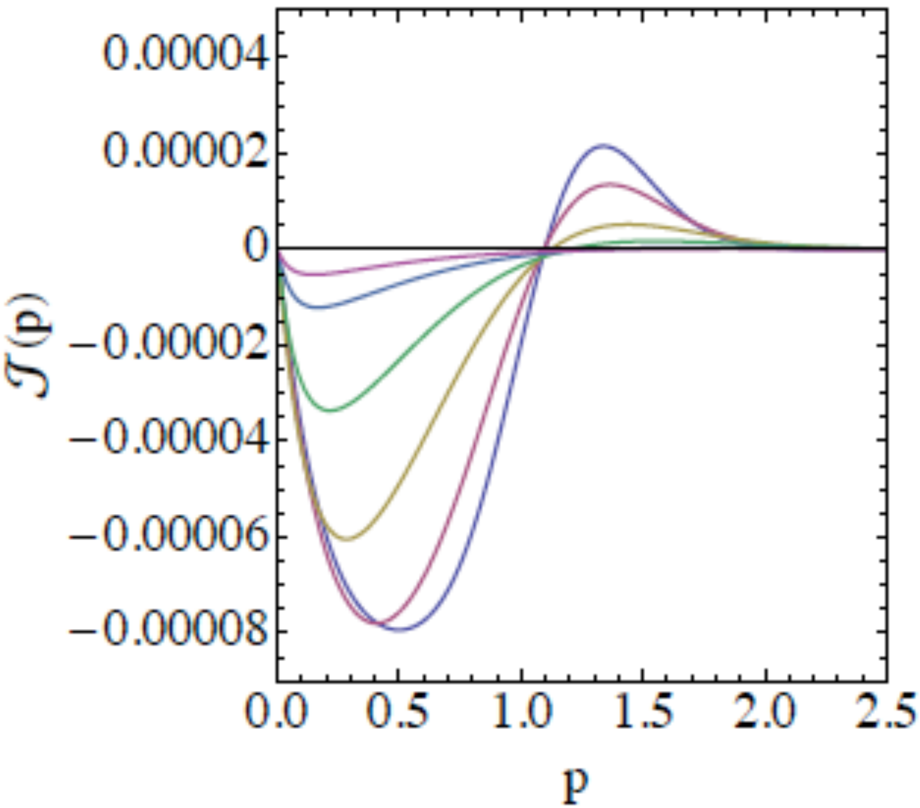
How Thermalization Proceeds



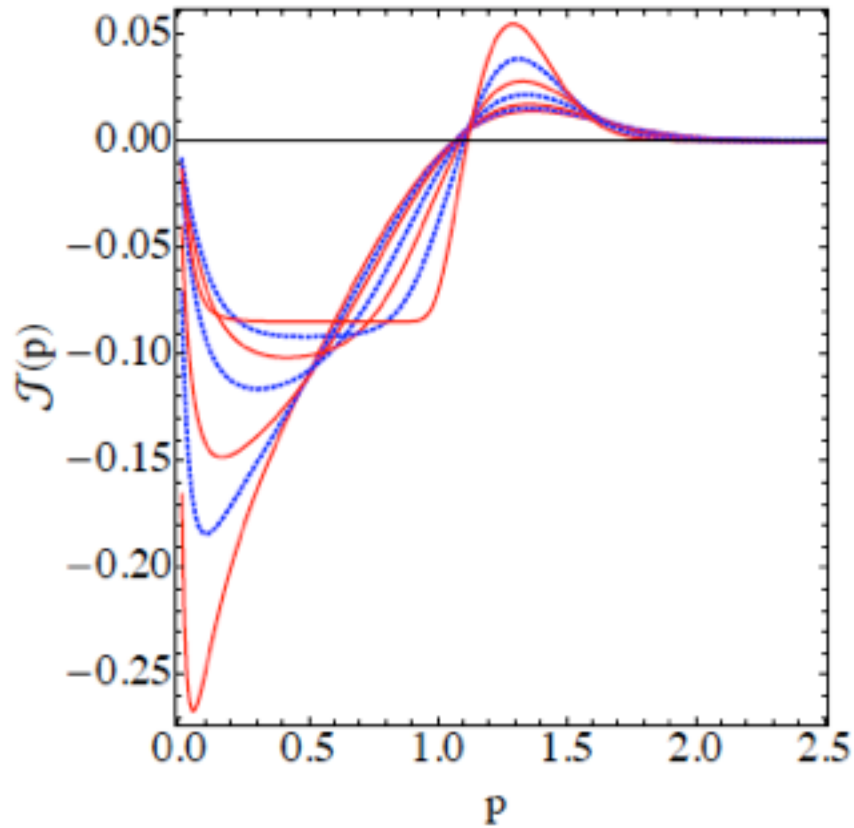
Initial glasma: $\Lambda \sim \Lambda_s \sim Q_s$ → Thermalized weakly-coupled QGP: $\Lambda \sim T$, $\Lambda_s \sim \alpha_s * T$

separation of two scales toward thermalization $\boxed{\frac{\Lambda_s}{\Lambda} \sim \alpha_s}$

IR and UV Cascade



f_0=0.1
(underpopulated)

