Gluon spectral functions and transport coefficients in Yang–Mills theory

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Introduction

Measurements of the elliptic flow variable v_2 in heavy-ion experiments at RHIC and CERN suggest that the quark-gluon plasma (QGP) exhibits almost perfect fluidity. This is reflected by a value of the ratio of shear viscosity over entropy density, η/s, closely above the universal lower bound $(\eta/s)_{\text{KSS}} = 1/(4\pi)$ found in AdS-CFT [1]. As a consequence, the QGP can be described well by hydrodynamics, a formulation which is expressed in terms of transport coefficients.
Transport coefficients, like the shear viscosity, can be obtained from the spectral function of the energy-momentum tensor via Kubo relations [2]. However, in the genuinely non-perturbative regime around the confinement-deconfinement transition, so far, only Euclidean correlation functions can be computed accurately. The spectral function, $\rho(\omega, p)$ (frequency ω, spatial momentum p), is then obtained by inverting an integral equation containing the Euclidean correlator, see eq. (1) below.

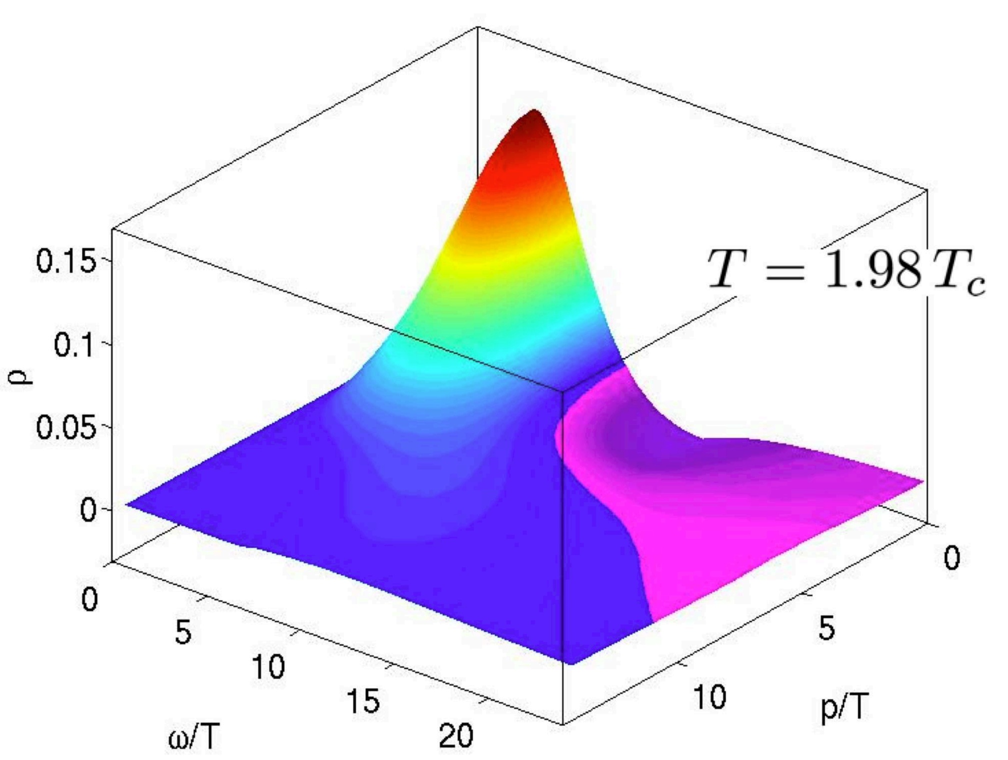
We focus on temperatures around the confinement-deconfinement phase transition, $T = 0.4 \div 4.5\,T_c$, where T_c is the critical temperature. We compute gluon spectral functions (SPFs) from non-perturbative, Euclidean propagators in Landau gauge finite temperature Yang–Mills theory [3]. Gluons show positivity violations, hence, their SPFs can be negative. We derive a generalised version of the standard maximum entropy method (MEM) [4] that allows for negative parts in SPFs. Via a Kubo relation, we compute shear viscosity from a closed expression in terms of SPFs. At $T = 1.25\,T_c$ the minimal value of $(\eta/s)_{\text{min.}} = 0.115$ closely above $(\eta/s)_{\text{KSS}}$ we recover that close to criticality the system is an almost perfect fluid. This presentation is based on [5].

