Approach to equilibrium in weakly coupled nonabelian plasmas

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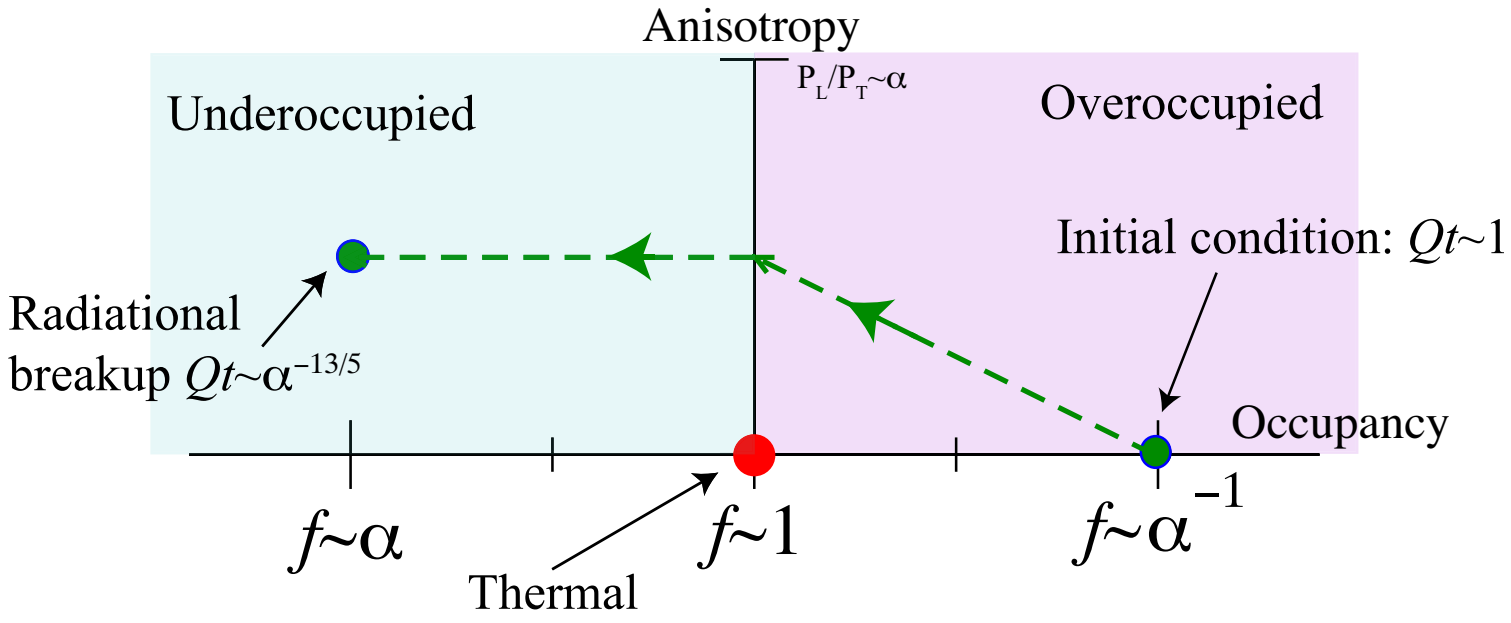
- What: Thermalization in $\alpha \ll 1$ nonabelian gauge theory
- How: Using combination of classical field theory and kinetic theory
- New: Smooth shift of d.o.f from fields to particles,
  first numerical estimates of bottom-up thermalization

