Phase diagram of lattice QCD in auxiliary filed Monte-Carlo method in the strong coupling region

A. Ohnishi[B] and T. Ichihara[A,B] Kyoto Univ.[A], YITP[B]

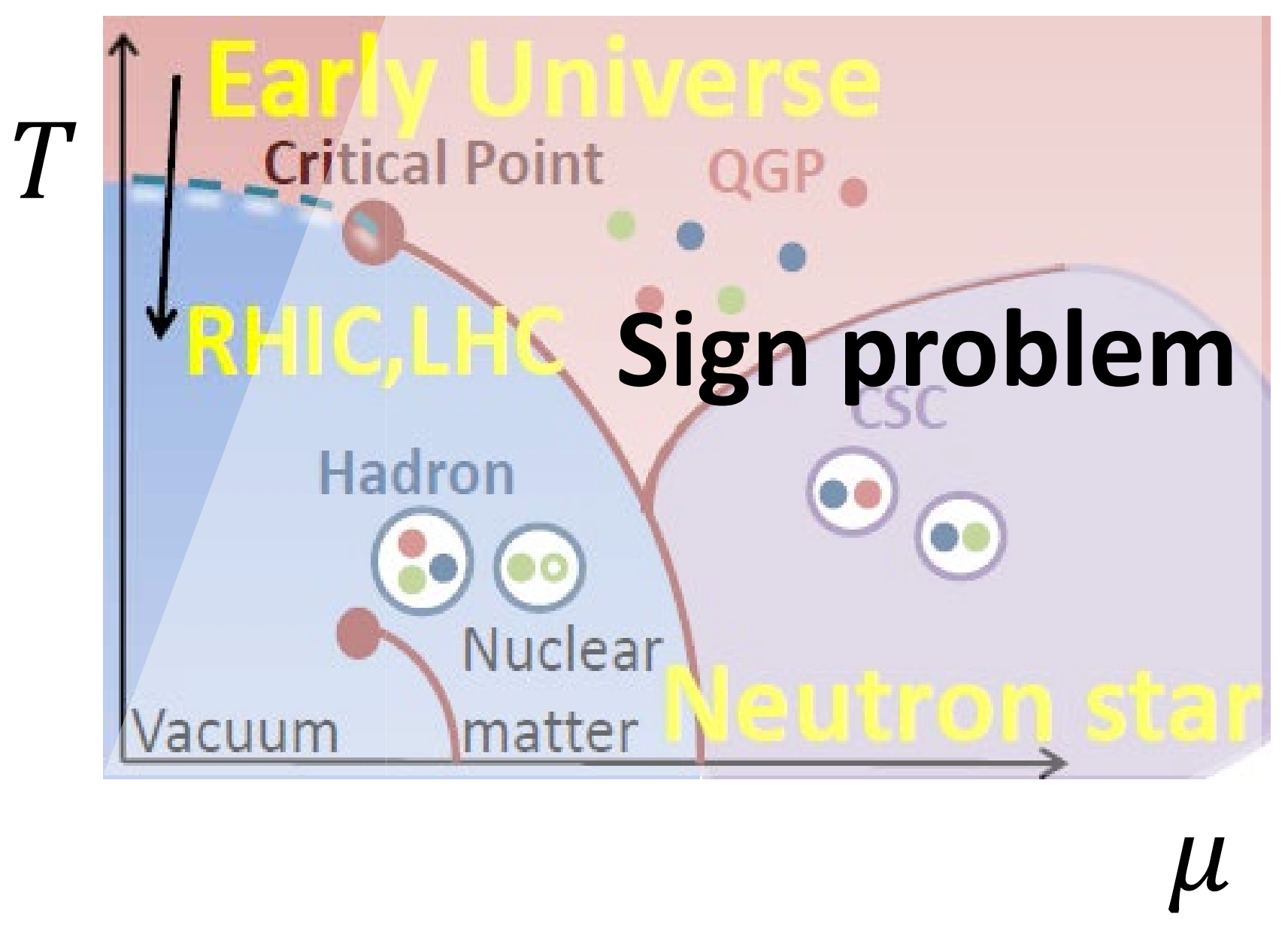
Abstract: We study the QCD phase diagram in the strong coupling region by using the auxiliary field Monte-Carlo method. We have the sign problem in both AFMC and monomer-dimer-polymer (MDP) simulations when we include fluctuation effects. Although we have the sign problem, we could investigate the QCD phase diagram in AFMC, which is consistent with MDP simulations in the strong coupling limit. Preliminary results seems to suggest that finite coupling effects make the sign problem more severe than in the strong coupling limit.

Introduction

The sign problem in QCD phase diagram

- MC simulation in lattice QCD

$$Z = \int \mathcal{D}[\bar\psi,\psi,U]\exp[-\bar\psi\mathcal{D}\psi - S_{\mathrm{G}}] = \int \mathcal{D}[U]\,\underline{\det\mathcal{D}}\exp[-S_{\mathrm{G}}]$$



- **Weight of MC integral becomes complex at finite real μ**
- γ_5 Hermiticity

$$\gamma_5\mathcal{D}(\mu)\gamma_5 = [\mathcal{D}(-\mu^*)]^\dagger$$
$$\det[\mathcal{D}(\mu)] = [\det\mathcal{D}(-\mu^*)]^*$$

- Partition function and sign problem $Z = \mathrm{Tr}e^{-\beta H}$

  We calculate Z in the eigen states $|\psi\rangle$ of H → semi-positive definite → no sign problem

  We adopt **states which are close to the eigen states of H**

  Hadronic degrees of freedom → Reducing sign problem (?)