Spectral functions

From a dispersion relation we find the relation between the Euclidean propagator, $G(\tau, p)$ for imaginary time τ, and the SPF, $\rho(\omega, p)$,

$$G(\tau,\, p) = \int_0^\infty \frac{d\omega}{2\pi} K_T(\tau, \omega)\,\rho(\omega, p)\,,\quad K_T(\tau,\,\omega) = (1 + n(\omega))\,e^{-\omega\tau} + n(\omega)e^{\omega\tau}\,, \qquad (1)$$

with the thermal Bose-distribution $n(\omega) = 1/\left(e^{\omega/T} - 1\right)$.
By inversion of eq. (1) we compute ρ, however, this inversion is not unique. MEM is based on Bayes' theorem in probability theory, hence, it provides a method to include given knowledge (in the present case the perturbative ultraviolet behaviour) about the SPF in the inversion of eq. (1) directly. By construction, the standard definition of MEM allows for positive SPFs only. Gluons, however, do not have this property, as they violate positivity. We overcome this obstacle by writing the SPF as a difference of two positive model functions ρ_s and s, i.e. $\rho(\omega, p) = \rho_s(\omega, p) - s(\omega, p)$. The dependence of final results on the particular choice of splittings turns out to be very weak.



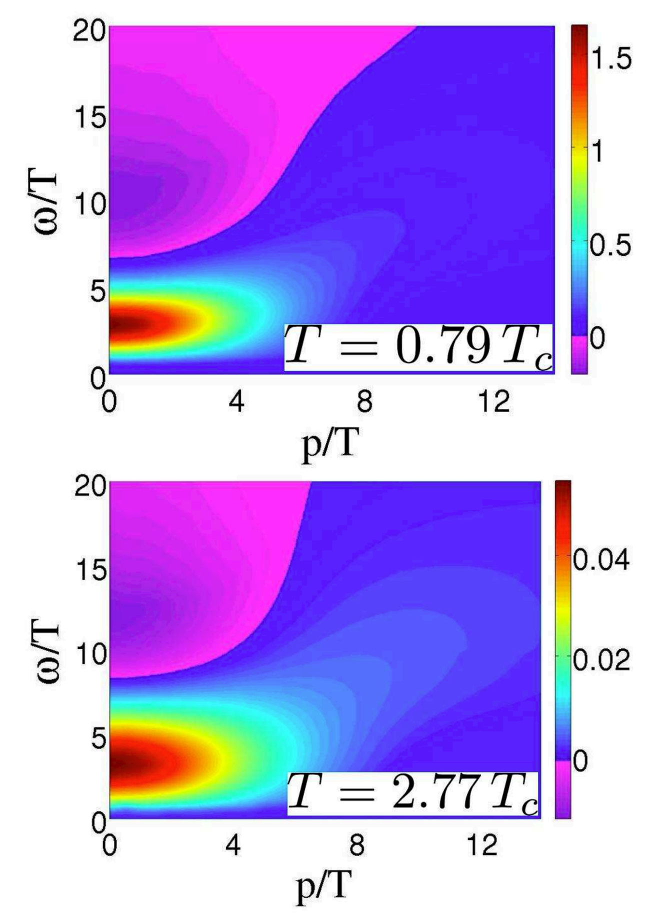


Figure 1: Spectral function of chromomagnetic gluon (chromoelectric qualitatively similar).

Figure 1 shows MEM-results for chromomagnetic gluon SPFs from Euclidean propagators, chromoelectric SPFs are qualitatively similar. The Euclidean propagators were computed with the functional renormalisation group [3], and are in quantitative agreement with results from lattice gauge theory.

The distinct features are a broad maximum at $\omega/T \approx 2 \div 3$ and violation of positivity at small momenta, as expected. At larger momenta the peak smears out and approaches the line $\omega = p$. With increasing temperature the peak broadens further.