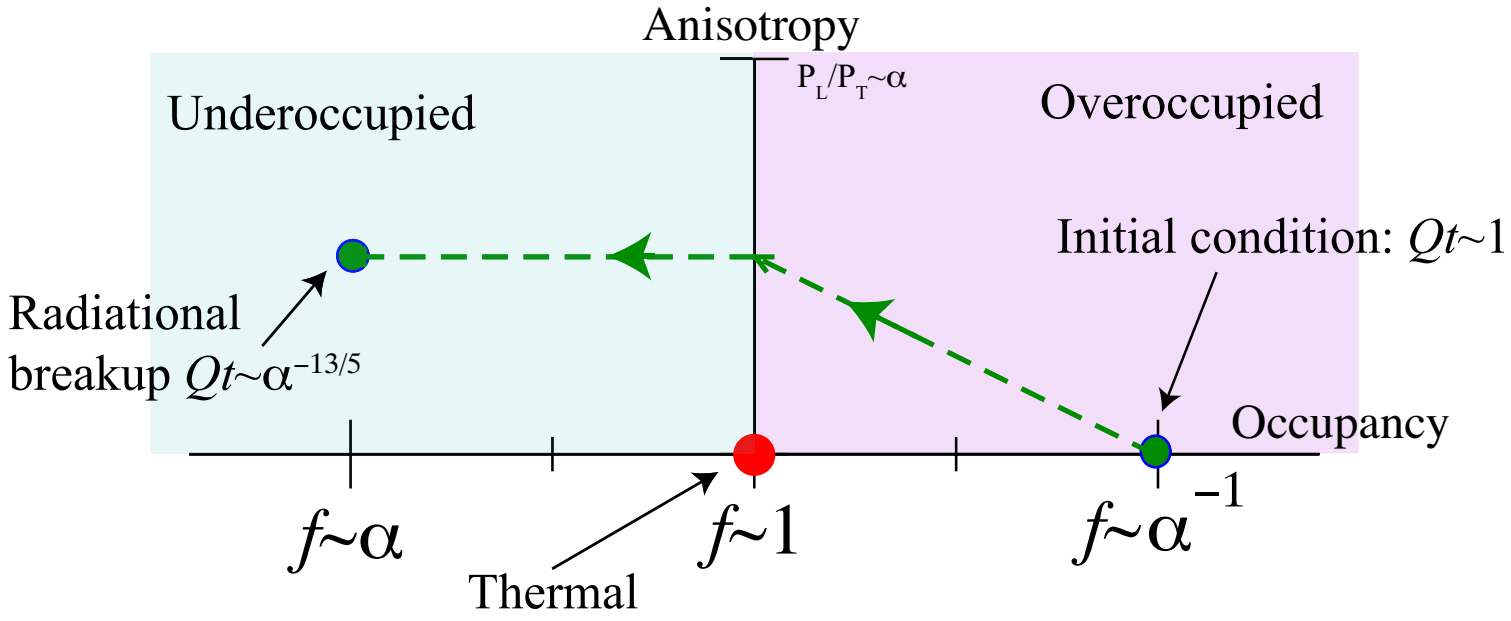
Motivation: Bottom-up thermalization



- CGC: Initial condition overoccupied $f(Q) \sim 1/\alpha$
- Expansion makes system underoccupied ($f(Q) \ll 1$) before thermalizing

Baier et. al hep-ph/0009237, AK, Moore 1108.4684

Motivation: Bottom-up thermalization



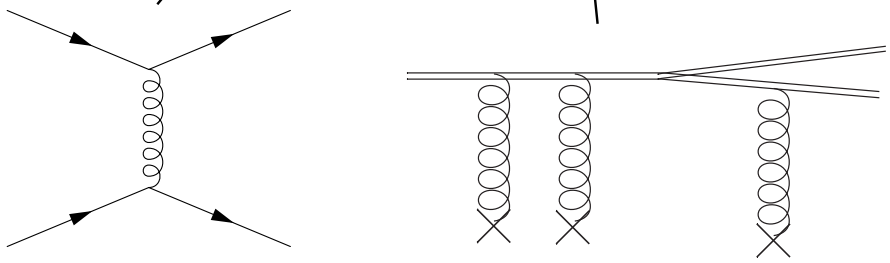
- Degrees of freedom:
  - Overoccupied: Classical field theory, $f \gg 1$
  - Underoccupied: (Semi-)classical particles, eff. kinetic theory, $f \ll 1/\alpha$

Talks by Schlichting, Epelbaum

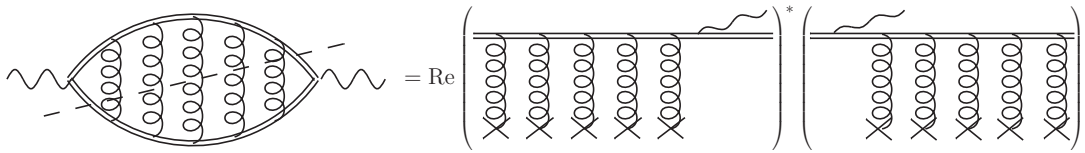