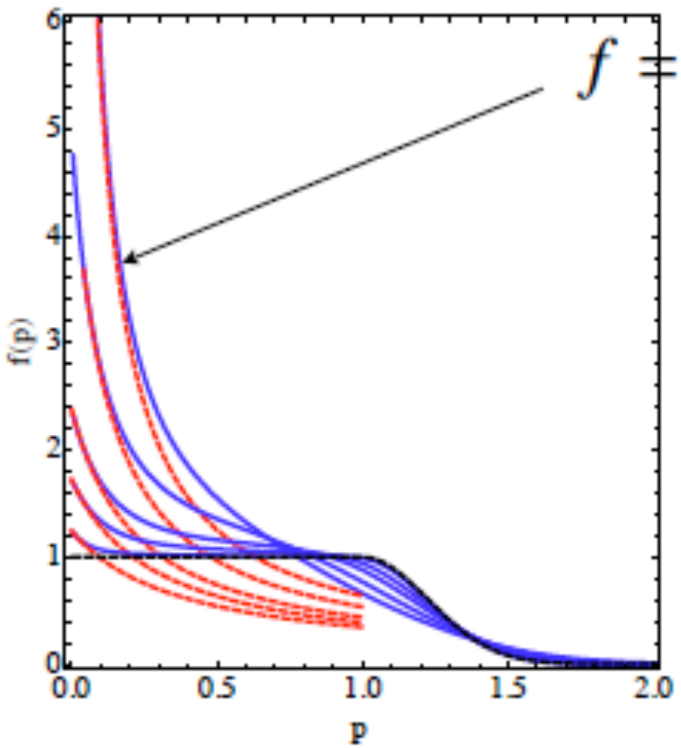


f_0=1
(overpopulated)

Blaizot, JL, McLerran, 1305.2119, NPA2013

How BEC Onset Occurs Dynamically?

A crucial step: rapid IR local thermalization



$$f = \frac{T^*}{p - \mu^*} \qquad (\mu^* < 0)$$

Very strong particle flux toward IR,
leading to rapid growth
and almost instantaneous
local thermal distribution
of very soft modes

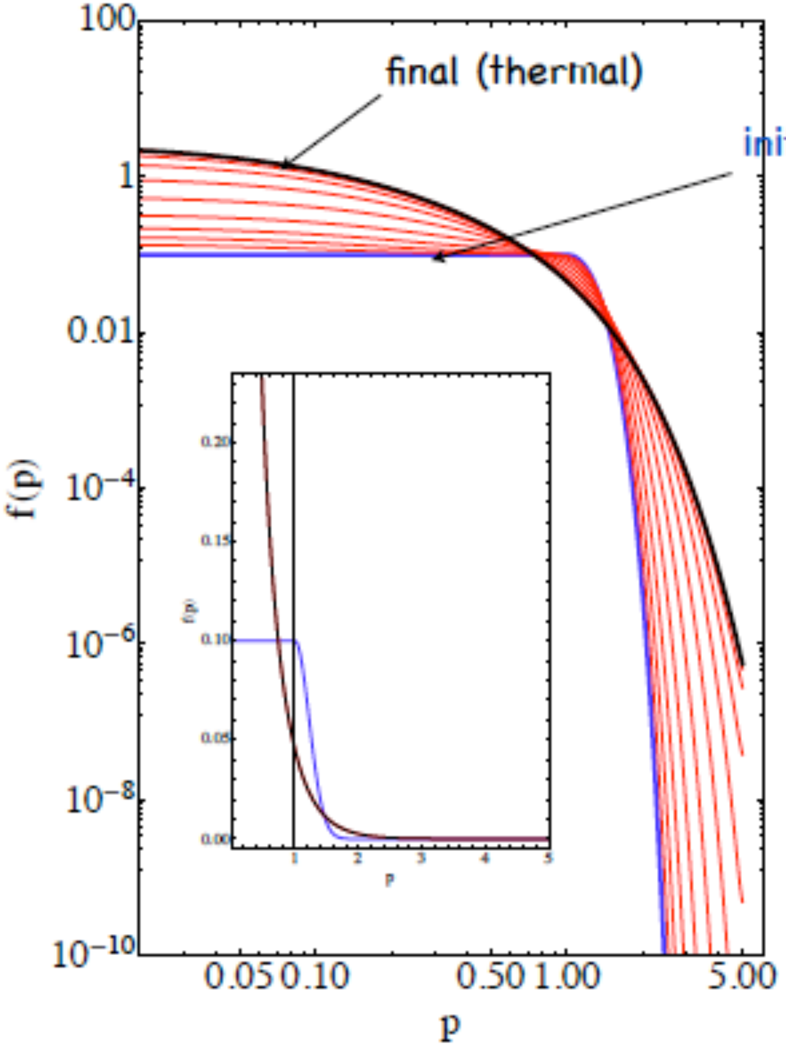
What happens next depends on INITIAL CONDITION:

underpopulation v.s. overpopulation

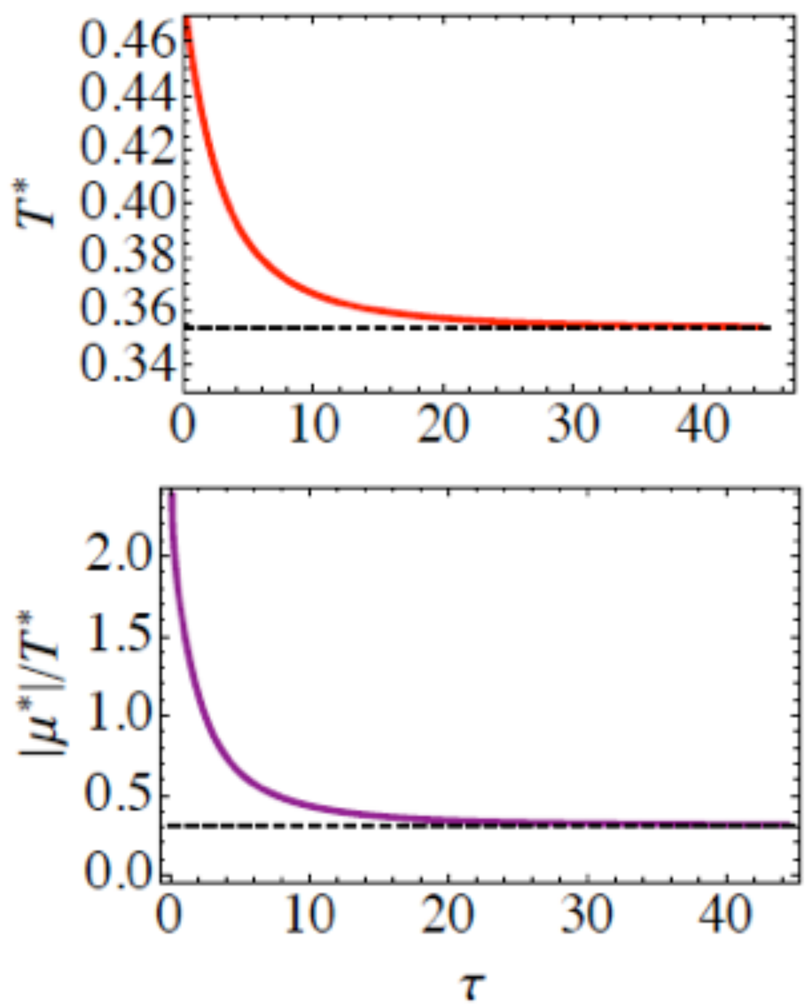
Blaizot, JL, McLerran, 1305.2119, NPA2013

Underpopulated Case

f 0 = 0.1



In underpopulated case, the system thermalizes to thermal BE distribution.

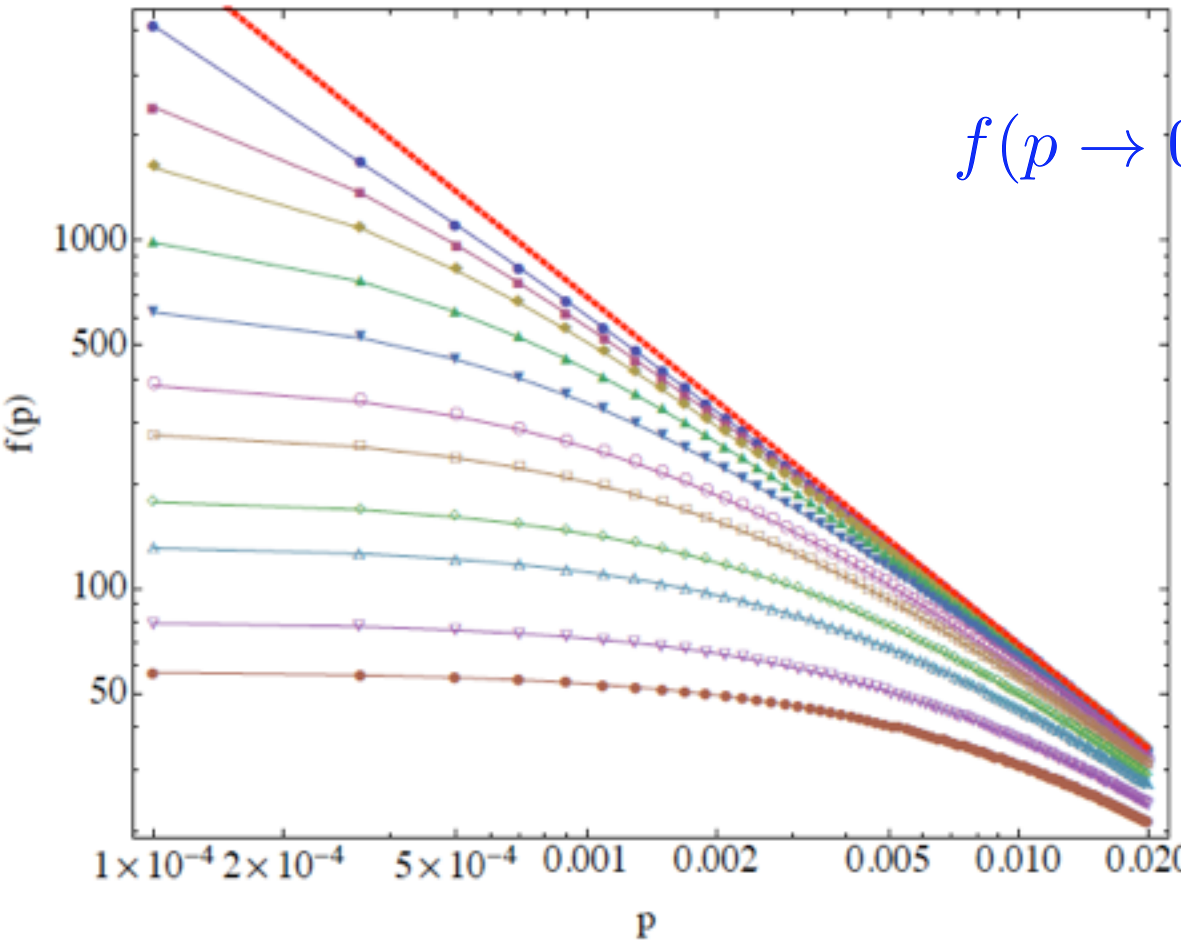


Overpopulated Case: How Onset of BEC Develops?