Strong coupling lattice QCD

$$S_{\mathrm{LQCD}} = S_{\mathrm{F}} + \boxed{S_{\mathrm{G}}}$$

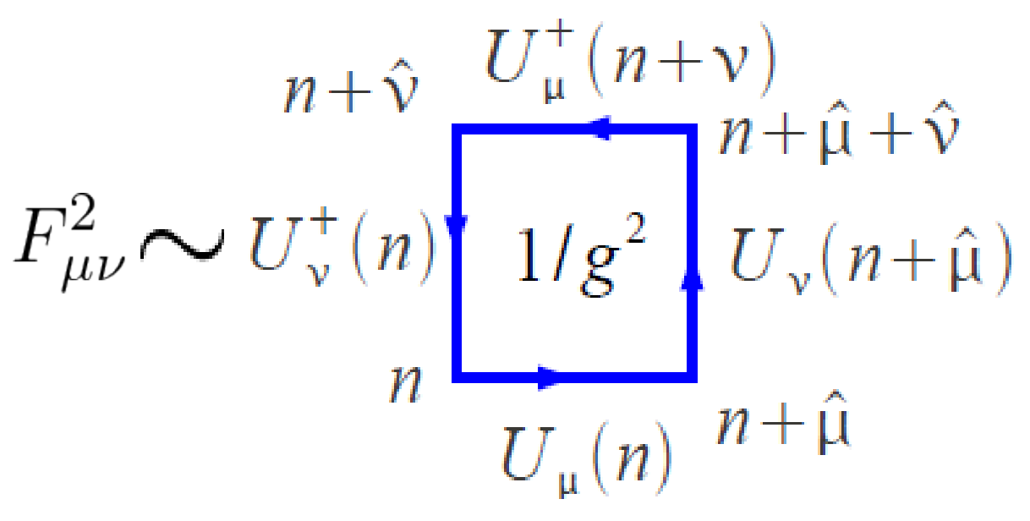
- 1/g² expansion in lattice QCD
- Integrating out link & Grassmann variables analytically
- MC integral by **Hadronic DOF** → Reduced sign problem

Gluon field → Link variables

$$U_\mu(x)\simeq\exp(igA_\mu)$$

Gluon action → Plaquette action

$$S_G = \frac{2N_c}{g^2}\sum_{plaq.}\left[1-\frac{1}{N_c}\mathrm{Re\,tr}\,U_{\mu\nu}(n)\right]$$



"The sign problem" with fluctuations

- Monomer-Dimer-Polymer simulations

  W. Unger, Ph. de Forcrand, *J. Phys. G: Nucl. Part. Phys.* **38** 124190 (2011)

- Auxiliary field Monte-Carlo method

  A. Ohnishi, T. I. and T. Z. Nakano : PoS LATTICE2012, 088 (2012).
  T. I. , T. Z. Nakano, and A. Ohnishi arXiv:1311.5352

We study the QCD phase diagram in the strong coupling region including fluctuation effects by using auxiliary feld Monte-Carlo (AFMC) method.

Formalism

Strong coupling lattice QCD action

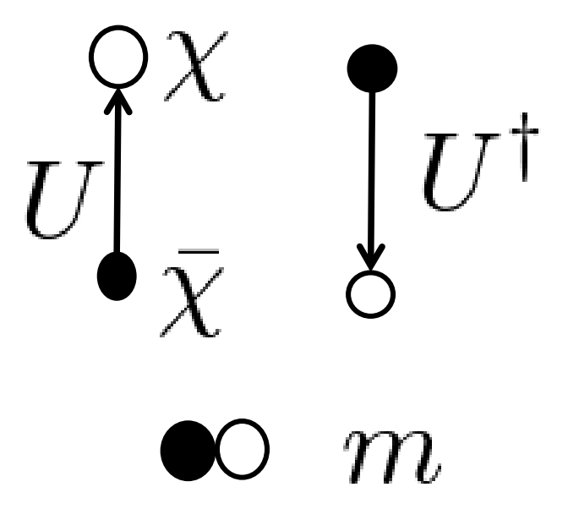
- Unrooted staggered fermion, anisotropic lattice, strong coupling limit
- $1/d$ expansion, $\boldsymbol{U_j}$ **integration** N.Bilic et al. (1992), G. Faldt et al.(1986)

$$S_{\mathrm{SCL-LQCD}} = \frac{1}{2}\sum_{\nu=0}^{d}\sum_{x}\left[\eta^+_{\nu,x}\bar\chi_x U_{\nu,x}\chi_{x+\hat\nu} - \eta^-_{\nu,x}(\mathrm{H.C.})\right] + m\sum_x \bar\chi_x\chi_x \cancel{+S_G}$$

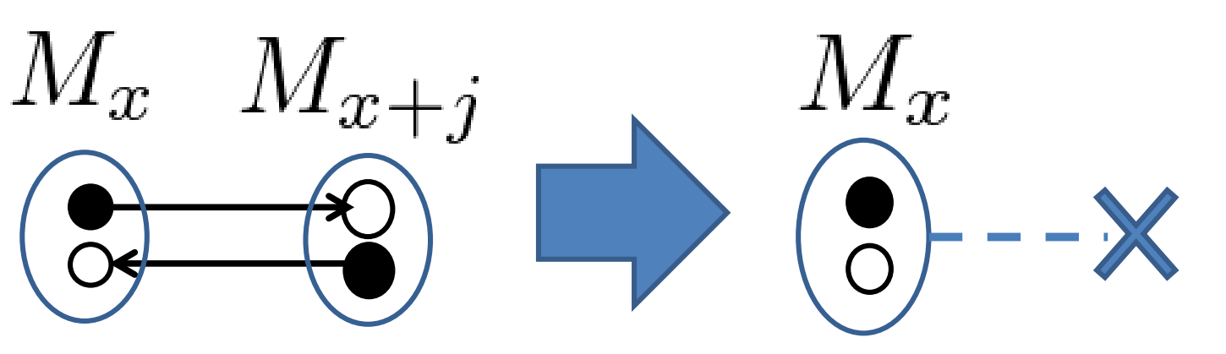
$$\eta^\pm_{\nu,x} = (\boxed{\gamma}e^{\pm\mu a_\tau}, (-1)^{x_1+\cdots+x_{\nu-1}})$$

