The Quark *Mass Gap* in *Strong* Magnetic Fields

($|eB| \gg \Lambda_{QCD}^2$)

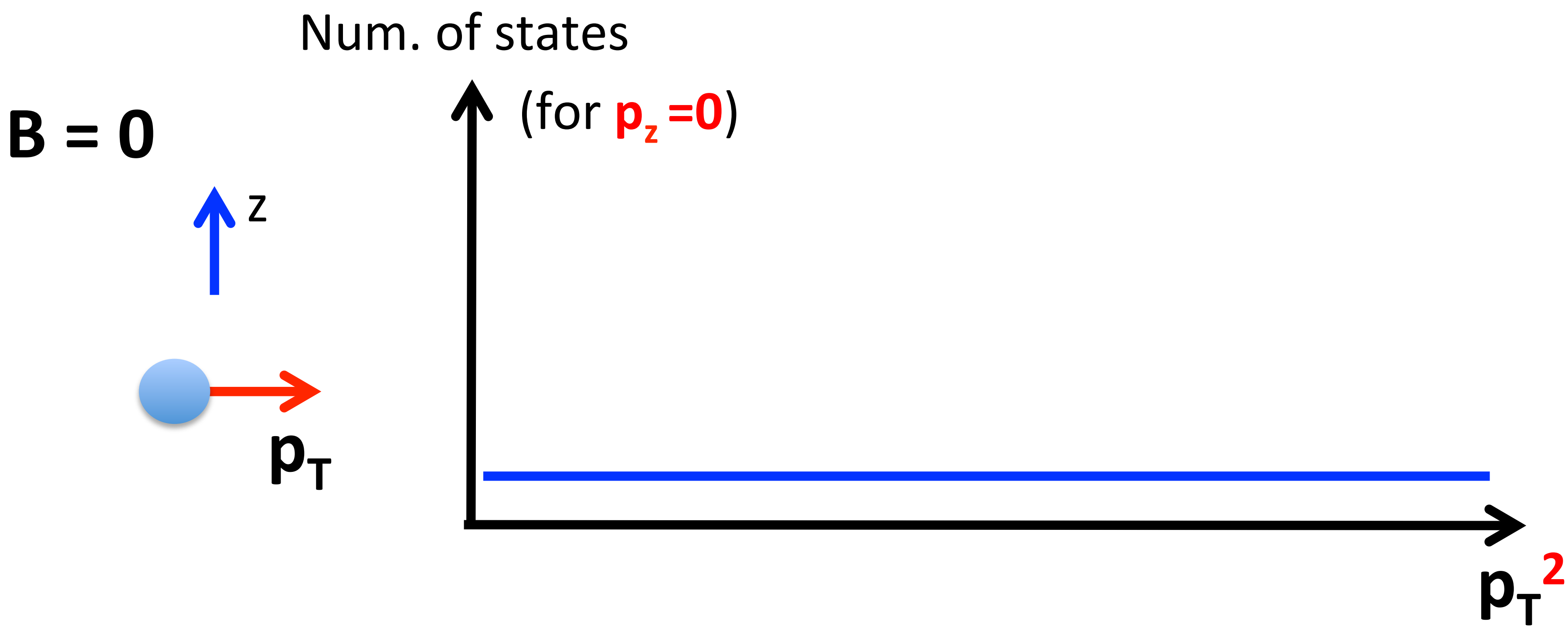
Toru Kojo (UIUC)

with Nan Su (Bielefeld)

Ref) PLB720 (2013), PLB726 (2013)

Quantum mechanics in mag. fields

(spinless, free particles)