Viscosity

A Kubo relation relates shear viscosity, η, to the slope of the spectral function $\rho_{\pi\pi}$ of the spatial, traceless part of the energy-momentum tensor π_{ij} at vanishing frequency,

$$\eta = \lim_{\omega\to 0} \frac{1}{20}\frac{\rho_{\pi\pi}(\omega, 0)}{\omega}\,, \text{ with } \rho_{\pi\pi}(\omega, p) = \text{Fourier-tr.}\left\{\langle[\pi_{ij}(x), \pi_{ij}(0)]\rangle\right\}\,. \qquad (2)$$

We decompose the correlator of the energy-momentum tensor in eq. (2) and express it in terms of propagators, $G_{A\phi_i}$, and field derivatives, leaving an equation involving two classes of diagrams. The first class is given in figure 2: it consists of one- to three-loop diagrams with gluon propagators connecting the two 2-gluon vertices of the energy-momentum tensor, $\pi^{(2)}_{ij}$. The other class can be written as effective vertex corrections, see figure 3.

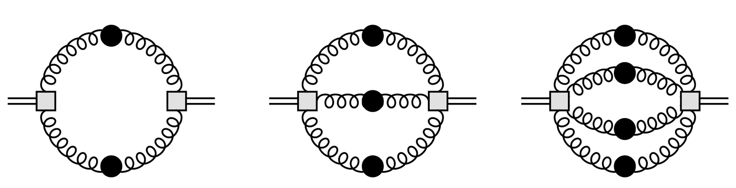


Figure 2: Class 1: π_{ij}'s are connected by G_{AA}.

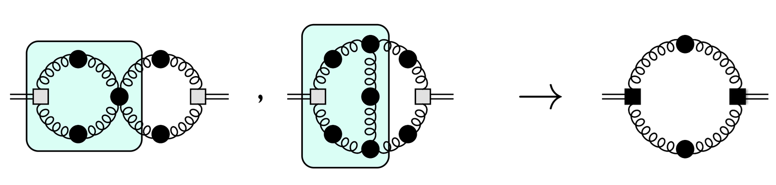


Figure 3: Class 2: effective vertex corrections.

At temperatures close to the critical value we can construct a renormalisation scheme such that contributions from higher loop orders are minimised [6]. This scheme was confirmed in a full 2-loop computation [7]. As a consequence, in the regime considered here the SPF of the energy-momentum tensor is approximated well by the one-loop truncation of figure 2. Finally, with the Kubo relation given in eq. (2), shear viscosity can be computed in terms of gluon SPFs only.