Kinetic

Mass



- Bosonization: Extended HS transformation



4 fermi-like

effective mass term

Auxiliary fields

$$\sigma_x = -\langle\bar\chi_x\chi_x\rangle$$
$$\pi_x = \boxed{\langle i\epsilon\bar\chi_x\chi_x\rangle}$$

Hadronic DOF

$$\epsilon = (-1)^{x_0+\cdots+x_d} \sim\gamma_5$$

Notice !

$$\exp[\alpha AB] = \int\mathcal{D}[\phi,\varphi]\,\exp\left[-\alpha\left[\phi^2+\varphi^2+(A+B)\varphi - \boxed{i}(A-B)\phi\right]\right]$$

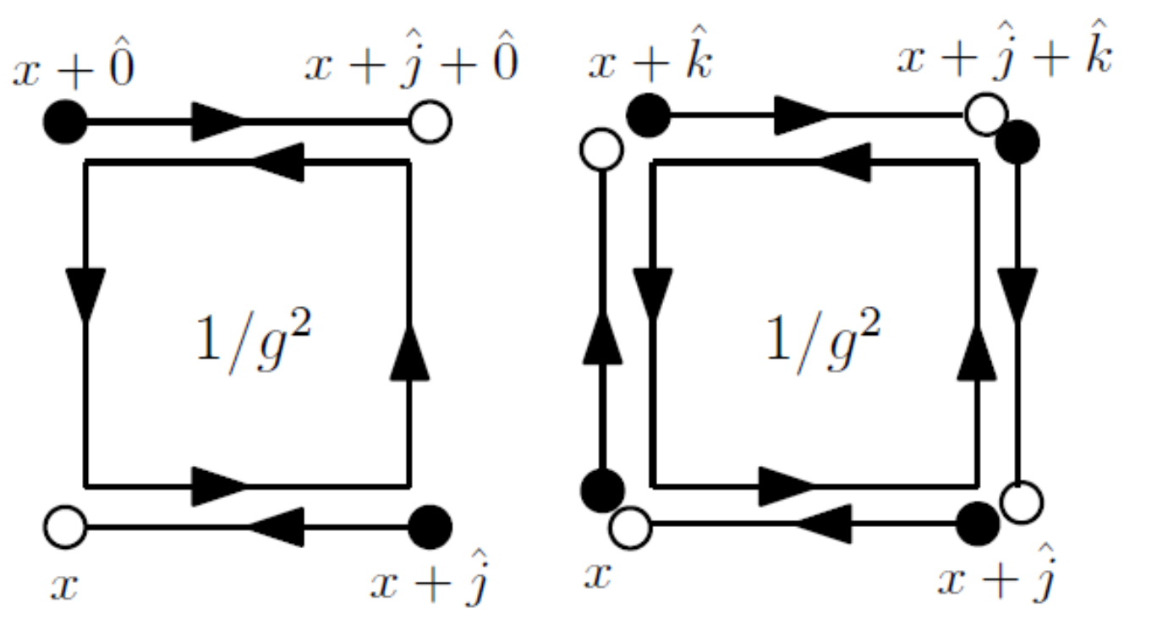
- Auxiliary Field Monte-Carlo

$$Z = \int\mathcal{D}\boxed{[\sigma_{\boldsymbol{k},\tau},\pi_{\boldsymbol{k},\tau}]\,e^{-S^{\mathrm{AF}}_{\mathrm{eff}}}}$$

NLO effective action

- Spatial plaquette terms
  - 8-fermion fields → 4-fermion fields → 2-fermion fields
  - Effective mass modification
- Temporal plaquette terms
  - 4-fermion fields → 2-fermion fields
  - Effective chem. pot. + wave fn. ren.

$$S_{\mathrm{NLO}} = S_{\mathrm{SCL}} + \Delta S^{\mathrm{s}}_{\mathrm{NLO}} + \Delta S^{\tau}_{\mathrm{NLO}}$$



$V^-_x V^+_{x+\hat j}$ 2 Mesonic hopping $\Delta S^\tau_{\mathrm{NLO}}$

$M_x M_{x+\hat j} M_{x+\hat k} M_{x+\hat k+\hat j}$ 4 Mesonic hopping $\Delta S^{\mathrm{s}}_{\mathrm{NLO}}$

