Second order hydro-coefficients from lattice QCD

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Introduction

The properties of Abelian and non-Abelian plasmas are the subject of intensive experimental and theoretical investigation. It is well-known that the electric force is screened by the quasi-free charges of a plasma. The static force between currents however remains unscreened. The coupling constants of both forces are enhanced at long distances compared to the vacuum. In a medium the Coulomb and Ampère forces receive a factor $(1+e^2\kappa_l)$ and $(1+e^2\kappa_t)$ to their intensities. The quantity κ_t enters the constitutive equation of the electric current at second order in a 'hydrodynamic' description. It describes the enhancement of the Ampère force. We compute κ_l, κ_l and the combination $(\kappa_l - \kappa_t)$ using *ab initio* lattice QCD simulations. We update our recent analysis [1] by increasing statistics and adding new lattice ensembles, covering a large temperature region around the deconfinement cross-over.

Definition of κ_l, κ_l and $(\kappa_l - \kappa_t)$

At finite temperature $T \equiv 1/\beta$, the tensor decomposition of the polarization tensor into a transverse $\Pi_T(q_0^2,\vec{q}^2)$ and longitudinal $\Pi_L(q_0^2,\vec{q}^2)$ part reads

$$\Pi_{\mu\nu}(q) = P^L_{\mu\nu}(q)\,\Pi_L(q_0^2,\vec{q}^2) + P^T_{\mu\nu}(q)\,\Pi_T(q_0^2,\vec{q}^2).$$

The 'matter' part is

$$\Pi_{\mu\nu}(q) \equiv \Pi^{\rm vac}_{\mu\nu}(q) + \Pi^{\rm mat}_{\mu\nu}(q),$$
$$\Pi^{\rm mat}_L(q) \equiv \Pi_L(q) - q^2\Pi^{\rm vac}(q^2),$$
$$\Pi^{\rm mat}_T(q) \equiv \Pi_T(q) - q^2\Pi^{\rm vac}(q^2).$$

With these conventions, we may define:

$$\kappa_t = \frac{\partial}{\partial(\vec{q}^2)}\Pi^{\rm mat}_T(0,\vec{q}^2)\Big|_{\vec{q}=0}, \qquad (1)$$

$$\kappa_\ell = \frac{\partial}{\partial(\vec{q}^2)}\Pi^{\rm mat}_L(0,\vec{q}^2)\Big|_{\vec{q}=0}. \qquad (2)$$

2nd order hydro-coeff. from lattice

In lattice QCD Eqs. (1,2) lead to definitions of κ_l, κ_l and $(\kappa_l - \kappa_t)$ in terms of integrals over lattice correlation functions:

$$\kappa_t = -\int_0^\infty dx\, x^2\,\Delta G_t(x,T), \qquad (3)$$
$$\Delta G_t(x,T) \equiv G_t(x,T) - G(x,0),$$

$$\kappa_l = -\int_0^\infty dx\, x^2\,\Delta G_l(x,T), \qquad (4)$$
$$\Delta G_l(x,T) \equiv G_l(x,T) - G(x,0),$$

$$\kappa_l - \kappa_t = -\int_0^\infty dx\, x^2\,\Delta G_{l-t}(x,T), \qquad (5)$$
$$\Delta G_{l-t}(x,T) \equiv G_l(x,T) - G_t(x,T),$$

whereby the correlators $G_l(x,T)$, $G_t(x,T)$ and $G(x,0)$ are defined in Eqs. (6-8).

More details in

[1] B. B. Brandt, A. Francis and H. B. Meyer, "Antiscreening of the Ampere force in QED and QCD plasmas," Phys. Rev. D **89** (2014) 034506

Numerical setup

All numerical results were obtained on a number of lattice ensembles using the Wilson-Clover action with fixed bare parameters. This enables a temperature scan by varying the temporal extent of the lattice.

