The "ripples" on relativistically expanding fluid with applications in heavy-ion collisions

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Abstract

We present **complete** and **analytic** *sound wave* solutions on top of both *Bjroken* flow and *Hubble* flow background. We visualize the evolution pattens for *Gaussian* perturbation.

Motivation

- Hydrodynamics works well for QGP in relativistic heavy-ion collisions.
- Fluctuations, including initial and thermal ones, play important role in heavy-ion collisions.
- We study analytically how fluctuation evolves on top of known background.
- For high energy collisions at LHC and RHIC, the initial expansion is close to Bjorken flow, while the later expansion is close to Hubble flow.

Working Frame

- hydro equation: $T^{\mu\nu}{}_{;\mu} = 0.$
- ideal hydro: $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - g^{\mu\nu} p.$
- E.o.S. $\epsilon = c_s^{-2} p.$
- perturbation: $p = p_0 + p_1,\ u^\mu = u_0^\mu + u_1^\mu.$
- keep only 1st-order term $\Rightarrow$ linearize hydro equation.
- We solve the equations in the most convenient coordinate.

Coordinate

Bjorken:

$$t = \tau_z \cosh\eta_z \quad z = \tau_z \sinh\eta_z \quad x = \rho\cos\phi \quad y = \rho\sin\phi$$

Hubble:

$$t = \tau_r \cosh\eta_r \quad z = \tau_r \sinh\eta_r \cos\theta \quad x = \tau_r \sinh\eta_r \sin\theta\cos\phi \quad y = \tau_r\sinh\eta_r\sin\theta\sin\phi$$

background flow:

Bjorken: $p_0 \propto \tau_z^{-4/3}$, $u_0^\mu = (1,0,0,0)$

Hubble: $p_0 \propto \tau_r^{-4}$, $u_0^\mu = (1,0,0,0)$

(I) Perturbation on Top of Longitudinal Bjorken Wave

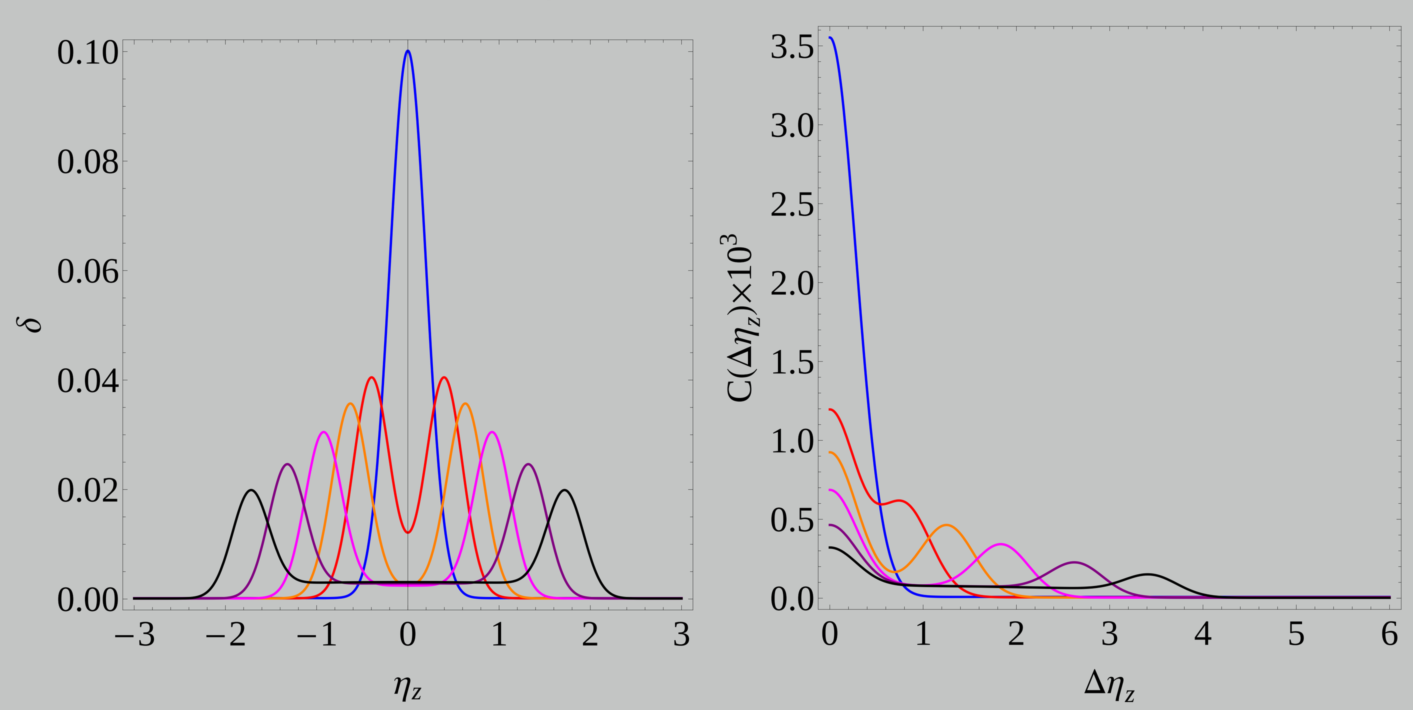
- We take Gaussian initial perturbation $\frac{\xi}{\sqrt{2\pi}\sigma} e^{-\frac{(\eta-\eta')^2}{2\sigma^2}}$ as an example.
- Evolution for pressure and velocity

