Shear viscosity from a large-N_c NJL model at next-to-leading order

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Motivation

Heavy-ion collisions at RHIC and LHC are one possible approach to explore the QCD phase diagram [1].

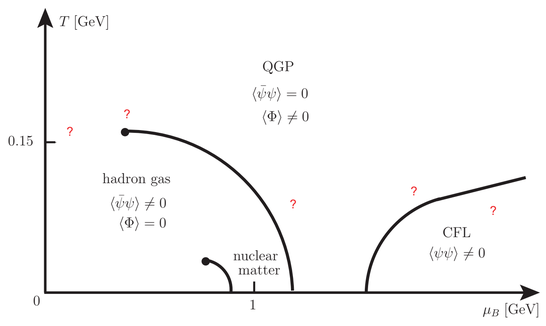


Figure: Sketch of the QCD phase diagram

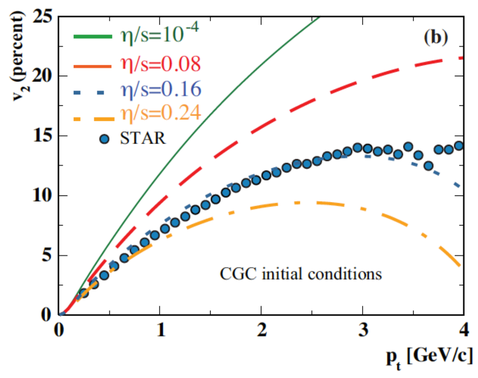


Figure: Extraction of a **constant** η/s from hydrodynamic simulations

The quark gluon plasma is an **almost perfect fluid** with a very small value of η/s[2].

Aim of this work: Calculate the thermal dependencies of the shear viscosity $\eta(T,\mu)$ within the NJL model.

NJL model and large-N_c expansion (e.g. [3, 4])

Two-flavor NJL Lagrangian: QCD $\oplus$ integrate all gluonic degrees of freedom

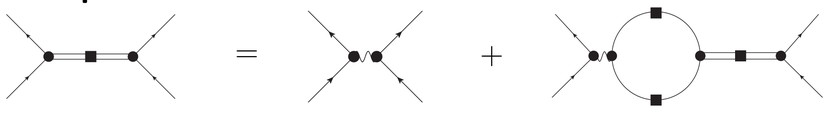
$$\mathcal{L}^{2f}_{\rm NJL} = \bar\psi\left(i\partial\!\!\!/ - m_0\right)\psi + \frac{G}{2}\left[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\vec\tau\psi)^2\right], \qquad \text{with } G\sim 1/N_c$$

Gap equation at Hartree level

$\mathcal{O}(1)$:

Bethe-Salpeter equation

$\mathcal{O}(\frac{1}{N_c})$:



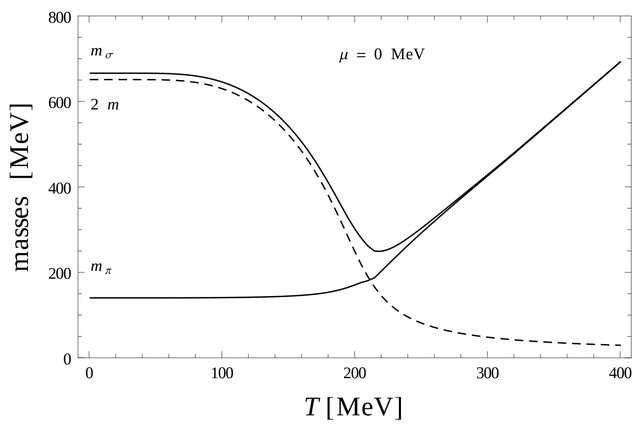


Figure: Thermal masses from the NJL gap equation for quarks and the BSE equation for mesons

