Introduction

Measurements of the elliptic flow variable $v_2$ in heavy-ion experiments at RHIC and CERN suggest that the quark-gluon plasma (QGP) exhibits almost perfect fluidity. This is reflected by a value of the ratio of shear viscosity over entropy density, $\eta/s$, closely above the universal lower bound $\eta/s_{\text{KSS}} = 1/4\pi$ found in AdS-CFT [1]. As a consequence, the QGP can be described well by hydrodynamics, a formulation which is expressed in terms of transport coefficients. Transport coefficients, like the shear viscosity, can be obtained from the spectral function of the energy-momentum tensor via Kubo relations [2]. Here, in the genuinely non-perturbative regime around the confinement-deconfinement transition, so far, only Euclidean correlation functions can be computed accurately. The spectral function, $\rho(\omega, p)$ (frequency $\omega$, spatial momentum $p$), is then obtained by inverting an integral equation containing the Euclidean correlator, see eq. (1) below.

\[ G(\tau, p) = \int_0^\infty \frac{d\omega}{2\pi} K_1(\omega) \rho(\omega, p) = (1 + n(\omega)) e^{-\omega\tau} + n(\omega)e^{\omega\tau}, \]  

with the thermal Bose-distribution $n(\omega) = 1/[e^{\omega/T} - 1]$. (1)

By inversion of eq. (1) we compute $\rho$, however, this inversion is not unique. MEM is based on Bayes' theorem in probability theory, hence, it provides a method to include given knowledge (in the present case the perturbative ultraviolet behaviour) about the SPF in the inversion of eq. (1) directly. By construction, the standard definition of MEM allows for positive SFs only. Gluons, however, do not have this property, as they violate positivity. We overcome this obstacle by writing the SPF as a difference of two positive model functions $\rho$, and $s$, i.e. $\rho(\omega, p) = s(\omega, p) - s(\omega, p)$. The dependence of final results on the particular choice of splittings turns out to be very weak.

Spectral functions

From a dispersion relation we find the relation between the Euclidean propagator, $G(\tau, p)$ for imaginary time $\tau$, and the SPF, $\rho(\omega, p)$,

\[ G(\tau, p) = \int_0^\infty \frac{d\omega}{2\pi} K_1(\omega) \rho(\omega, p) = (1 + n(\omega)) e^{-\omega\tau} + n(\omega)e^{\omega\tau}, \]  

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The distinct features are a broad maximum at $\omega/T \approx 2 - 3$ and violation of positivity at small momenta, as expected. At larger momenta the peak smears out and approaches the line $\omega = p$. With increasing temperature the peak broadens further.

Viscosity

A Kubo relation relates shear viscosity, $\eta$, to the slope of the spectral function $\rho_{\text{sp}}$ of the spatial, traceless part of the energy-momentum tensor $\pi_{ij}$ at vanishing frequency,

\[ \eta = \lim_{\omega \to 0} \frac{\rho_{\text{sp}}(\omega, 0)}{\omega}, \]  

with $\rho_{\text{sp}}(\omega, p) = \text{Fourier-tr.}\{[(\pi_0(x), \pi_0(0))]\}'. (2)

We decompose the correlator of the energy-momentum tensor in eq. (2) and express it in terms of propagators, $G_{ij}$, and field derivatives, leaving an equation involving two classes of diagrams. The first class is given in figure 2: it consists of one- to three-loop gluon propagators connecting the two co-gluon vertices of the energy-momentum tensor, $\pi_{ij}$. The other class can be written as effective vertex corrections, see figure 3.

Figure 1: Spectral function of chromomagnetic gluon (chromoelectric qualitatively similar).

Figure 2: Class 1: $\pi_{ij}$ are connected by $G_{ij}$.

Figure 3: Class 2: effective vertex corrections.

At temperatures close to the critical value we can construct a renormalisation scheme such that contributions from higher loop orders are minimised [6]. This scheme was confirmed in a full 2-loop computation [7]. As a consequence, in the regime considered here the SPF of the energy-momentum tensor is approximated well by the one-loop truncation of figure 2. Finally, with the Kubo relation given in eq. (2), shear viscosity can be computed in terms of gluon SFs only.

Outlook

The inclusion of dynamical quarks is straightforward and allows for a generalisation to full QCD. This provides direct access to non-equilibrium observables and, hence, phenomenological studies that permit direct comparison with experimental results from heavy-ion collisions. In particular, our focus is on a description of jet quenching. For this, we compute quark and gluon spectral functions also in the real time formalism directly.