$M_x = \bar\chi_x\chi_x$

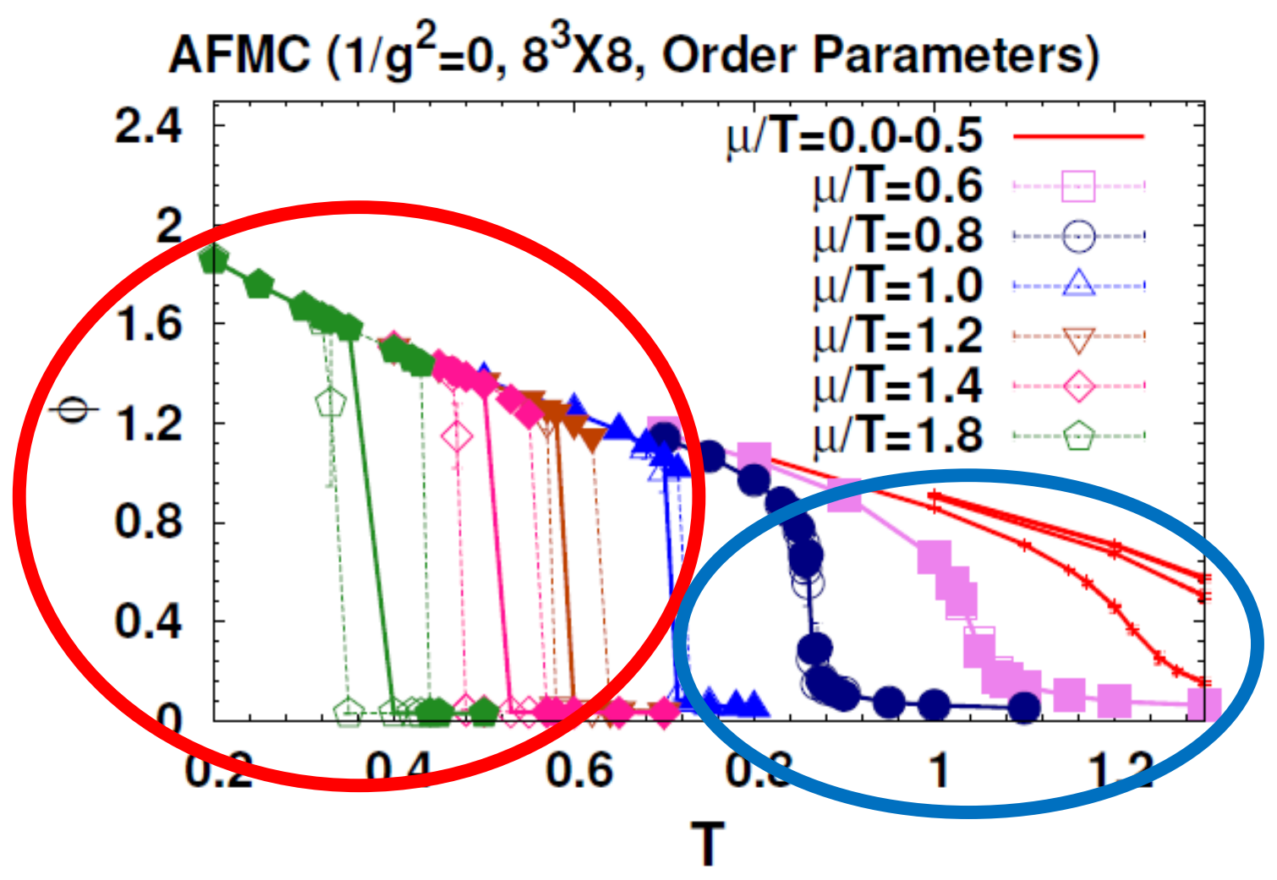
Results

Reservations:

Chiral limit, Jack knife errors, All results in lattice unit

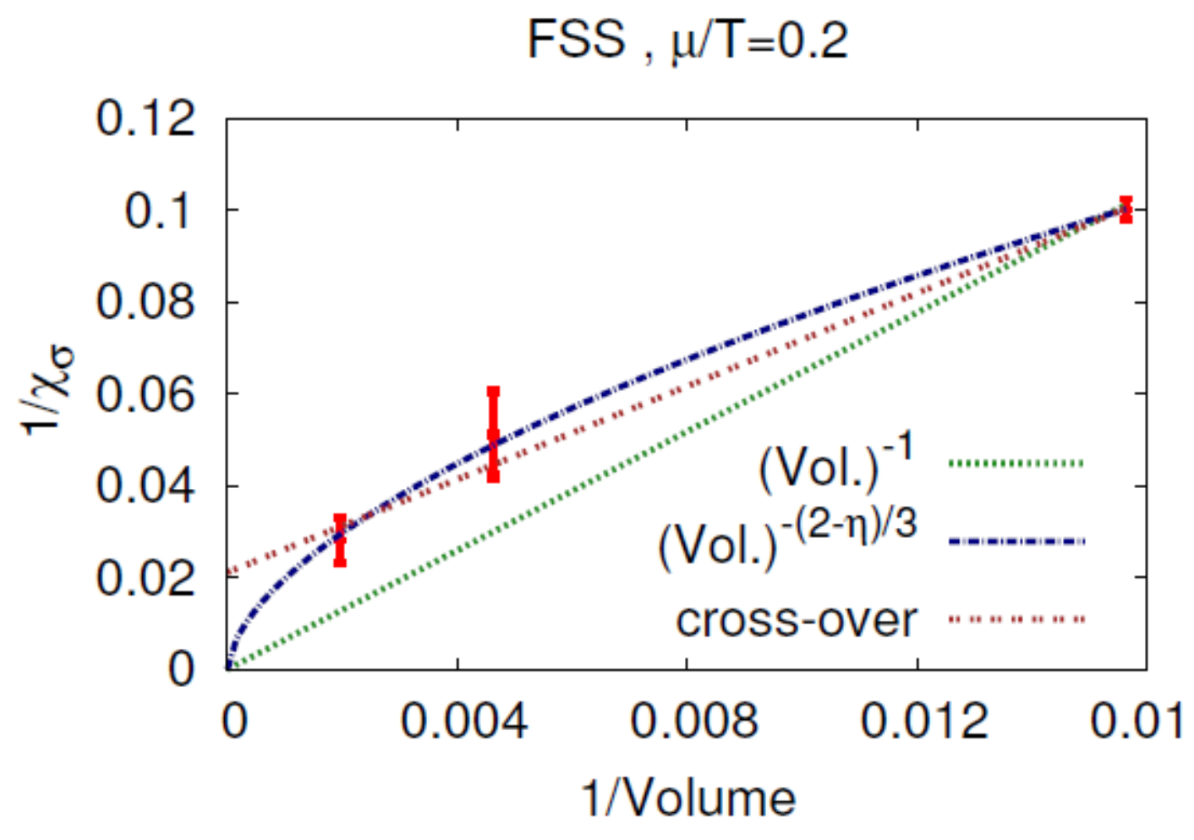
Behavior of order parameters

- Chiral condensate ϕ



Would-be 1st at high μ **Would-be 2nd at low μ**

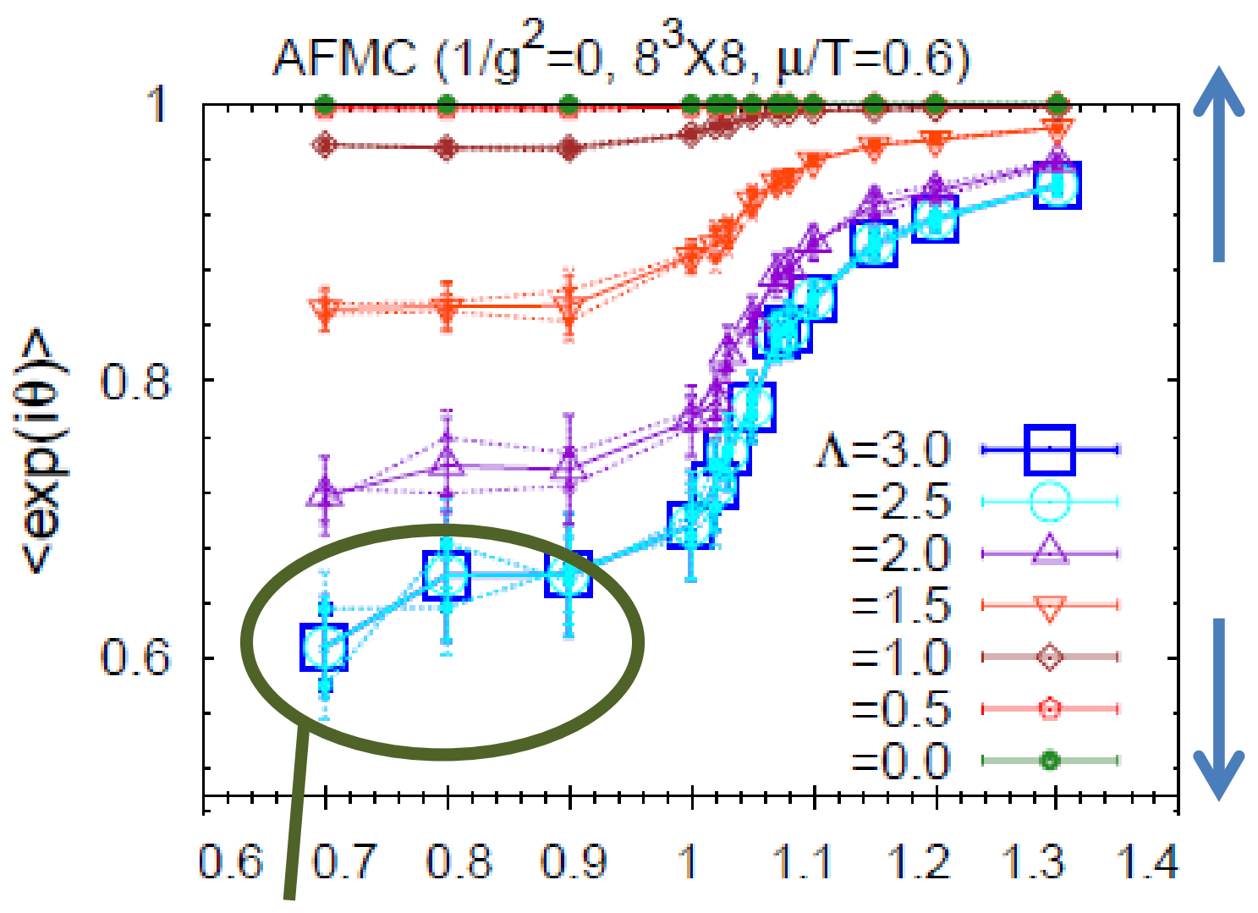
- Scaling behavior of chiral susceptibility at low μ



Would-be 2nd or cross over

Sign problem (SB)