Input parameters

$\Lambda = 651$ MeV
$m_0 = 5.50$ MeV
$G = 10.1$ GeV^{-2}

Values at $(T,\mu)=(0,0)$

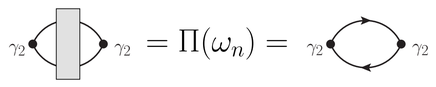
$\langle\bar\psi\psi\rangle = -(316\text{ MeV})^3$
$m_q = 325$ MeV
$m_\pi = 140$ MeV
$f_\pi = 94.0$ MeV

Shear viscosity from Kubo formalism[5, 6]: a non-perturbative approach

Kubo formula for the shear viscosity

$$\eta(\omega,\vec x,x_0) = \frac{1}{T}\int_0^\infty dt\, e^{i\omega t}\int d^3r\,\left(T_{21}(\vec r,t), T_{21}(\vec x,x_0)\right)$$

Organize the contributions to the **four-point correlator** by their large-N_c scalings[7, 8]:



$= \Pi(\omega_n) =$

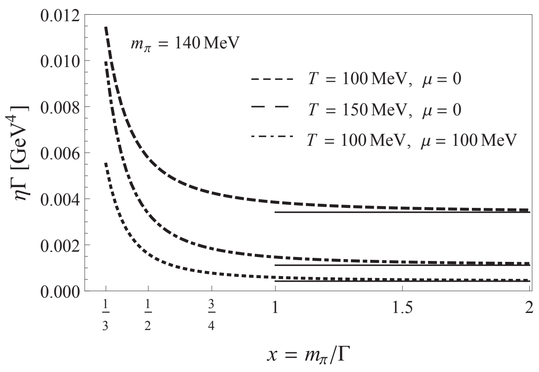
$+ \mathcal{O}(N_c^0)$.

static shear viscosity:

$$\eta = -\left.\frac{d\,\mathrm{Im}\,\Pi^R(\omega)}{d\omega}\right|_{\omega=0} = \frac{16N_cN_f}{15\pi^3T}\int_{-\infty}^{\infty}d\varepsilon\int_0^\infty dp\,p^6\,\frac{M^2\Gamma^2(p)\,n_F(\varepsilon)(1-n_F(\varepsilon))}{\left[\left(\varepsilon^2-p^2-M^2+\Gamma^2(p)\right)^2+4M^2\Gamma^2(p)\right]^2}$$

At very high T and also for $N_c\to\infty$ the spectral width becomes small, $\Gamma\to 0$:

$$\eta \to \frac{2N_cN_f}{15\pi^2T}\int_{|\epsilon|>M}d\epsilon\,\frac{(\epsilon^2-M^2)^{5/2}\,n_F(\varepsilon)(1-n_F(\varepsilon))}{M\,\Gamma(\sqrt{\epsilon^2-M^2})}\,.$$



$$\eta[\Gamma] = \frac{A_{-1}}{\Gamma} + A_0 + A_1\Gamma + A_2\Gamma^2 + \dots$$

$\Rightarrow$ NJL model is only for $\Gamma \ll m_\pi$ perturbative

Shear viscosity and strong cutoff effects

From a purely mathematical point of view, the shear viscosity $\eta[\Gamma(p)]$ does not require necessarily a momentum-cutoff[8]:

In order for the shear viscosity $\eta[\Gamma]$ as functional of $\Gamma(p)$ to be convergent, the asymptotic $\Gamma(p)$ should not converge too rapidly to zero:

$$\eta[\Gamma(p)] < \infty \quad\Leftrightarrow\quad p^3 e^{-\beta p/2} \in o(\Gamma(p))$$

$$\Gamma_{\rm const} = 100\text{ MeV}\,,$$
$$\Gamma_{\rm exp}(p) = \Gamma_{\rm const}\, e^{-\beta p/8}\,,$$
$$\Gamma_{\rm Lor}(p) = \Gamma_{\rm const}\,\frac{\beta p}{1+(\beta p)^2}\,,$$
$$\Gamma_{\rm div}(p) = \Gamma_{\rm const}\sqrt{\beta p}\,.$$