$$\frac{p_1}{p_0} = \frac{\xi}{2\pi}\left(\frac{\tau'}{\tau}\right)^{\frac{1}{3}} \int_{-\infty}^{\infty} dk\ e^{-\frac{\sigma^2 k^2}{2}} \cos[k(\eta-\eta')] \cos[c_s\sqrt{k^2-\frac{1}{3}} \ln\frac{\tau}{\tau'}]\ .$$

$$u_1^\eta = \frac{\xi}{4\pi}\left(\frac{\tau'}{\tau}\right)^{\frac{4}{3}} \int_{-\infty}^{\infty} dk\ e^{-\frac{\sigma^2 k^2}{2}} \frac{\sin[k(\eta-\eta')]}{k} \times \left(\frac{1}{2}\cos[c_s\sqrt{k^2-\frac{1}{3}}\frac{\tau}{\tau'}] - \frac{\sqrt{3}\sqrt{k^2-\frac{1}{3}}}{2}\sin[c_s\sqrt{k^2-\frac{1}{3}}\ln\frac{\tau}{\tau'}]\right)\ .$$

- We define $\delta \equiv p_1/p_0$, and we can also calculate correlations

$$C(\Delta\eta_z) = \int \delta(\eta_z)\delta(\eta_z+\delta\eta_z)d\eta_z\ .$$

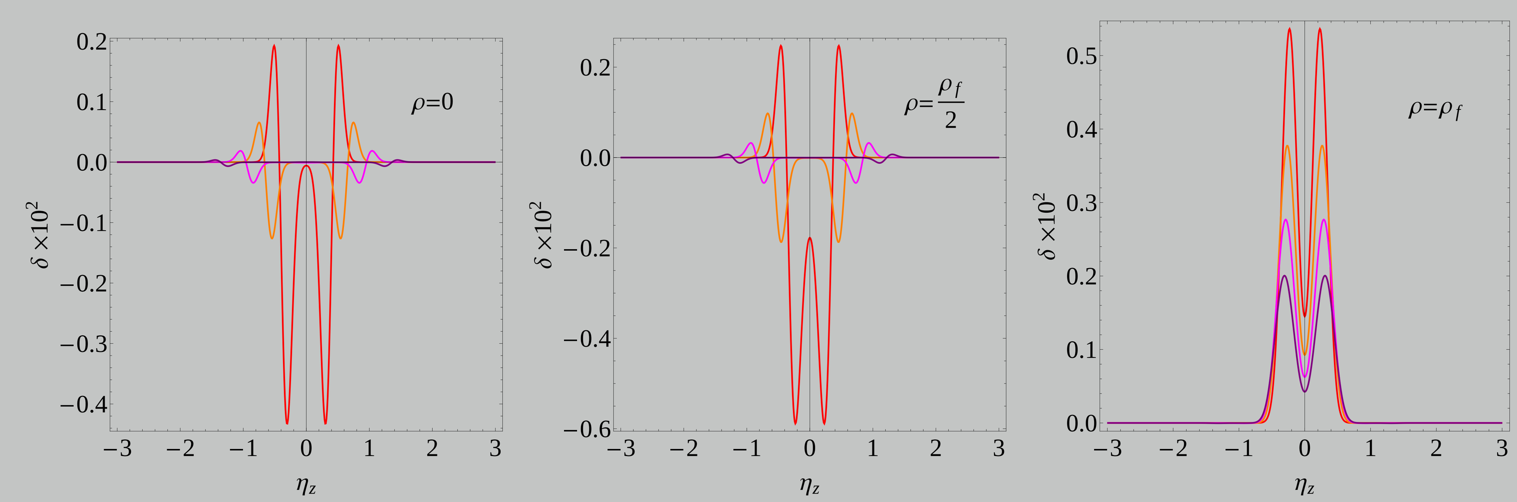