$6/g_0^2$	5.50	m_π[MeV]	270
κ	0.13671	Z_V	0.768(5)
c_{SW}	1.751496	a[fm]	0.0486(4)(5)

$N_t \times 64^3$	N_{conf}	N_{src}	T[MeV]
128	132	4	32(1)
24	206	64	169(3)
20	297	64	203(4)
16	313	65	253(4)
12	269	65	338(6)

Note: New ensembles ($N_t = 24, 20, 12$) and increased statistics ($N_{src} > 1$) compared to [1]

Lattice correlators

We compute the local-conserved vector correlation functions:

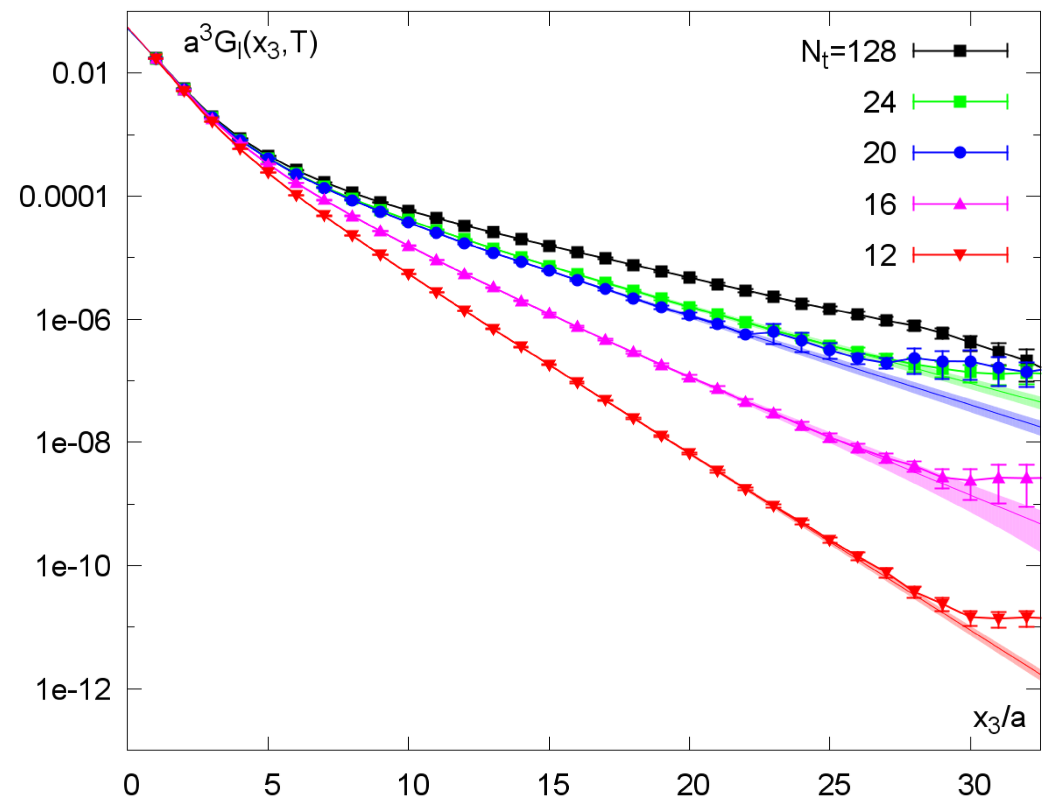
$$G(x_0) = \frac{1}{3}\,a^3\,Z_V\sum_{(\vec{x})}\langle J^c_k(x)J^\ell_k(0)\rangle \qquad (6)$$