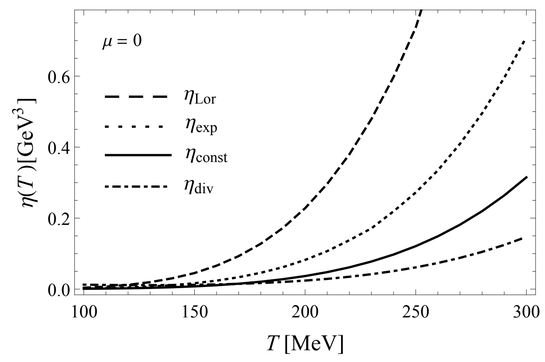


Figure: Shear viscosity **without** a cutoff

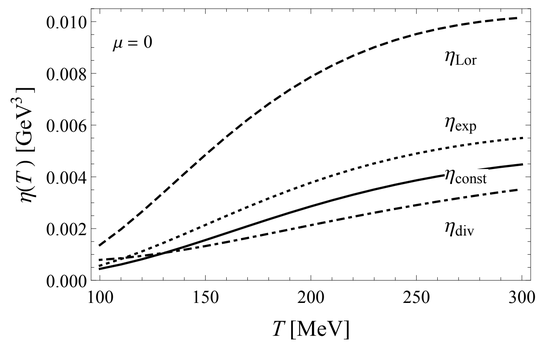
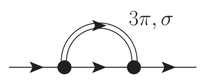


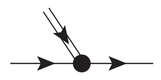
Figure: Shear viscosity **with** a physical NJL cutoff

Leading-order results[8]: mesonic fluctuations

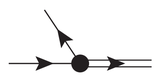
Quark self energy and **quark spectral width**

$\Sigma^{\pi,\sigma}_\beta(\vec p,\nu_n) =$





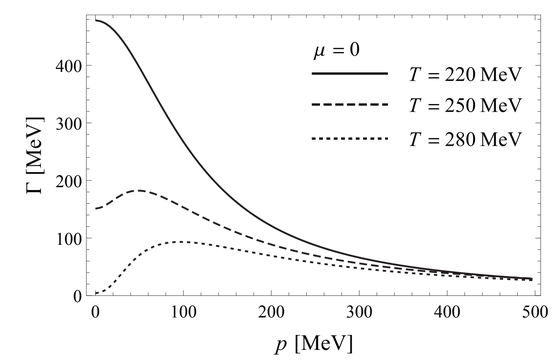
Landau damping

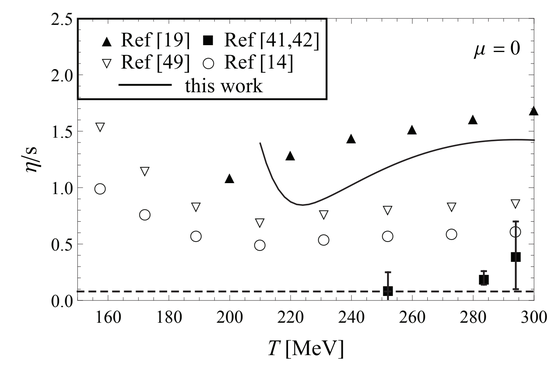


recombination

$$\Rightarrow \Gamma^{\pi,\sigma}_q(p_0,\vec p) := -\lim_{\varepsilon\to 0}\mathrm{Im}\,\Sigma^{\pi,\sigma}_\beta(\vec p,-ip_0+\varepsilon) = \frac{\mp 2\,m g^2_{Mqq}}{4\pi|\vec p|}\int_{E_{\min}}^{E_{\max}} dE_f\left[n_B(E_b)+n_F^-(E_f)\right]$$

with $E_{\min,\max}(p) = \frac{1}{2m^2}\left[\left(m_M^2-2m^2\right)\sqrt{m^2+p^2} \mp p\,m_M\sqrt{m_M^2-4m^2}\right]$