- Average phase factor, $\langle e^{i\theta}\rangle$

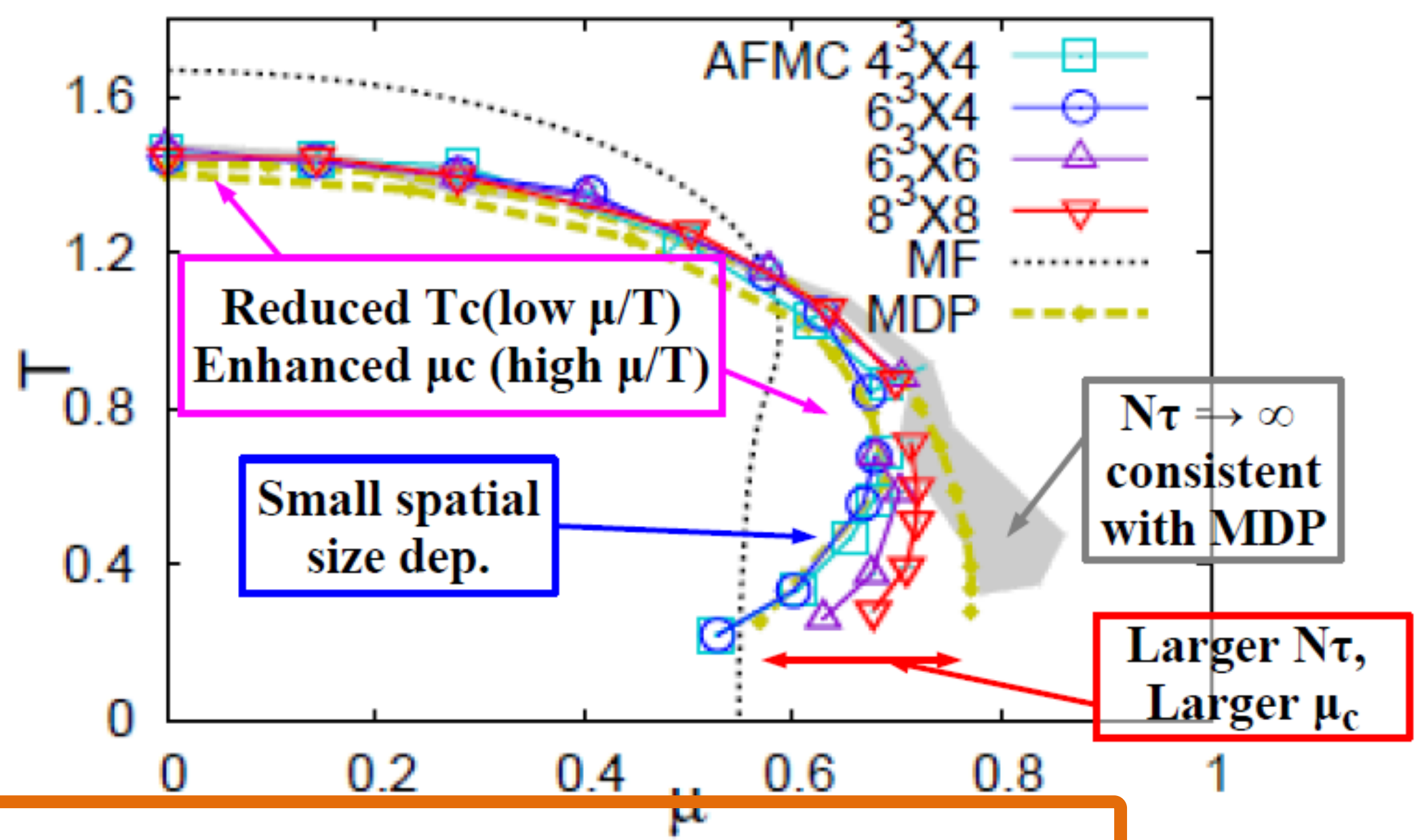


No SB

Severe SB $\langle e^{i\theta}\rangle\sim0$

Full configuration results

The QCD phase diagram in the strong coupling limit



Reduced Tc(low μ/T)
Enhanced μc (high μ/T)

Small spatial size dep.

Nτ → ∞ consistent with MDP

Larger Nτ, Larger μc

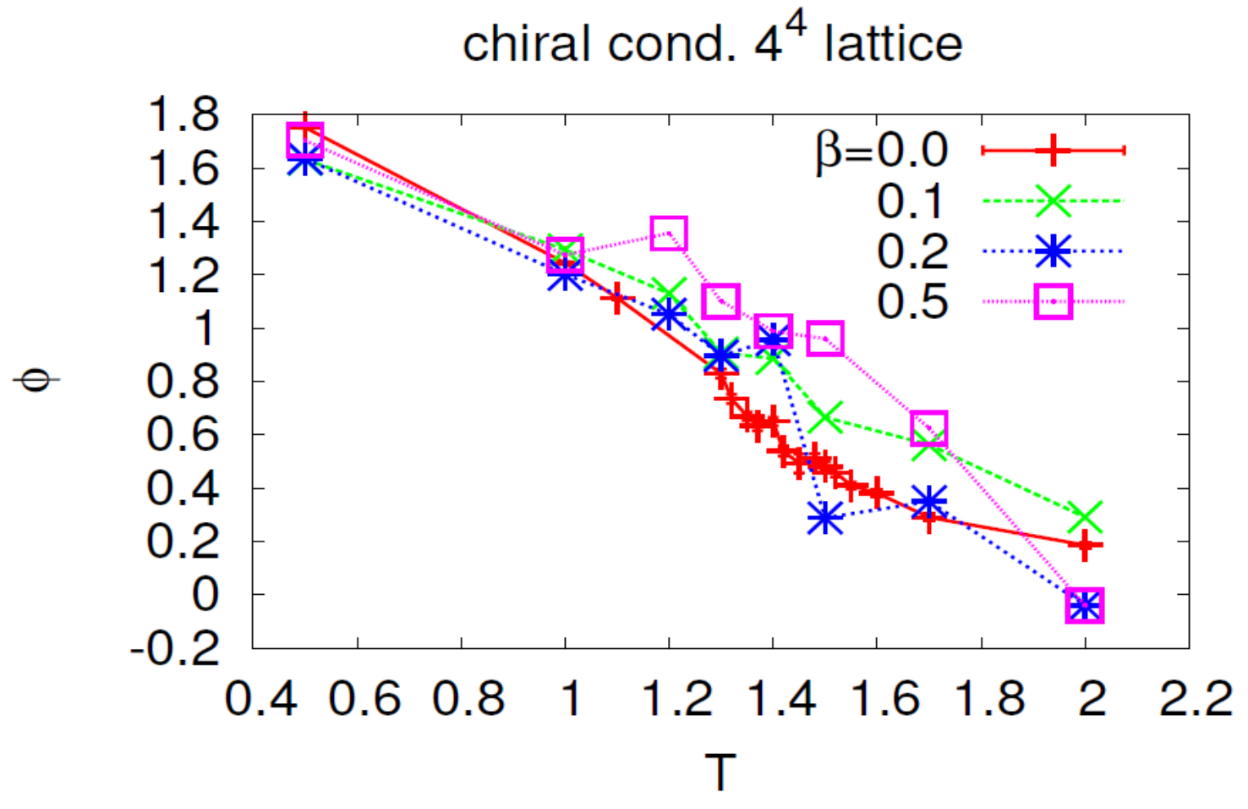
- The would-be 1st order at high μ and the would be 2nd at low μ