- Full description: Need change of d.o.f. from fields to particles
- Overlapping domain of validity $1 \ll f \ll 1/\alpha$: Field-particle duality

Effective kinetic theory of Arnold, Moore and Yaffe hep-ph/0209353

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$



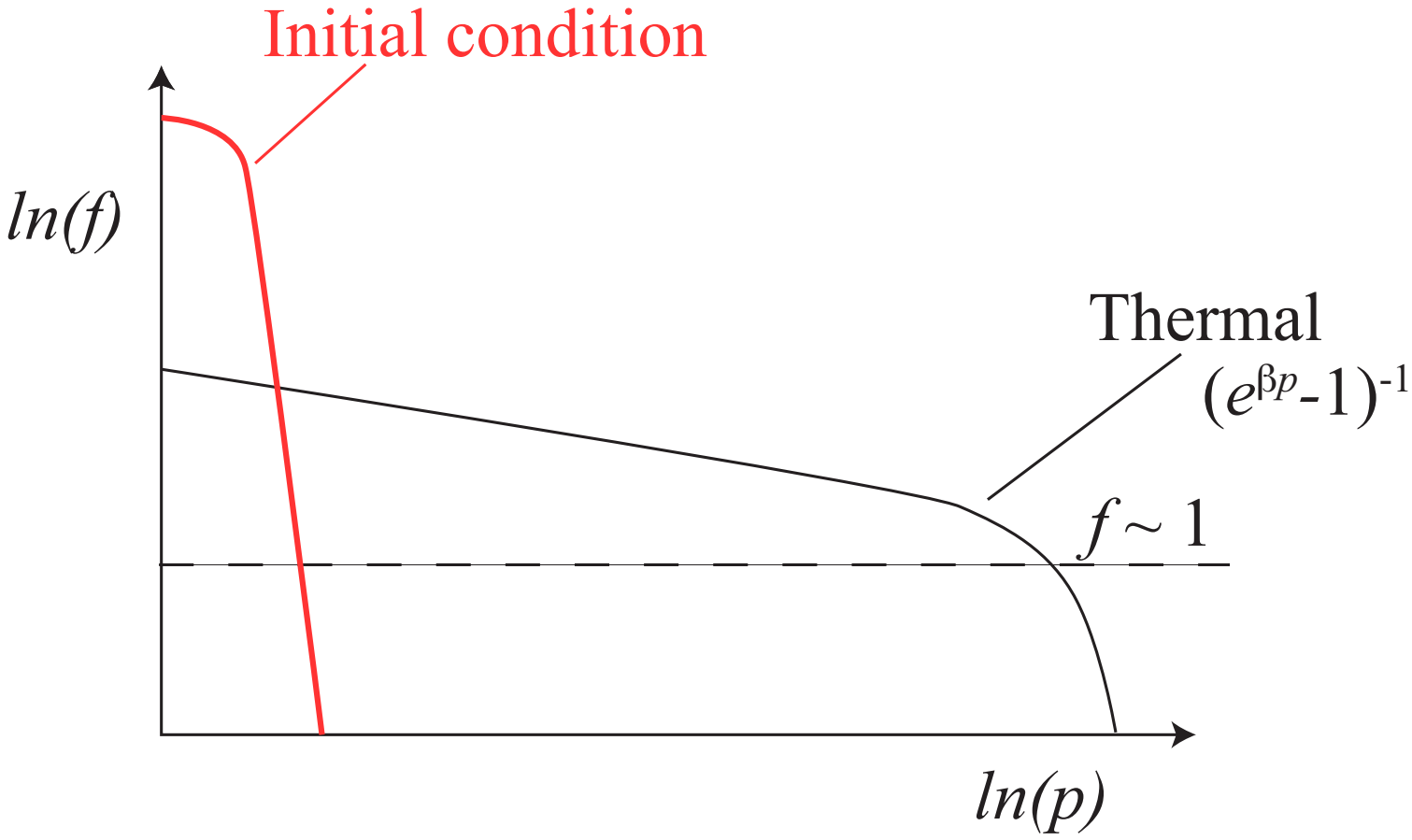
- Soft and collinear divergences lead to nontrivial matrix elements:
  - Coulombic divergence in t, u-channels regulated by screening:
    Hard-loop resummed matrix element
  - Collinear divergence regulated by LPM-suppression:
    Ladder resummed effective $1 \leftrightarrow 2$ matrix element