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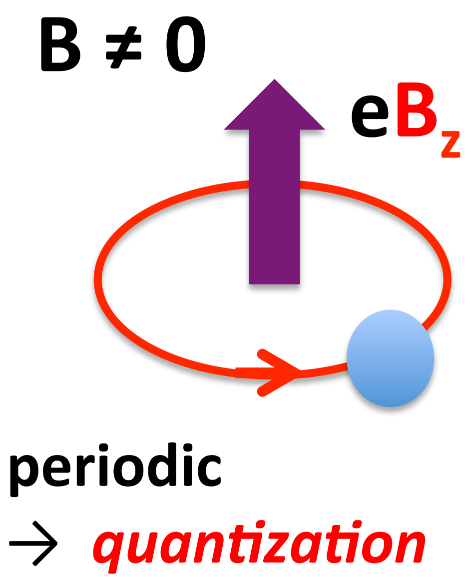
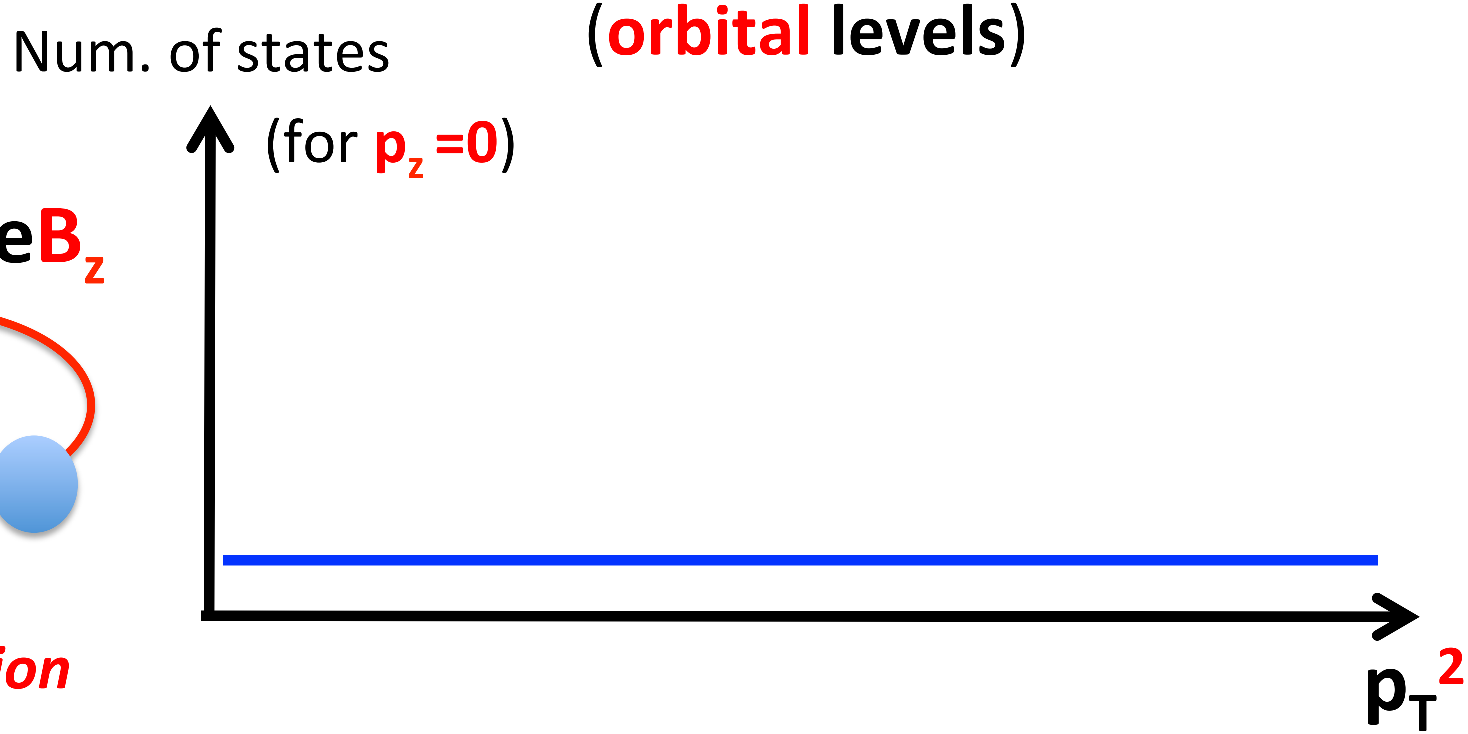