- curves in different color mean quantities at different τ.
- sound spreads out inside sound horizon and fills up the interval region.
- correlation patten shows a peak associated with the sound horizon generates together with the short-range peak.

(II) Perturbation on Top of General Bjorken Wave

- Evolution of Gaussian initial perturbation $\frac{\xi}{(2\pi)^{3/2}\sigma^3} e^{-\frac{\bar\eta^2+\bar\rho^2/\tau'^2}{2\sigma^2}}$:

$$\frac{p_1}{p_0} = \int_0^\infty d\omega \int_{-\infty}^{\infty} dk\ e^{ik\bar\eta} J_0(\omega\bar\rho) W_{\omega,k}(\tau)$$

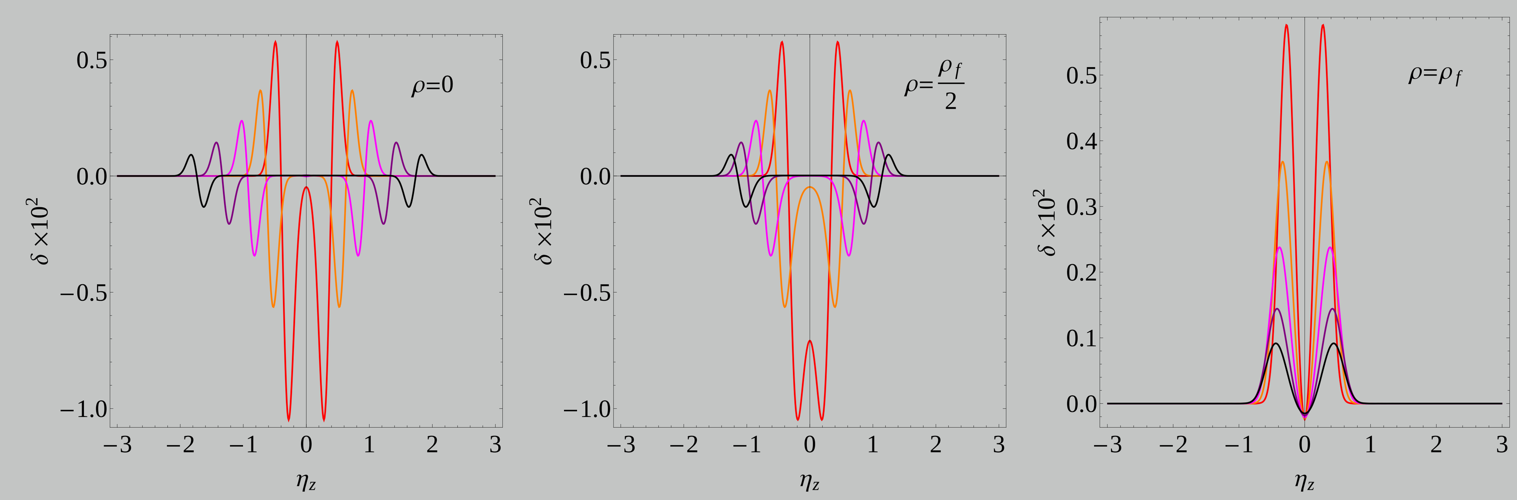


- $\rho_f = c_s(\tau-\tau')$ is the transverse wave-front.
- wave at transverse wave-front has nearly vanishing longitudinal velocity.