- No free parameters; LO accurate in the $\alpha \to 0$, $\alpha f \to 0$ limit.
- Used *e.g.* for LO transport coefficients in QCD

Overoccupied cascade

What happens if you have too many soft gluons, $f \sim 1/\alpha$.
No longitudinal expansion.



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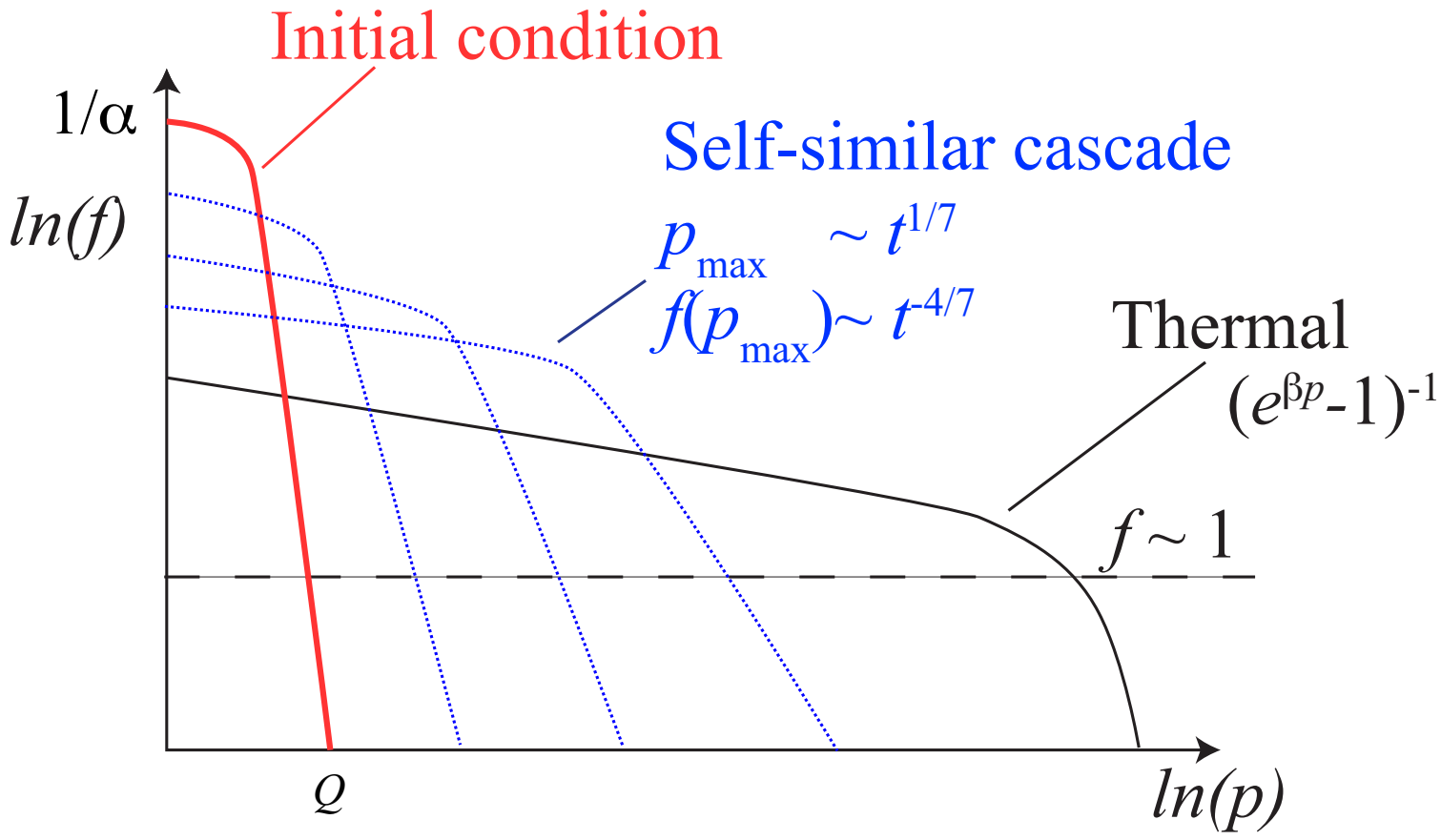
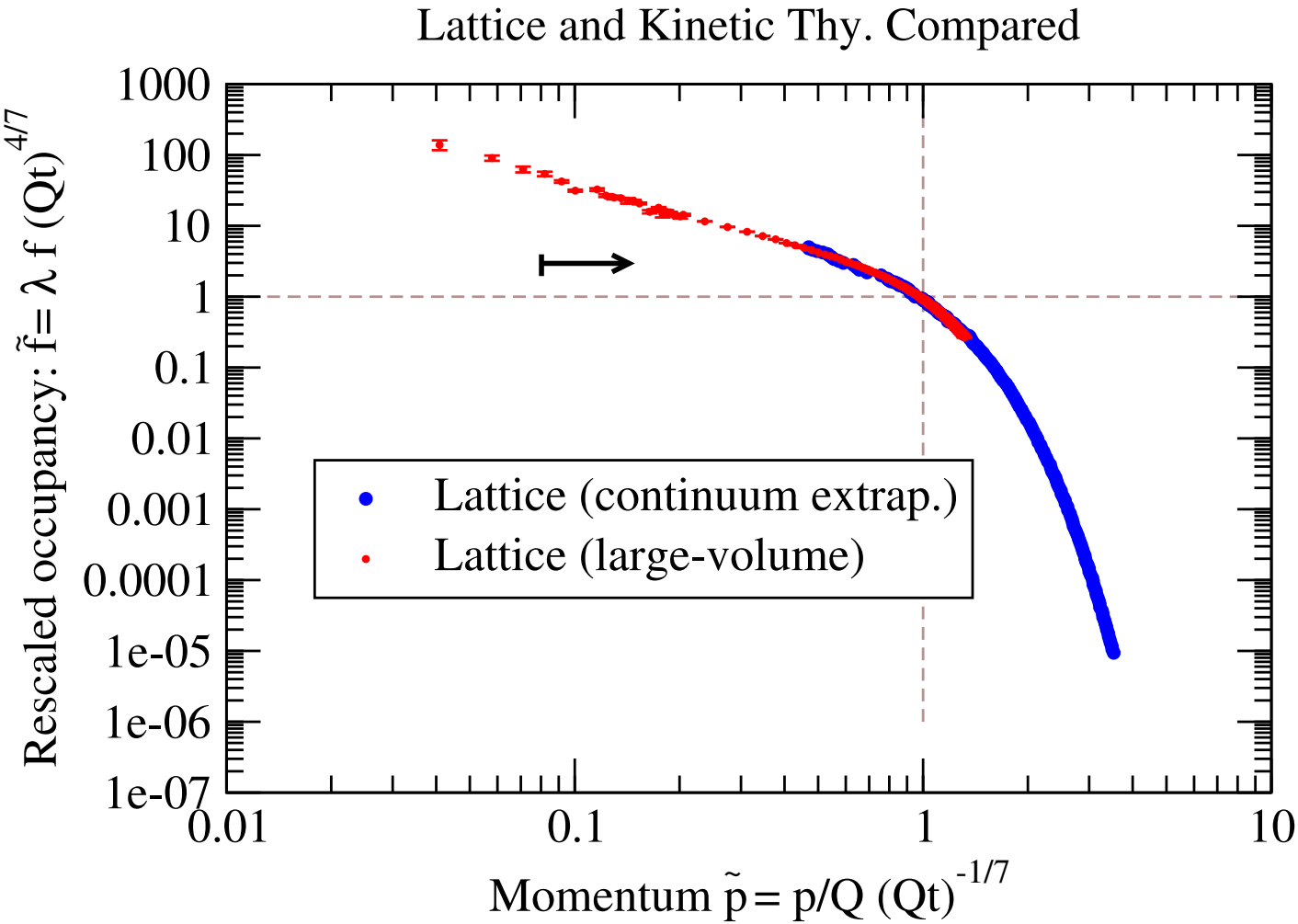