NLO results

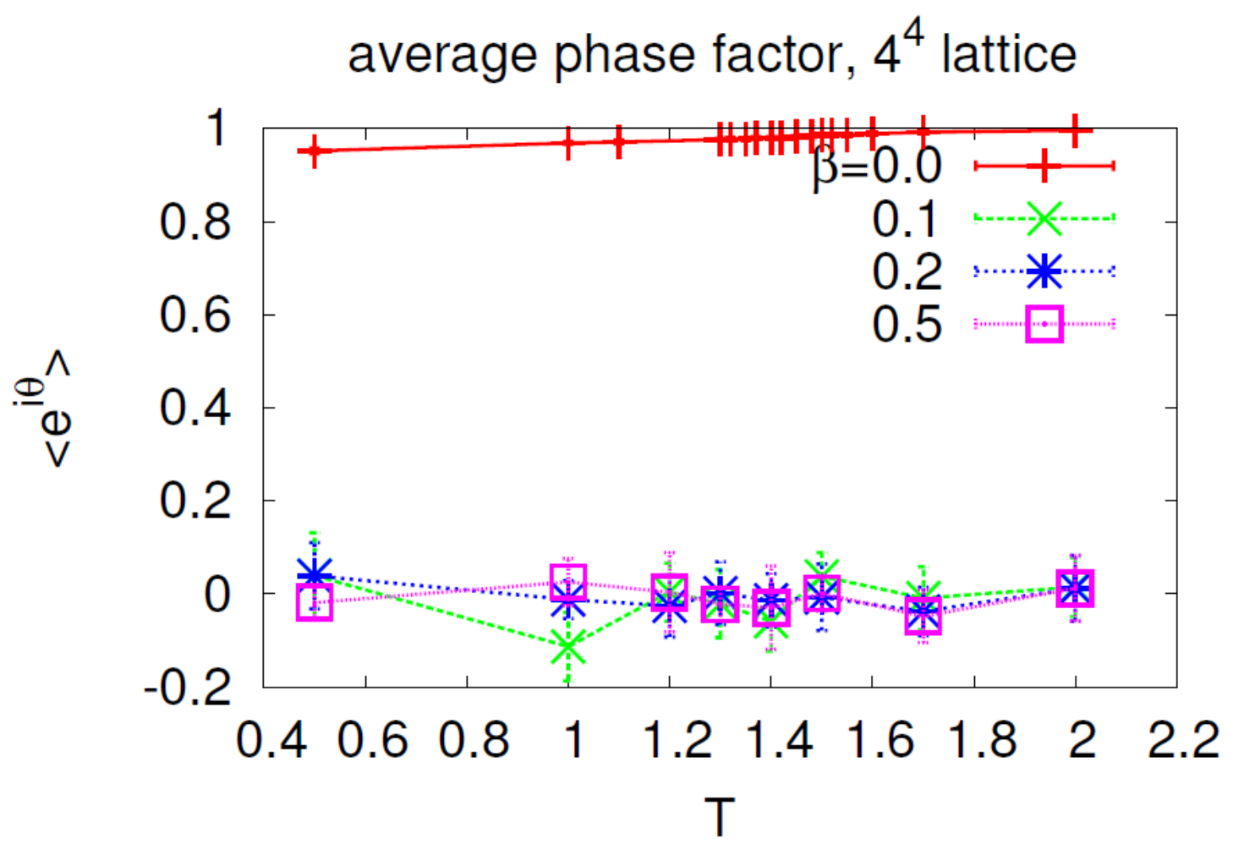
- Temporal corrections
  - Might be severe weight cancellation (SB) due to complex auxiliary fields.

$$\Delta\alpha^-_x = C_\tau\left\{-i\omega^*_{x+\hat j} - i\omega^*_{x-\hat j} - (\epsilon\Omega^*)_{x+\hat j} - (\epsilon\Omega^*)_{x-\hat j}\right\}$$
$$\Delta\alpha^+_x = C_\tau\left\{i\omega_{x+\hat j} + i\omega_{x-\hat j} + (\epsilon\Omega)_{x+\hat j} + (\epsilon\Omega)_{x-\hat j}\right\}$$

- Results of only spatial corrections
  - Chiral condensate ϕ



  - Average phase factor, $\langle e^{i\theta}\rangle$



Reducing sign problem?