Before it could reach equilibrium, onset of BEC occurs!

A **critical** IR distribution develops, i.e. Mu* vanishes.

(In thermal BEC: global distribution must be critical.)

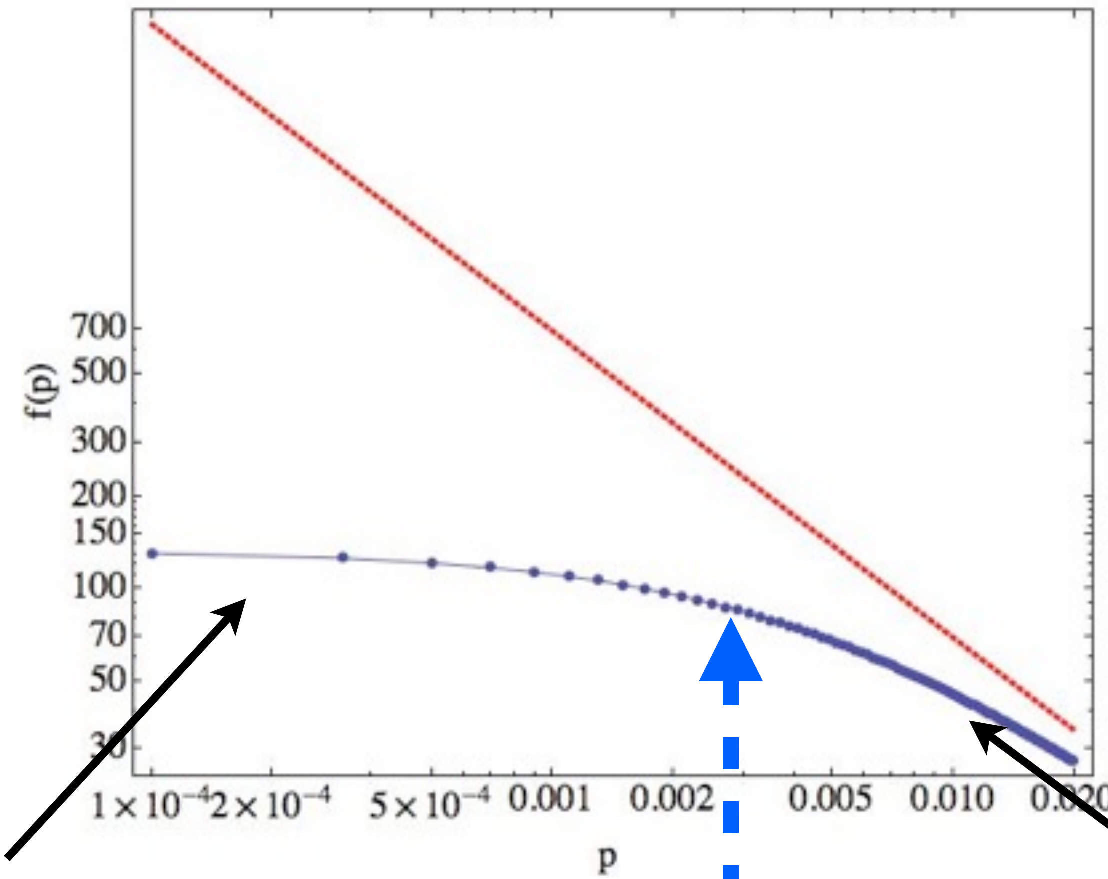


$$f(p \to 0) \to \frac{T^*}{p - \mu^*}$$

$$\mu^* \to 0$$

f_0=1

Overpopulated Case:
How Onset of BEC Develops?