Overoccupied cascade

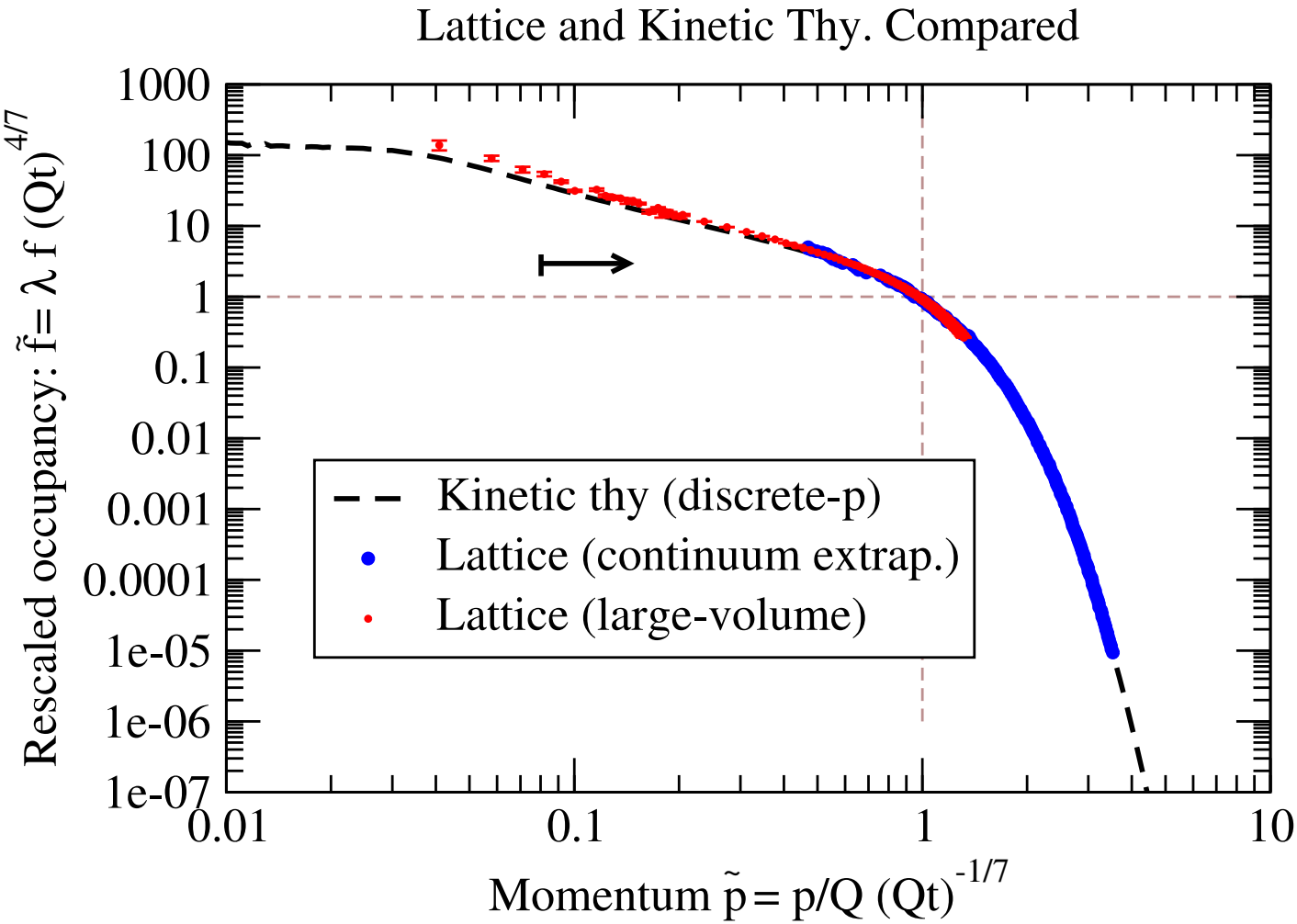


Form of turbulent cascade from classical lattice simulation,

$$1 \ll f \lesssim 1/\alpha$$

Large-volume: (Qa)=0.2, (QL)=51.2, Cont. extr.: down to (Qa)=0.1, (QL)=25.6, Qt=2000, $\tilde{m} = 0.08$

Overoccupied cascade



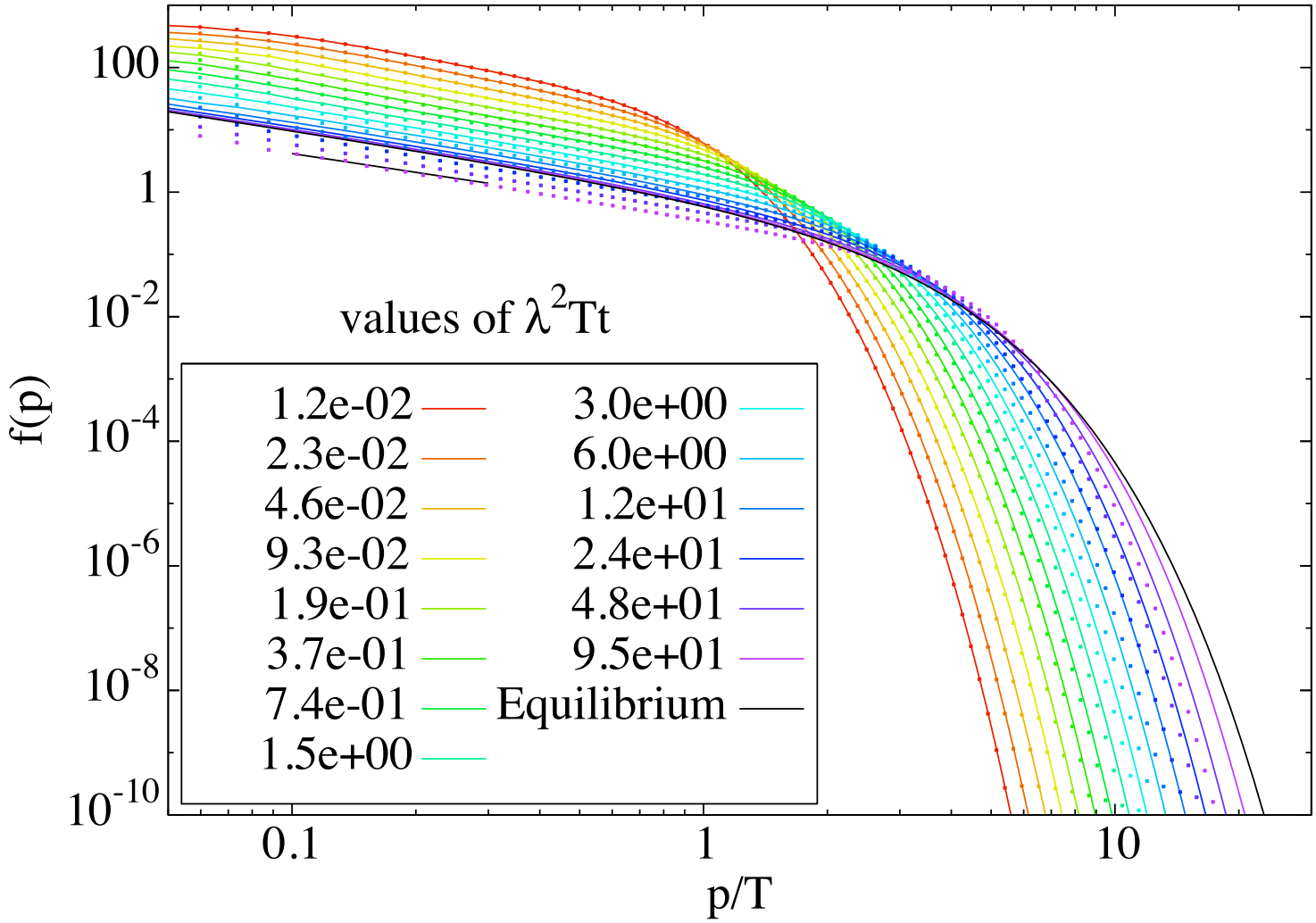
Same system, very different degrees of freedom