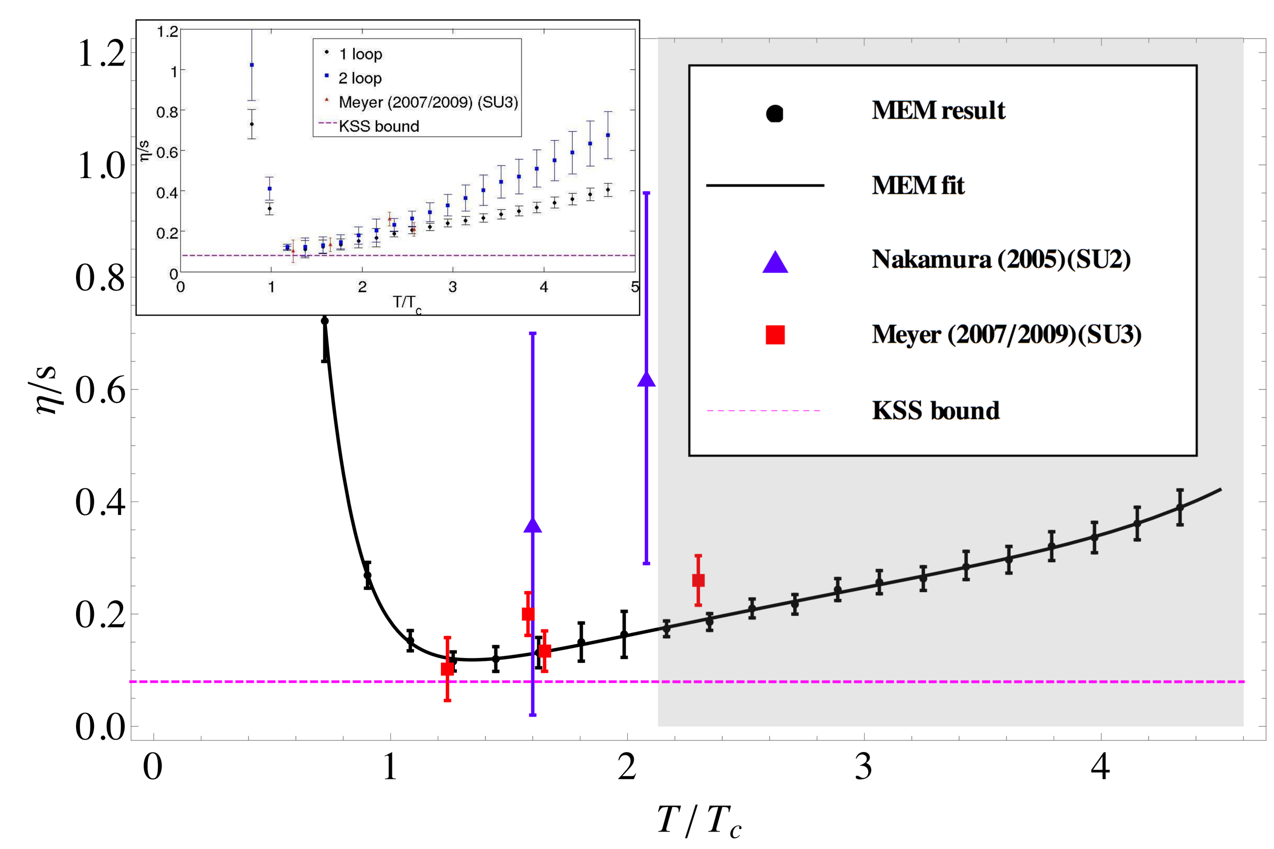


Figure 4: Temperature dependence of ratio of shear viscosity over entropy density, η/s. The inlay shows the two-loop result of ref. [7].

Figure 4 shows the ratio of shear viscosity over entropy density, η/s, where the entropy is taken from lattice gauge theory [8]. The black error bars indicate the combined systematic error from both MEM computation and one-loop approximation as discussed above. In the shaded region only MEM errors are displayed. The error analysis exhibits a small systematic and statistical error for temperatures $T_c \leq T \lesssim 2\,T_c$.
The curve in figure 4 exhibits a clear minimum at $T = 1.25\,T_c$ with a value of $\eta/s = 0.115\,(17)$. The steep rise for temperatures below T_c is to be seen as a qualitative result.

Outlook

The inclusion of dynamical quarks is straightforward and allows for a generalisation to full QCD. This provides direct access to non-equilibrium observables and, hence, phenomenological studies that permit direct comparison with experimental results from heavy-ion collisions. In particular, our focus is on a description of jet quenching. For this, we compute quark and gluon spectral functions also in the real time formalism directly.

[1] P. Kovtun, D. Son, A. Starinets, Phys. Rev. Lett. **94**, 111601 (2005).
[2] R. Kubo, Journal of the Physical Society of Japan **12**, 570 (1957).
[3] L. Fister, J. M. Pawlowski, arXiv: 1112.5440 [hep-ph].
[4] M. Asakawa, T. Hatsuda, Y. Nakahara, Prog. Part. Nucl. Phys. **46**, 459 (2001).
[5] M. Haas, L. Fister, J. M. Pawlowski, arXiv: 1308.4960 [hep-ph].
[6] L. Fister, J. M. Pawlowski, Phys. Rev. **D88** (2013) 045010.
[7] N. Christiansen, M. Haas, J. M. Pawlowski, N. Strodthoff, *in prep.*
[8] F. Karsch, Lect. Notes Phys. **583**, 209 (2002). H. B. Meyer, Phys. Rev. **D80**, 051502 (2009). S. Borsanyi, G. Endrodi, Z. Fodor, S. Katz, and K. Szabo, JHEP **1207**, 056 (2012).