p << *Mu**

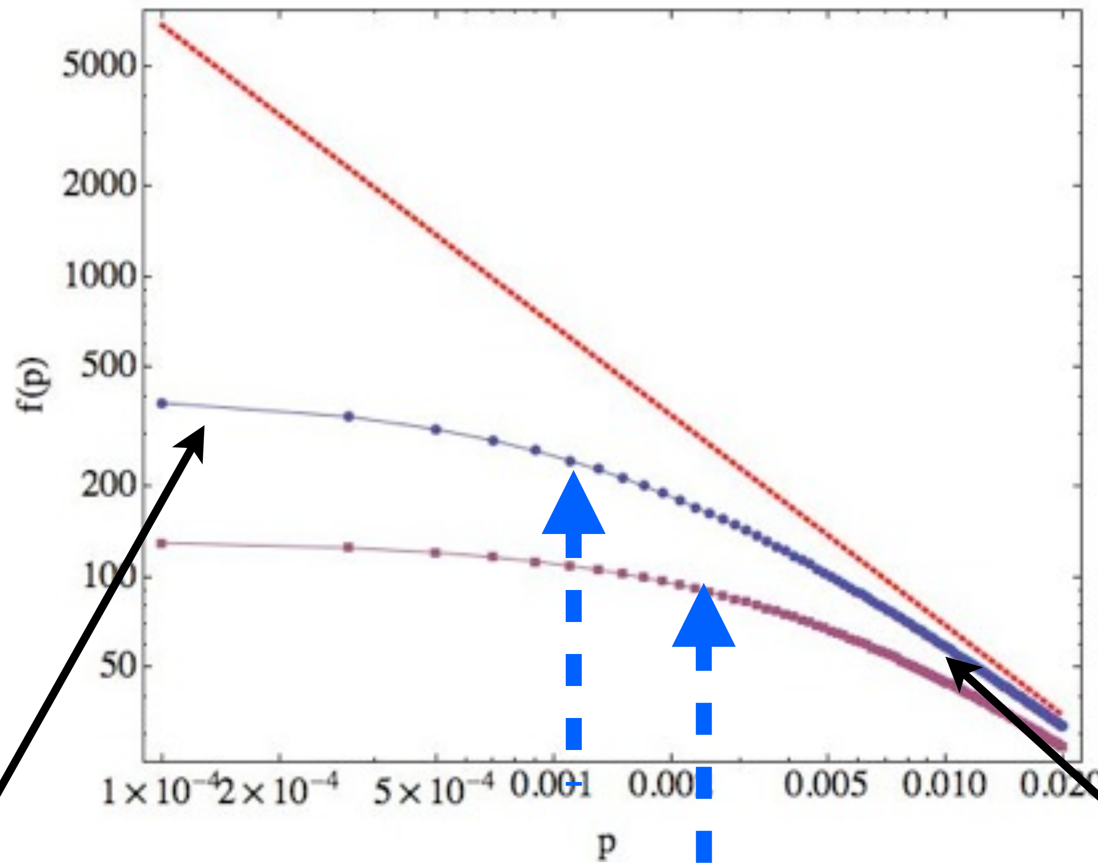
$$f \to T^*/|\mu^*|$$

p >> *Mu**

$$f \to T^*/p$$

$$f(p \to 0) \to \frac{T^*}{p - \mu^*}$$

Overpopulated Case: How Onset of BEC Develops?



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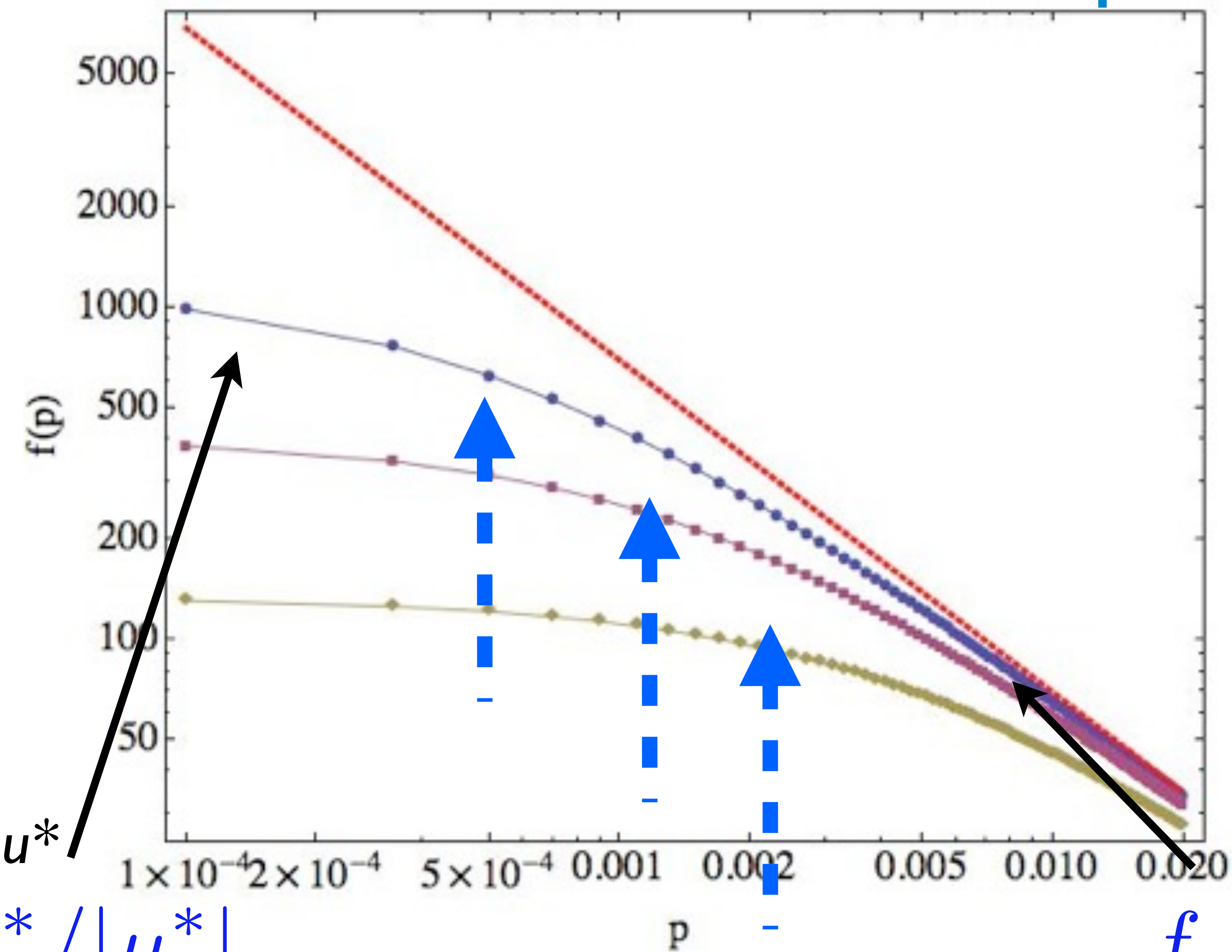