$$1 \lesssim f \ll 1/\alpha$$

Sensitive to the details of the collision terms

Overoccupied cascade

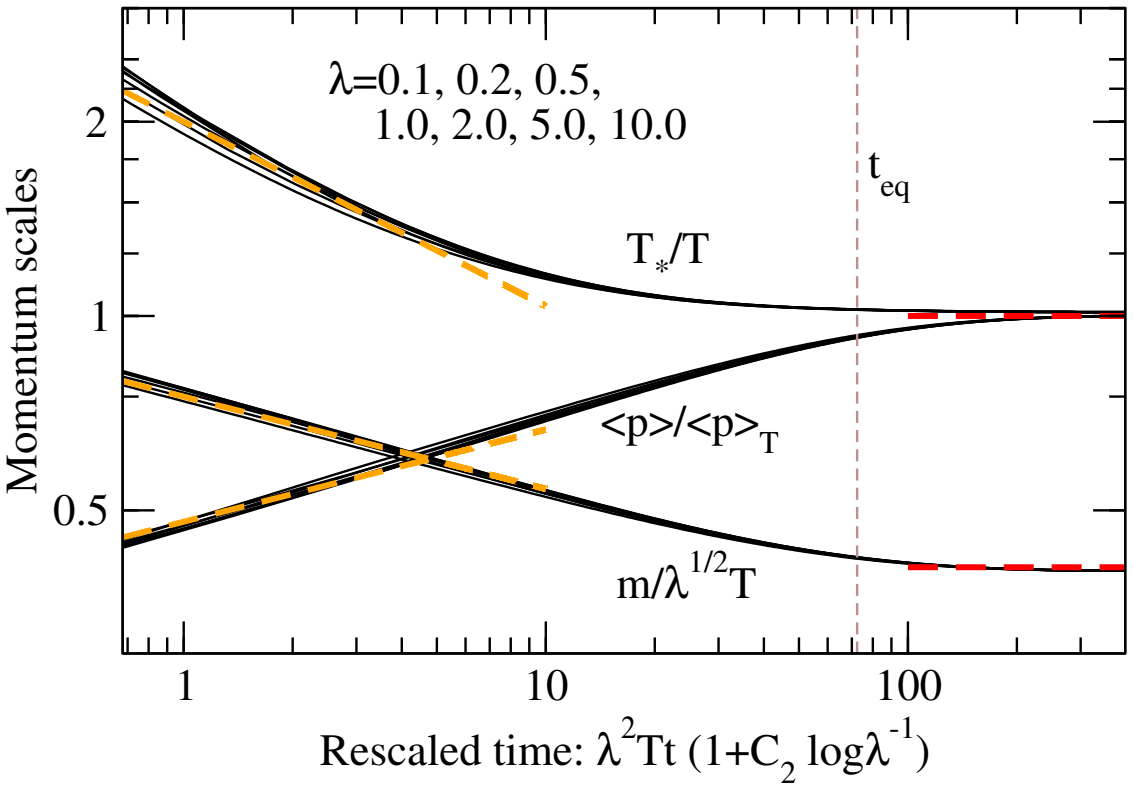
to appear AK, Lu



Thermal equilibrium reached once $f \sim 1$.

Overoccupied cascade

to appear AK, Lu



$$m^2 = \lambda \int_{\mathbf{p}} \frac{f(p)}{p}$$

$$T_* = \frac{\lambda}{2} \int_{\mathbf{p}} f(p)[1+f(p)]/m^2$$

$$\langle p \rangle = \frac{1}{n} \int_{\mathbf{p}} p\, f(p)$$

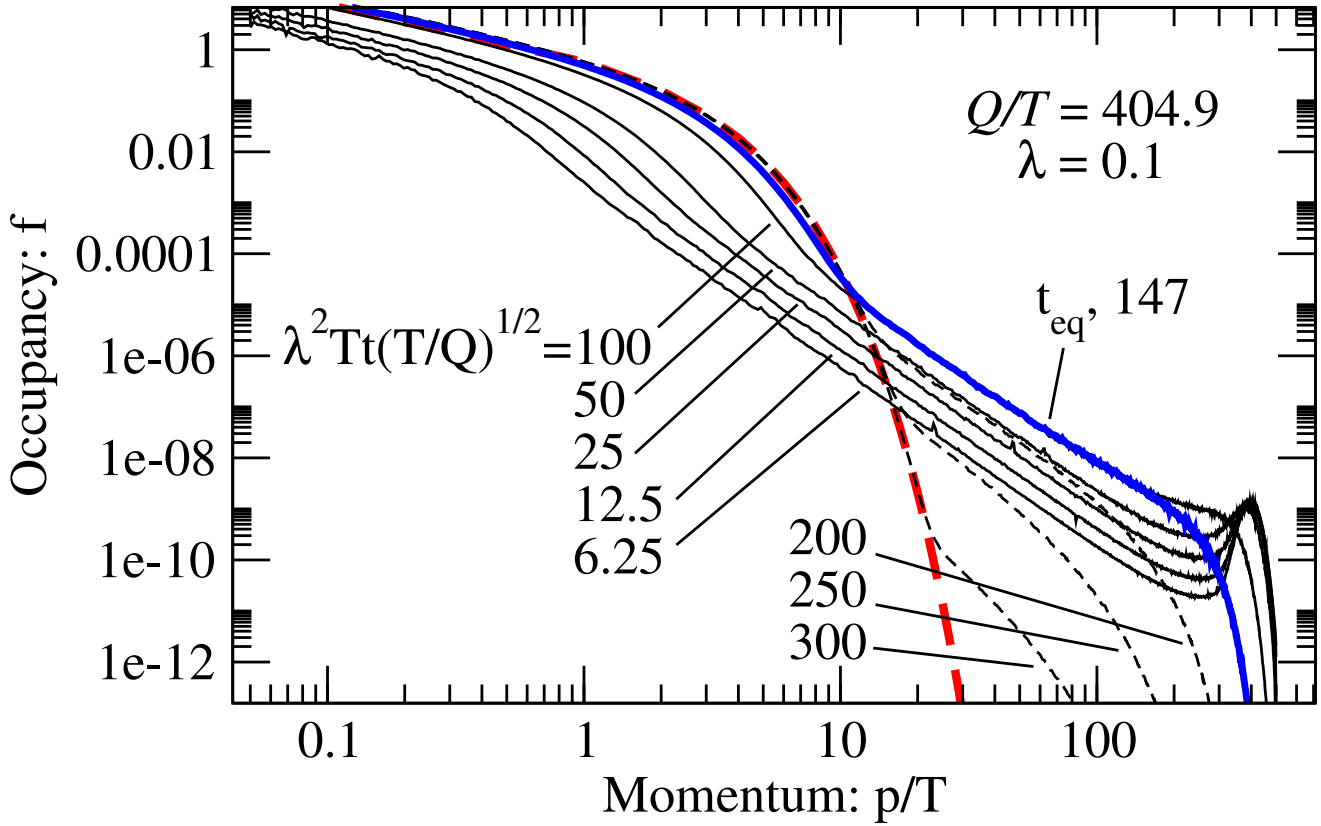
Therm. time through the approach of $\langle|p|\rangle - \langle|p|\rangle_T \sim \exp(-t/t_{\rm eq})$