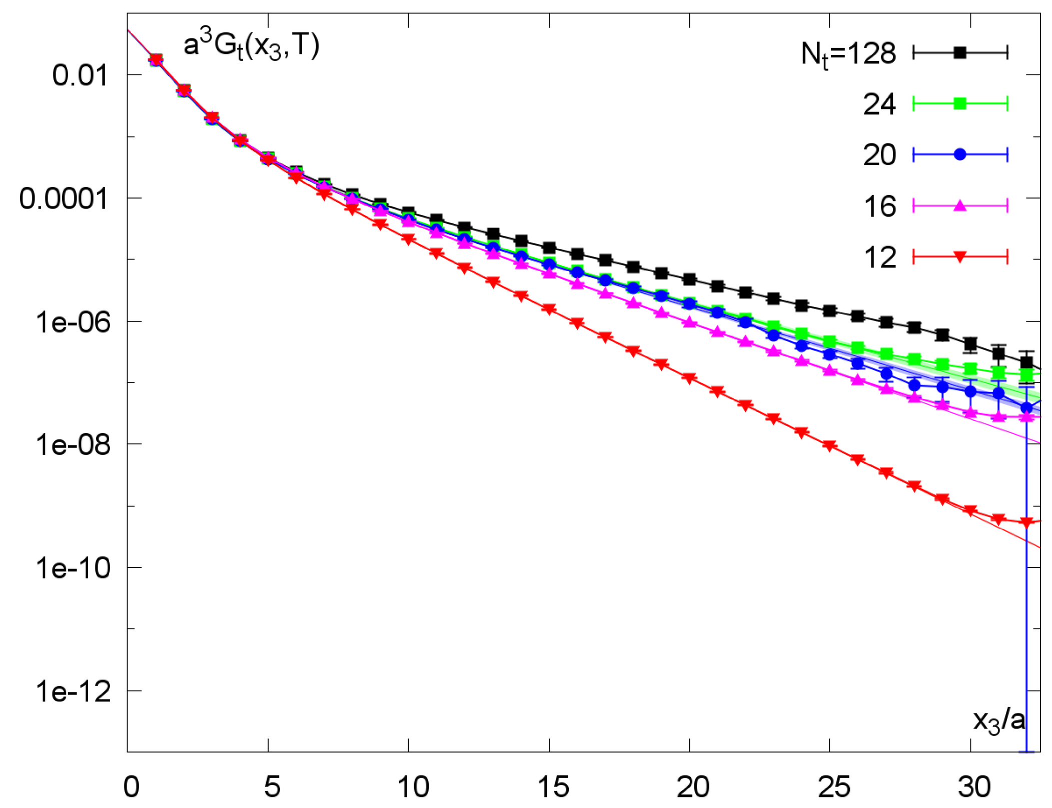
$$G_t(x_3) = \frac{1}{2}\,a^3\,Z_V\sum_{(\vec{x})}\langle J^c_i(x)J^\ell_i(0)\rangle \qquad (7)$$

$$G_l(x_3) = a^3 Z_V\sum_{(\vec{x})}\langle J^c_0(x)J^\ell_0(0)\rangle \qquad (8)$$

$$\text{with: } J^l_\mu(x) = \bar{q}(x)\gamma_\mu q(x)$$
$$J^c_\mu(x) = -\frac{1}{2}\Big(\bar{q}(x+a\hat{\mu})(1+\gamma_\mu)U^\dagger_\mu(x)q(x) - \bar{q}(x)(1-\gamma_\mu)U_\mu(x)q(x+a\hat{\mu})\Big).$$