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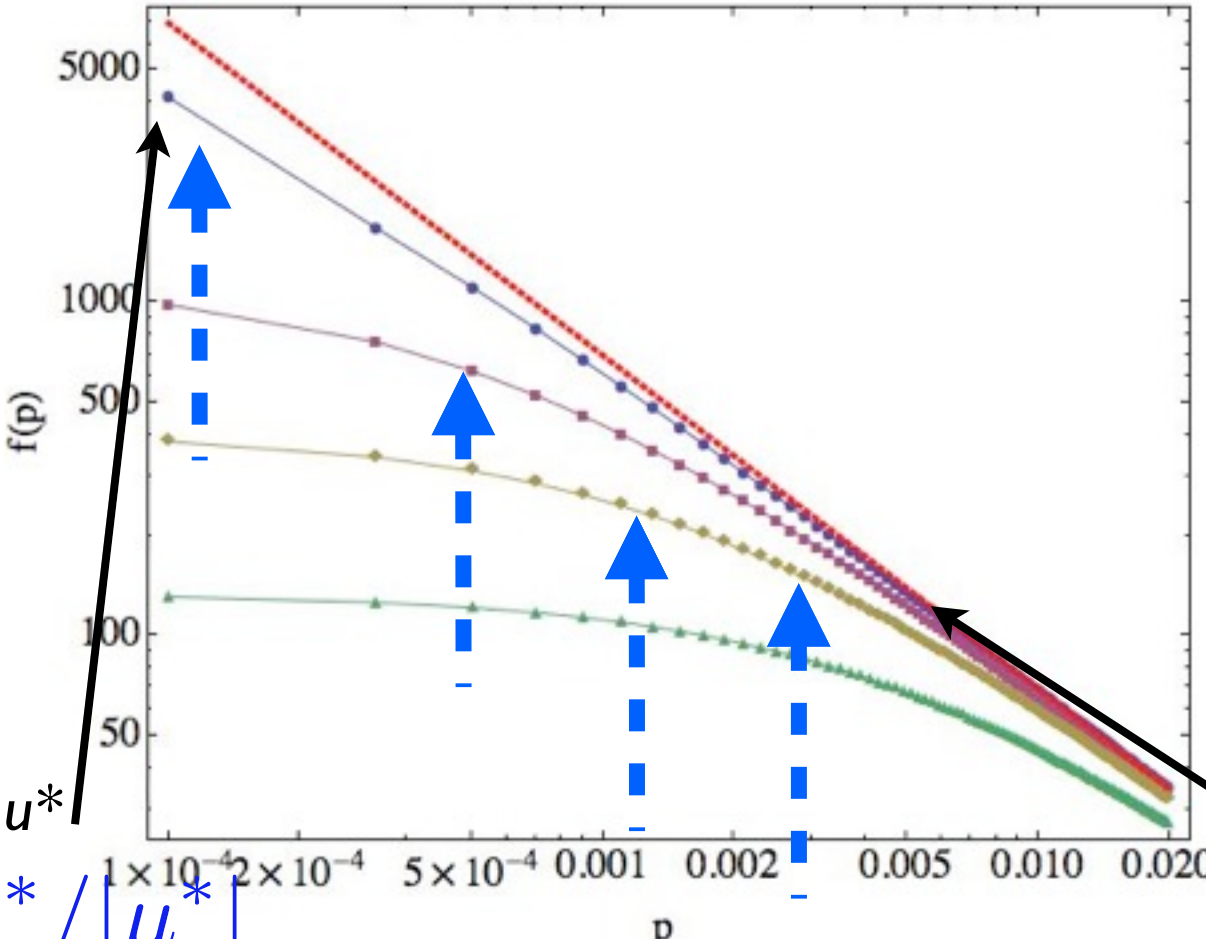
Overpopulated Case: How Onset of BEC Develops?