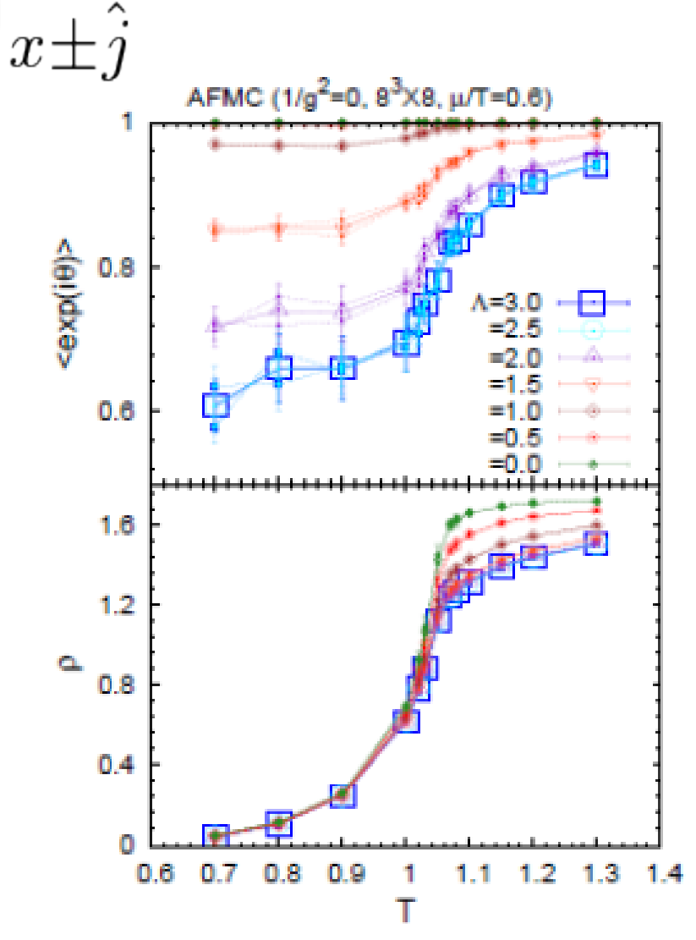
Effective mass term

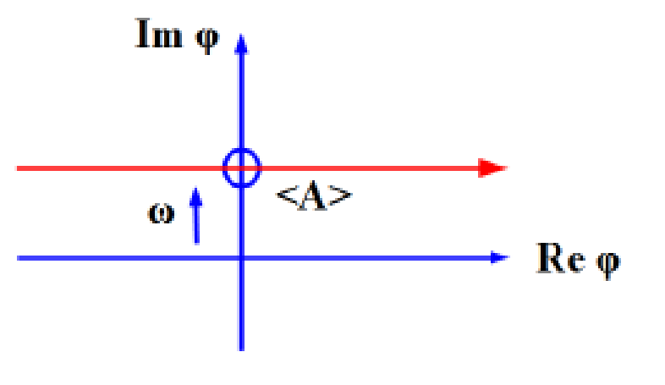
$$m_x = m + \frac{1}{4N_c}\sum_j(\sigma + \boxed{i\epsilon\pi})_{x\pm\hat j}$$

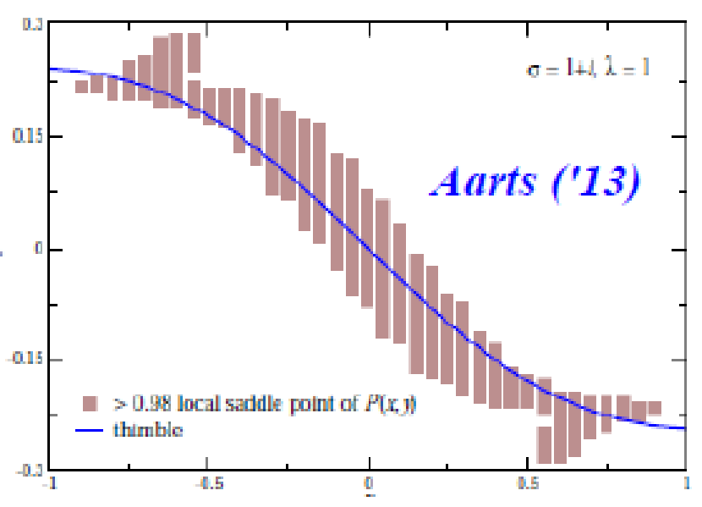
$$\boxed{\epsilon = (-1)^{x_0+\cdots+x_d}} \sim\gamma_5$$

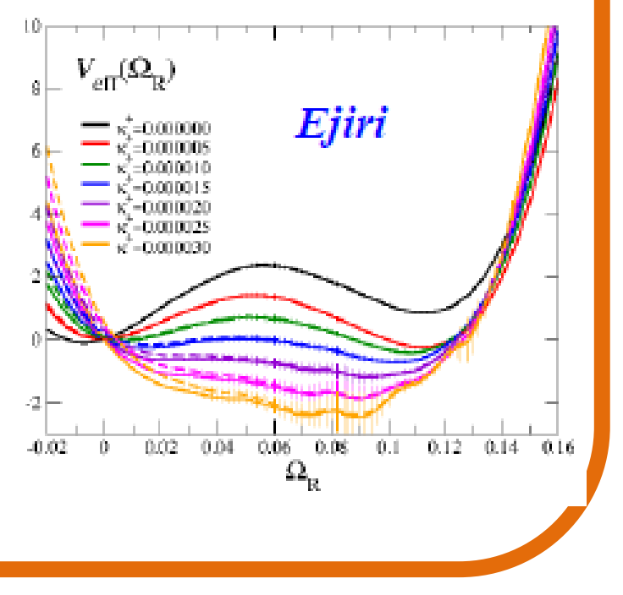
- Imaginary terms do not contribute as long as low momentum mode contribute → idea 1

- **Idea 1: Cutoff or Gauss integral of high momentum modes**
- **Idea 2: Change the integral path**
- **Idea 3: Combination of Fugacity exp. or Histogram method**









We investigated chiral phase transition in AFMC and deduced phase transition order from the behavior of the chiral condensate and finite size scaling analysis. We also show the average phase factor in AFMC, which originate from bosonization procedure, and some NLO effects. While we have the sign problem, we could show the QCD phase diagram in the strong coupling limit, which is consistent with MDP. Several prescriptions to weaken the sign problem were also discussed.