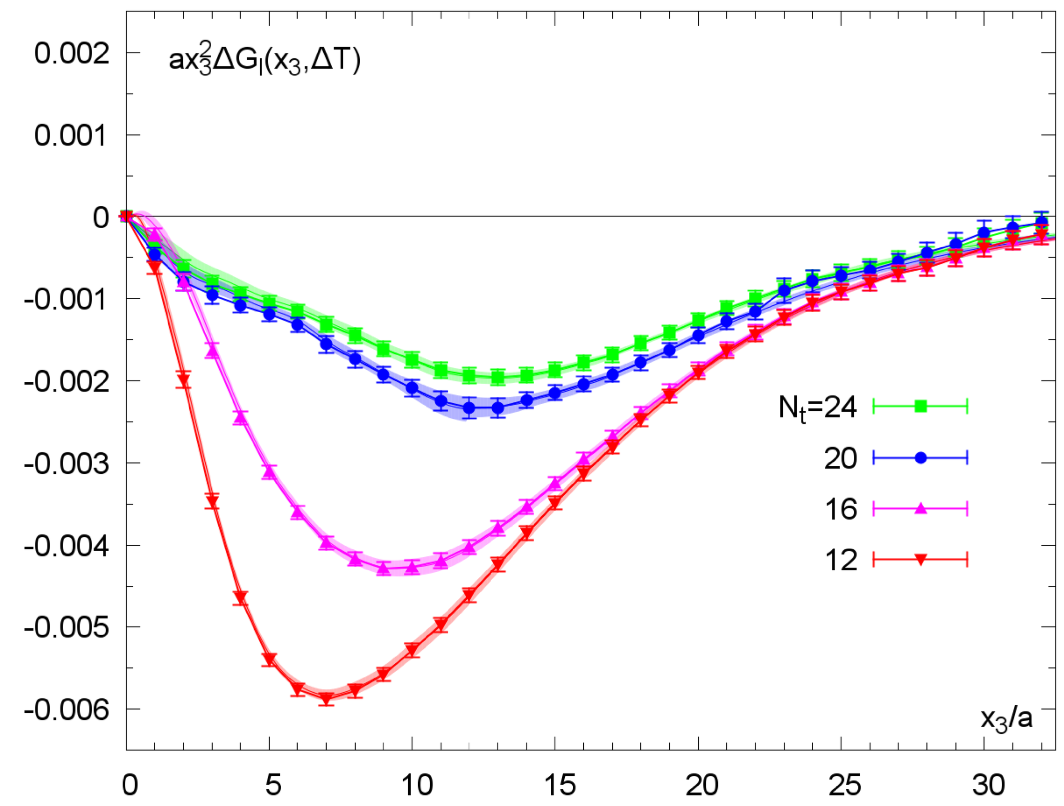
For the long tail of the integrations in Eqs. (3-5) the data is extended by a simple exponential Ansatz (shaded bands):