$\mu^* \to 0$

proceed in a self-similar scaling way

*p << Mu**

$f \to T^*/|\mu^*|$

*p >> Mu**

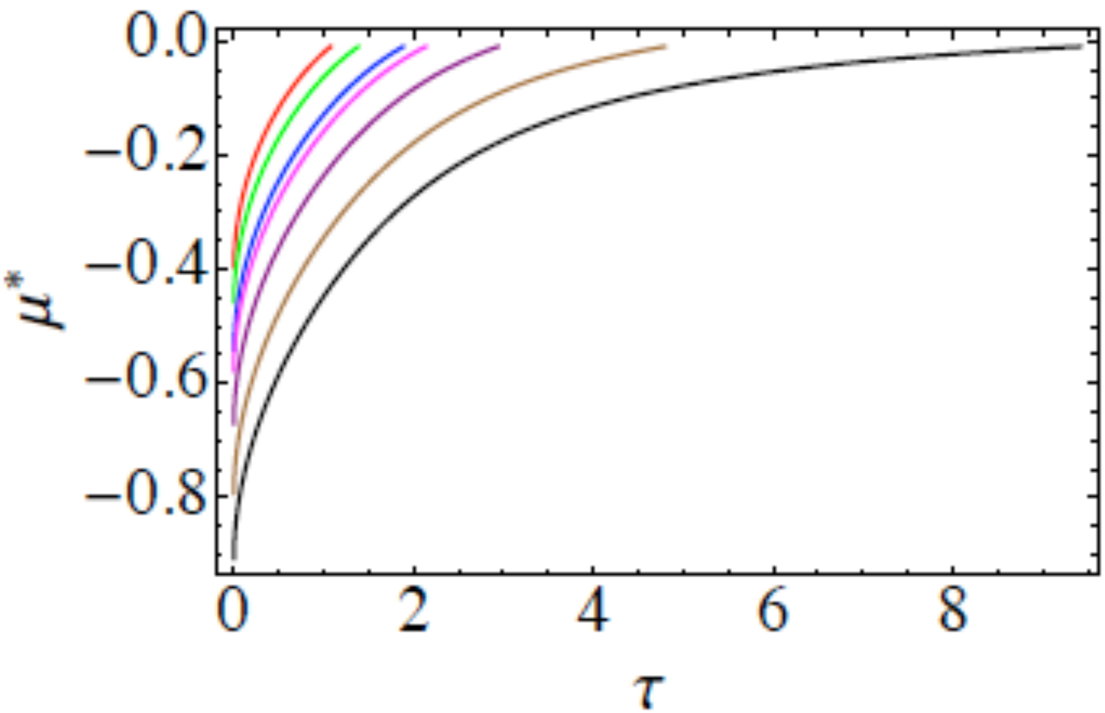
$f \to T^*/p$

$$f(p \to 0) \to \frac{T^*}{p - \mu^*}$$

Onset of Dynamical BEC

Onset of dynamical (out-of-equilibrium) BEC:

* occurring in a finite time
* local Mu* vanishes with a **scaling behavior**
* persistence of particle flux toward zero momentum



$$|\mu^*| = C(\tau_c - \tau)^\eta.$$

$$\eta \simeq 1$$

For different
f_0 = 0.2, 0.3, 0.5, 0.8, 1, 2, 5

Blaizot, JL, McLerran, 1305.2119, NPA2013

How Robust is the BEC Onset Dynamics?

There are a number of important aspects to explore about this dynamical process from initial overpopulation to the onset of BEC:

✦ How does that depend on the initial distribution shape?

--> the same onset dynamics (Blaizot, Liao, McLerran)

✦ How does that depend on a finite mass (e.g. from medium effect)

--> the same onset dynamics (Blaizot, Jiang, Liao)

✦ How is that influenced by the longitudinal expansion?

--> the same onset dynamics (Blaizot, Jiang, Liao, McLerran)

✦ How is that influenced by including quarks?

--> the same onset dynamics (Blaizot, Wu, Yan)

✦ How is that influenced by including inelastic collisions?

--> the same onset dynamics (Huang, Liao)

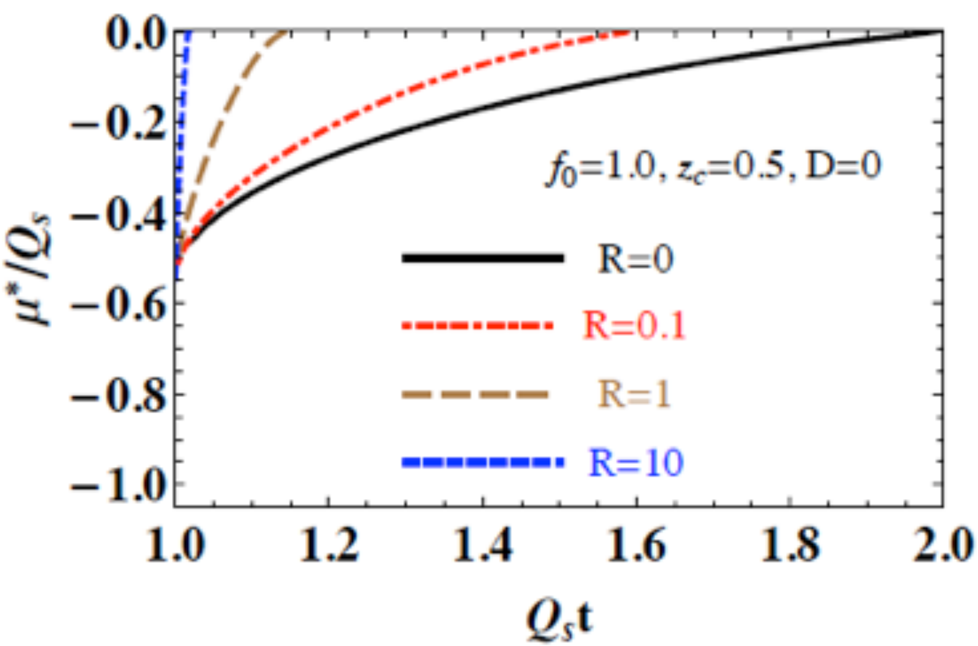
Including the Inelastic

An inelastic kernel including 2<-->3 processes
(Gunion-Bertsch, under collinear and small angle approxation)

$$\mathcal{D}_t f_p = \mathcal{C}^{\rm eff}_{2\leftrightarrow 2}[f_p] + \mathcal{C}^{\rm eff}_{1\leftrightarrow 2}[f_p],$$