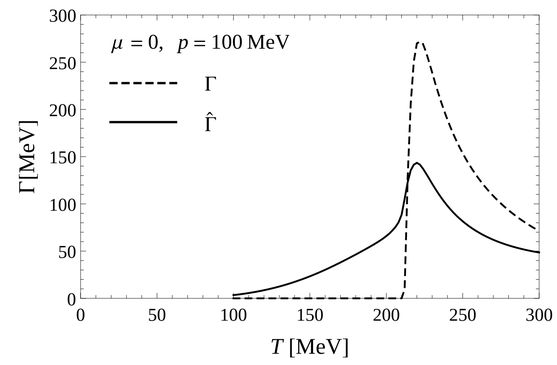
Next-to-leading order results $\mathcal{O}(N_c^{-1})$: mesonic decays

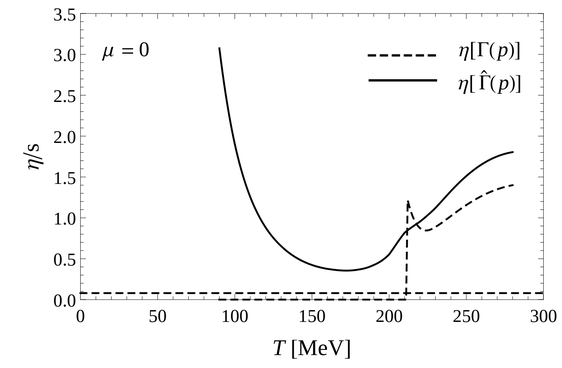
Mesonic spectral function

$$\rho(\omega) = g^2_{Mqq}\,\delta(\omega^2-m_M^2) + \frac{1}{\pi}\frac{4G^2N_c\omega^2\mathrm{Im}\,I_2(-i\omega)}{\left(\frac{m_0}{m}-4GN_c\omega^2\mathrm{Re}\,I_2(-i\omega)\right)^2+\left(4GN_c\omega^2\mathrm{Im}\,I_2(-i\omega)\right)^2}$$

Off-shell mesonic decays via convolution of Γ and ρ:

$$\hat\Gamma^{S/P}_q(p) = \mathcal{N}\int_{4m^2}^{4(m^2+\Lambda^2)} d\omega^2\;\Gamma^{S/P}_q(p)\Big|_{m_M\mapsto\omega}\;\rho(\omega)$$





Flat η/s around $T\approx 170-180$ MeV $\approx T_c$

References and Acknowledgement

[1] U. Heinz, Ch. Shen, and H. Song, AIP. Conf. Proc. **1441**, 766 (2012).
[2] H. Niemi, G.S. Denicol, P. Huovinen, E. Molnár, and D.H. Rischke, Phys. Rev. Lett. **106**(21), 212302 (2011).
[3] U. Vogl and W. Weise, Progr. Part. Nucl. Phys. **27** 195 (1991).
[4] M. Oertel, M. Buballa, and J. Wambach, Nucl. Phys. A **676** 247 (2000).
[5] R. Kubo, J. Phys. Soc. Jpn. **12**(6), 570 (1957).
[6] S. Jeon, Phys. Rev. D **52**, 3591 (1995).
[7] M. Iwasaki, H. Ohnishi, and T. Fukutome, Prog. Theor. Phys. **119**, 991–1004 (2008).
[8] R. Lang and W. Weise, Eur. Phys. J. A **50**, 63 (2014).
[9] S. Plumari, A. Puglisi, F. Scardina, and V. Greco, Phys. Rev. C **86**, 054902 (2012).
[10] Ch. Sasaki and K. Redlich, Nucl. Phys. A **832**(1–2), 62–75 (2010).
[11] S. Ghosh et al., Phys. Rev. C **86**, 068201 (2013).
[12] A. Nakamura, Phys. Rev. D **76**, 101701(R) (2007).
[13] H.B. Meyer, Phys. Rev. D **76**, 101701(R) (2007).

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