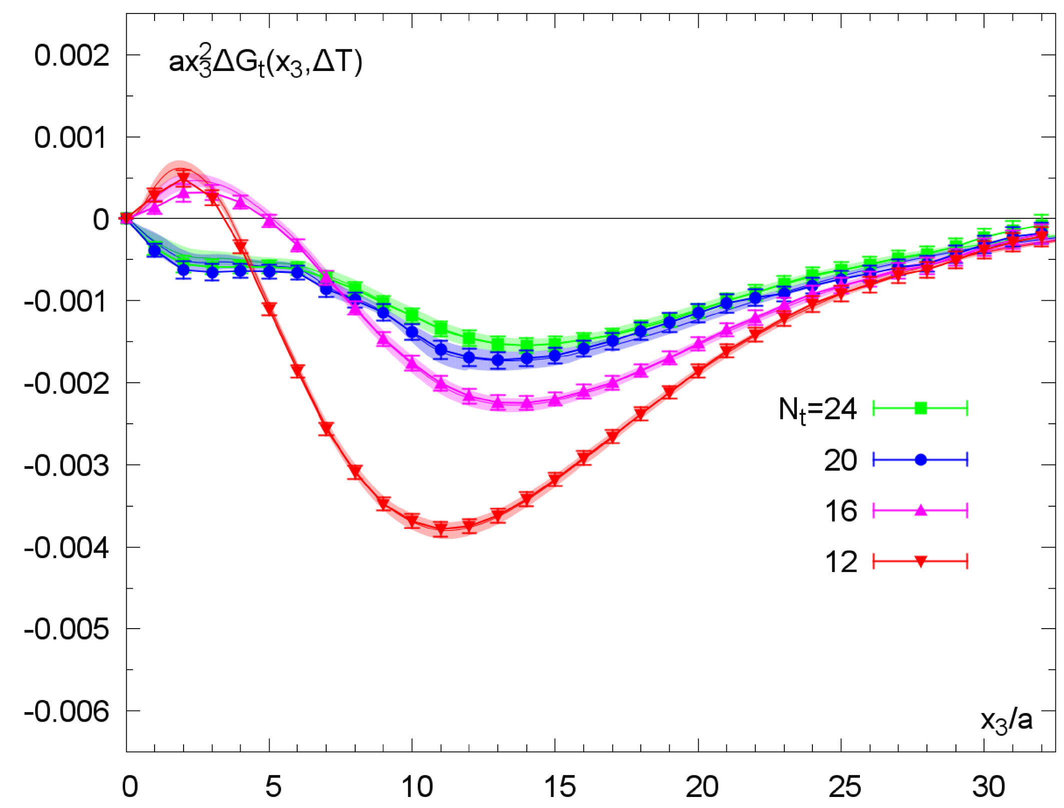


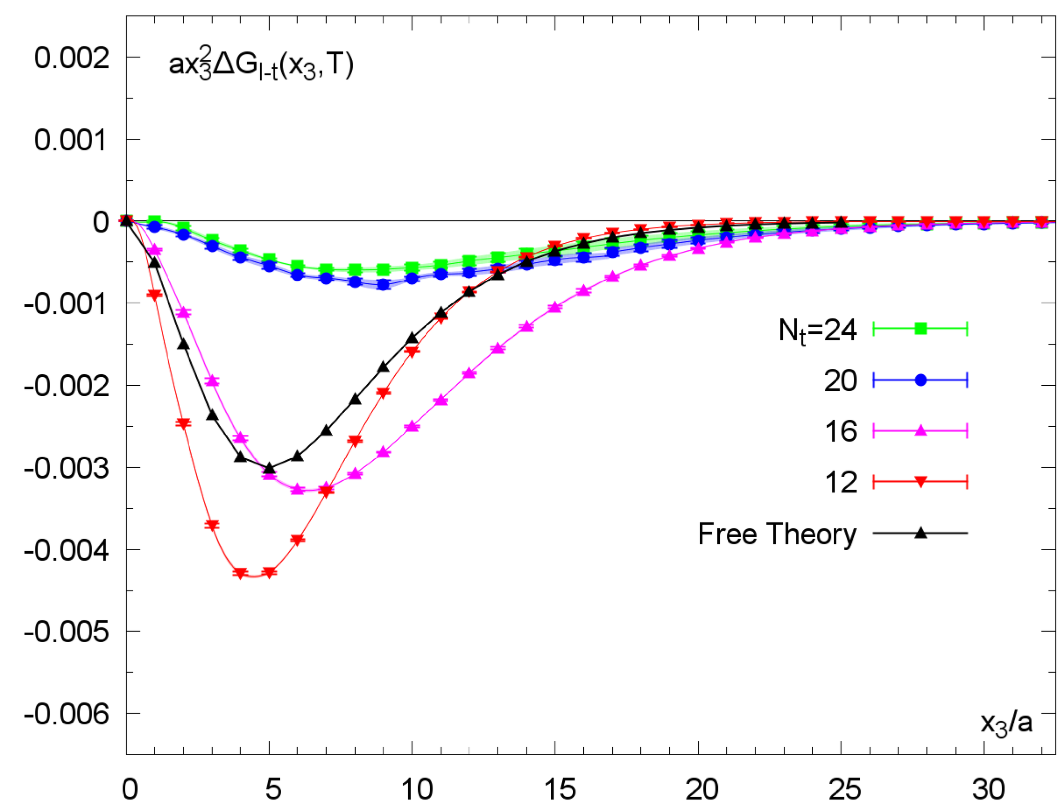


Integrands for κ_l, κ_l and $(\kappa_l - \kappa_t)$

To compute κ_l, κ_l and $(\kappa_l - \kappa_t)$ we form the appropriate differences of lattice correlators and fold with x^2 to obtain the integrands of Eqs. (3-5). The subsequent numerical integration yields our final results.







Results and summary

N_t	κ_t	κ_l	$(\kappa_l - \kappa_t)$
24	0.0283(15)	0.0365(10)	0.0082(9)
20	0.0318(15)	0.0425(12)	0.0107(7)
16	0.0343(14)	0.0701(14)	0.0358(2)
12	0.0557(14)	0.0889(14)	0.0323(1)
free			0.0253
16[1]	0.0400(39)	0.0750(34)	0.0350(24)

- We computed κ_l, κ_l and $(\kappa_l - \kappa_t)$ using lattice QCD with very high accuracy.
- We updated our recent analysis [1] by increasing statistics.
- We extended the covered temperature range across the cross-over region, $169 \lesssim T \lesssim 339$MeV.