(III) Perturbation on Top of Hubble Wave

- Evolution of Gaussian initial perturbation $\frac{\xi}{(2\pi)^{3/2}\sigma^3} e^{-\frac{\bar\eta^2}{2\sigma^2}}$:

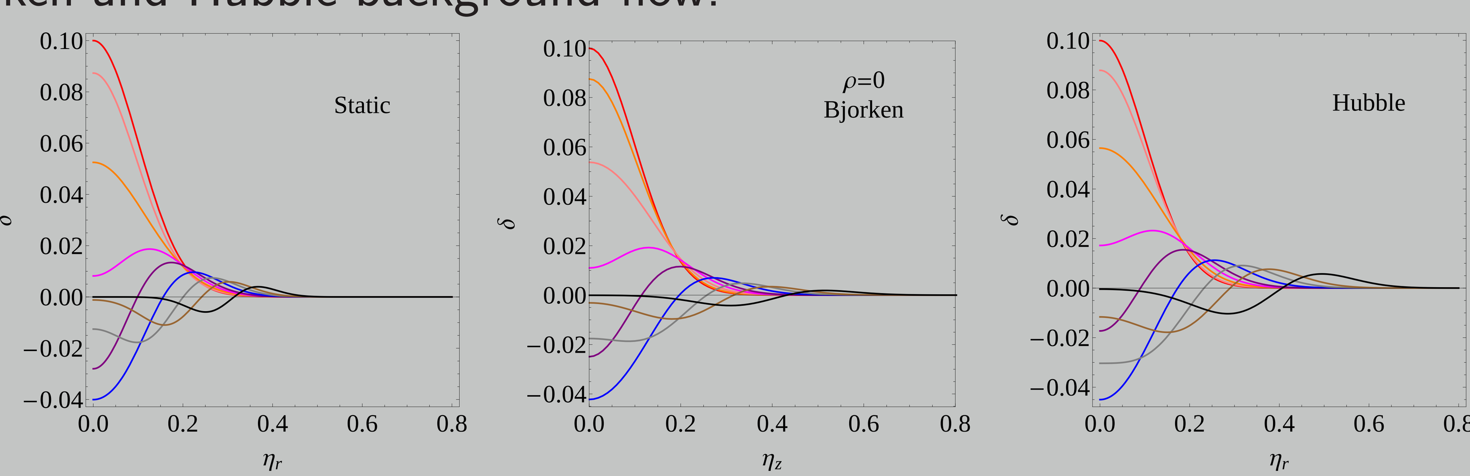
$$\frac{p_1}{p_0} = \frac{\xi}{4\pi^2}\int_{-\infty}^{\infty} \frac{\sin(k\sigma^2)}{k\sigma^2 e^{-\sigma^2/2}} e^{-\frac{\sigma^2 k^2}{2}} \frac{\sin(k\bar\eta)}{\sinh(\bar\eta)} \cos[\beta_k \ln\frac{\tau}{\tau'}] k\ dk\ .$$



- $\rho_f = \tau_r \sinh[c_s \ln(\tau/\tau')]$ is the transverse wave-front.
- negative perturbation is found in both 3D case, but not in longitudinal Bjorken case.

Early Evolution

- We visualize early evolution of Gaussian initial perturbation on top of static, Bjorken and Hubble background flow:



- Early evolution are similar to the static background as the initial local flow in the generating spot is weak.
- Background independent negative perturbation is seen in 3D flow.

Conclusion & Outlook

- Generally derive fluctuation propagation in different background.
- Explicitly show propagation for Gaussian perturbation.
- Visualize the patten of evolution.
- Further work on the application in heavy ion collision

References

J. I. Kapusta *et al*, Phys. Rev. C **85**, 054906 (2012)