$$t_{\rm eq} \approx \frac{72.}{1+0.12\log\lambda^{-1}} \frac{1}{\lambda^2 T}, \qquad \lambda = 4\pi N_c \alpha.$$

Underoccupied cascade

AK, Lu, to appear

Isotropic, underoccupied initial conditions, initial scale $\langle p^2 \rangle = Q^2$



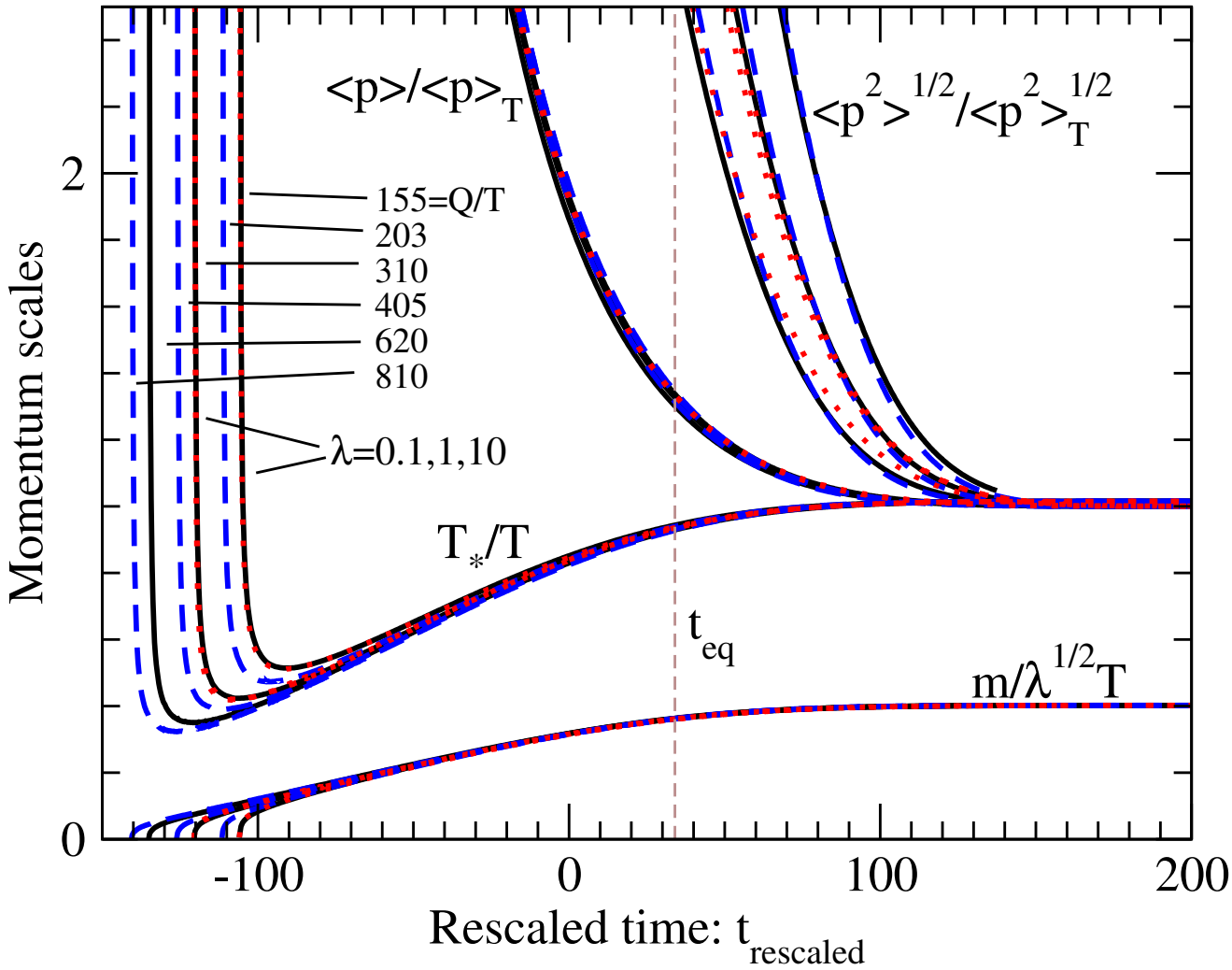
Thermalization time parametrically given by stopping time of jet of momentum Q:

AK, Moore 1107.5050

$$t_{\rm eq} \approx \left(\frac{Q}{T}\right)^{1/2} \frac{1}{\lambda^2 T}$$

Underoccupied cascade

Set of initial conditions with gaussian and step-cutoff initial conditions



$$t_{\rm eq} \approx \frac{34. + 21.\ln(Q/T)}{1 + 0.037\log\lambda^{-1}} \left(\frac{Q}{T}\right)^{1/2} \frac{1}{\lambda^2 T}$$

Connection to heavy-ion physics

Bottom-up thermalization a la BMSS: Baier et. al hep-ph/0009237, AK, Moore 1108.4684

- Underoccupied cascade, but expansion reduces the target temperature

$$\tau_{\rm eq} \sim \frac{1}{\alpha^2 T}\left(\frac{Q}{T}\right)^{1/2}, \quad \epsilon \sim T^4 \sim \frac{Q^4}{\alpha Q t} \Rightarrow Qt \sim \alpha^{-13/5}$$

- Rough estimate: replace parametric estimate by the numerical

$$t_{\rm eq} \lesssim 0.1 - 1 fm/c$$

Caveats: angular dependence, fermions, extrapolation

Conclusions

- Combination of classical simulations and effective kinetic theory allows to follow the time evolution from highly occupied initial condition to thermal equilibrium
- Thermalization times for simple systems faster than naively expected
- Inserting the underoccupied thermalization time to bottom-up thermalization yields a rough estimate for heavy-ion collisions

$$t_{\mathrm{eq}} \lesssim 0.1 - 1\mathrm{fm/c}$$

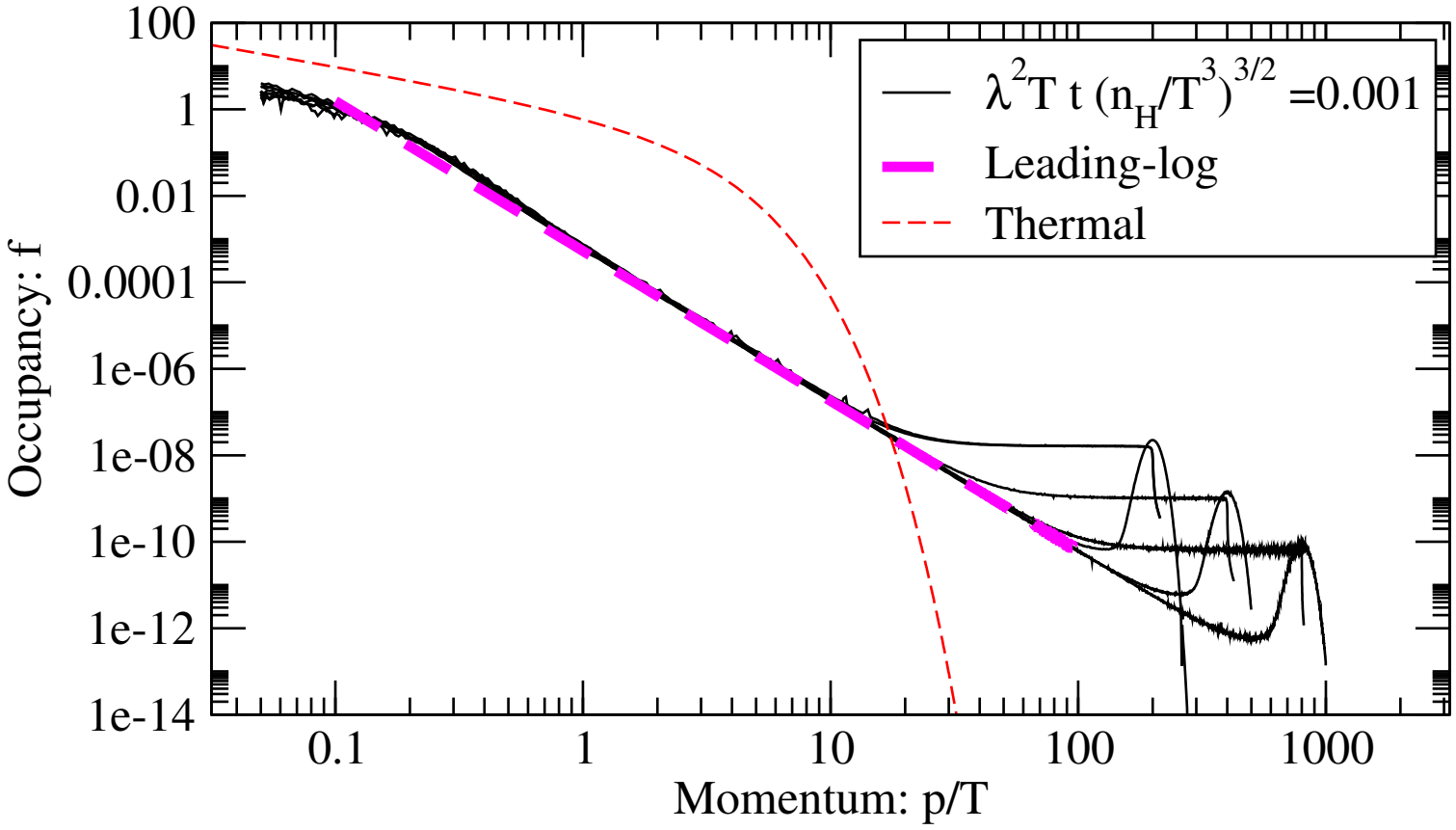
Outlook

- Proper treatment of expansion and angular dependence
- Implementation of fermions to kinetic theory
- Inclusions of plasma unstable modes AK, Moore 1108.4684
- NLO not inconceivable

p.s. No sign of BEC

Underoccupied cascade

Set of isotropic, underoccupied initial conditions, initial scale $\langle p^2 \rangle = Q^2$



At early times, connection to deep-LPM limit of $C_{1\leftrightarrow 2}$ by Arnold and Dogan

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