Huang & JL, arXiv:1303.7214

$$\begin{aligned}\mathcal{C}^{\rm eff}_{1\leftrightarrow 2} &= \xi\,\alpha_s^2\,R\,\frac{I_a}{I_b}\left\{\int_0^{z_c}\frac{dz}{z}\left[g_p f_{(1-z)p} f_{zp} - f_p g_{(1-z)p} g_{zp}\right]\right. \\ &\left.+\int_0^{z_c}\frac{dz}{(1-z)^4 z}\left[g_p g_{zp/(1-z)} f_{p/(1-z)} - f_p f_{zp/(1-z)} g_{p/(1-z)})\right]\right\}\end{aligned}$$

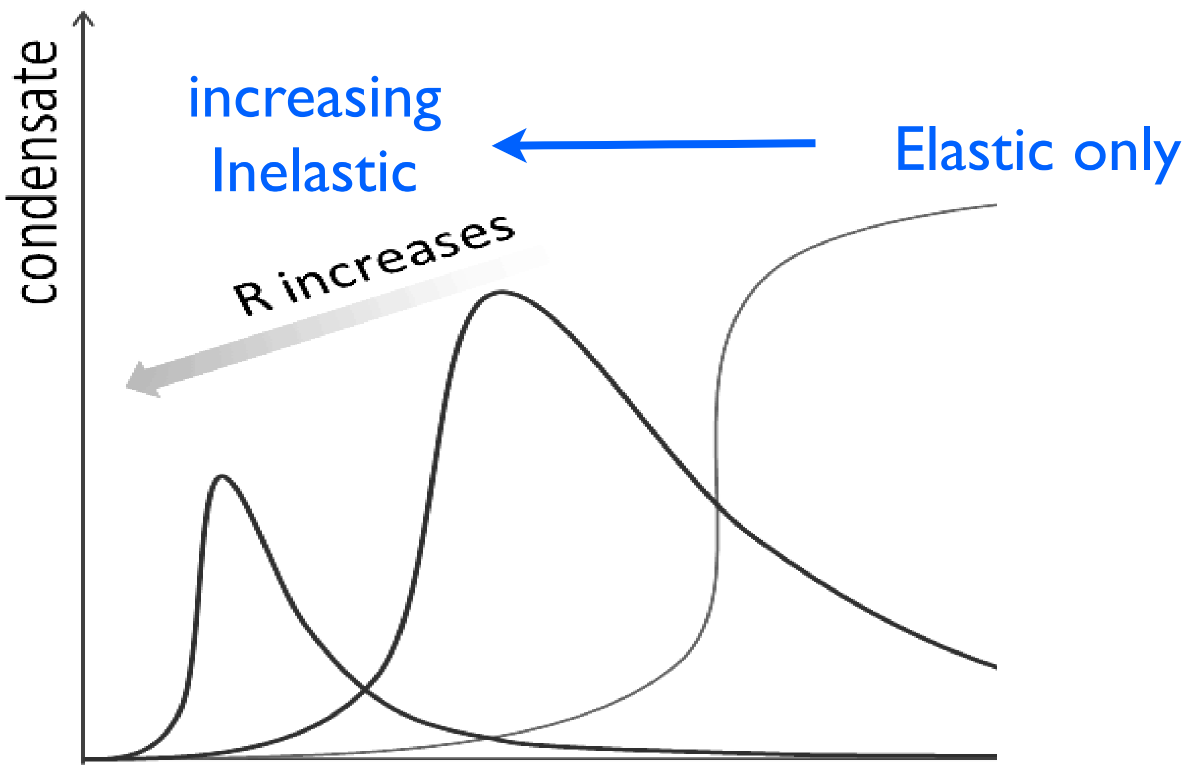


Local effect:
enhance IR growth,
accelerate the onset

Global effect:
reduce number density,
enhance entropy growth

The “Fuller” Picture

What we find: the inelastic process catalyzes
the onset of dynamical (out-of-equilibrium) BEC.
It might sound contradicting with common wisdom ...
but it is NOT.



Summary

* Initial gluon system at very early stage of a heavy ion collision is characterized by **high overpopulation.**

* Elastic process (alone) in highly overpopulated system can induce **very rapid growth of soft modes** and drive toward equilibration. This is a very robust feature and may lead to a transient **Bose-Einstein Condensate**.

* Inelastic processes may further enhance the rapid growth of soft modes and **catalyze the onset** of BEC (but will remove the condensate afterwards). The time window for a condensate could be sizable.

* Dynamical onset of BEC in a scaling way is found to be a very robust feature.

* An estimate with reasonable parameters indicate such phenomenon occurs on **a time scale 0.1 ~1 fm/c**, expected in early stage of heavy ion collisions.