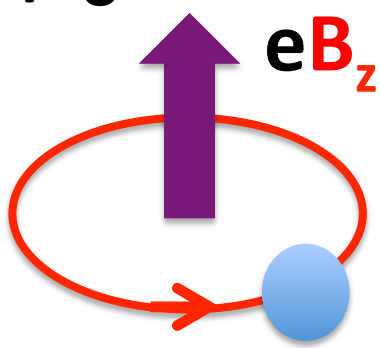


Quantum mechanics in mag. fields

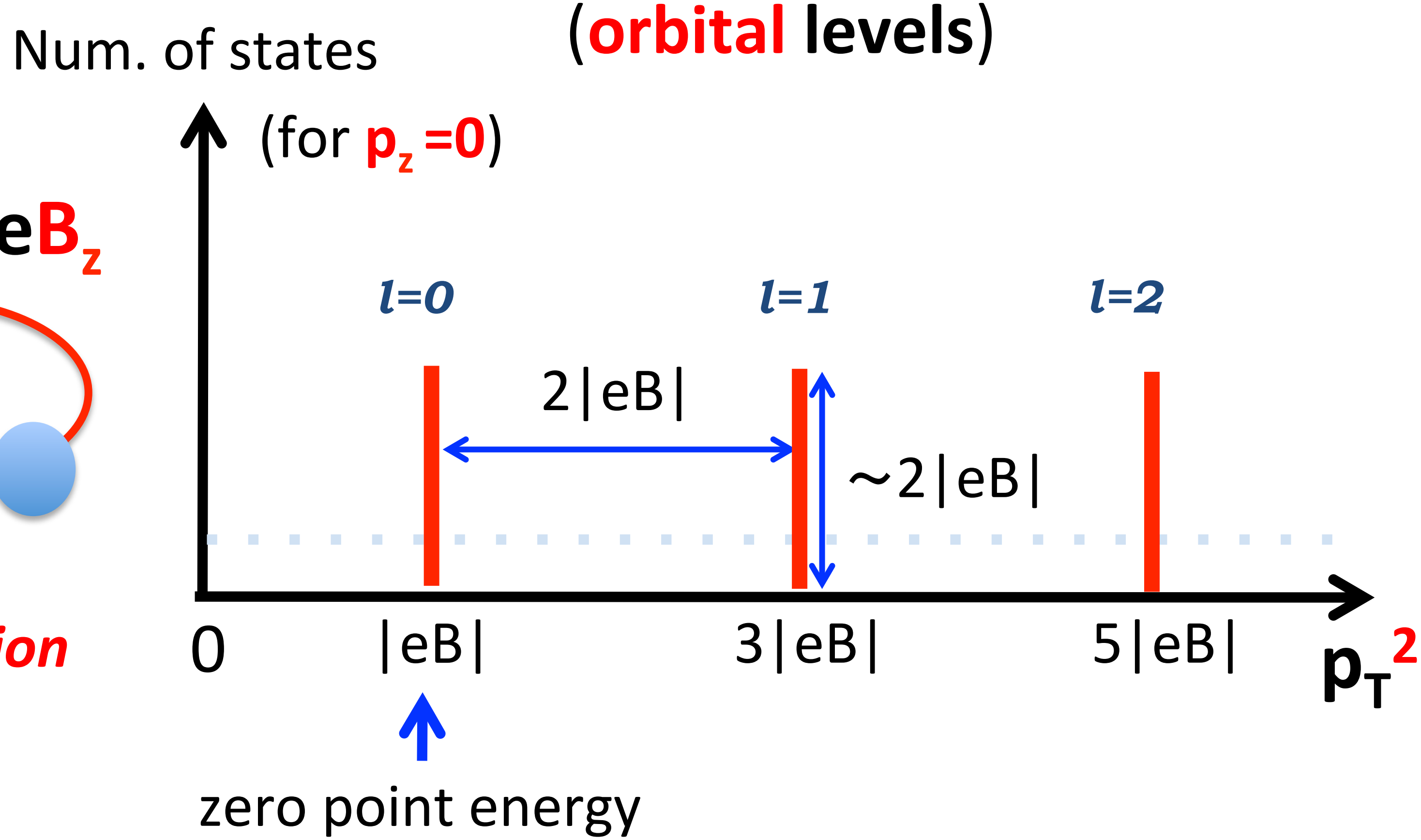
(spinless, free particles)

(orbital levels)

B ≠ 0

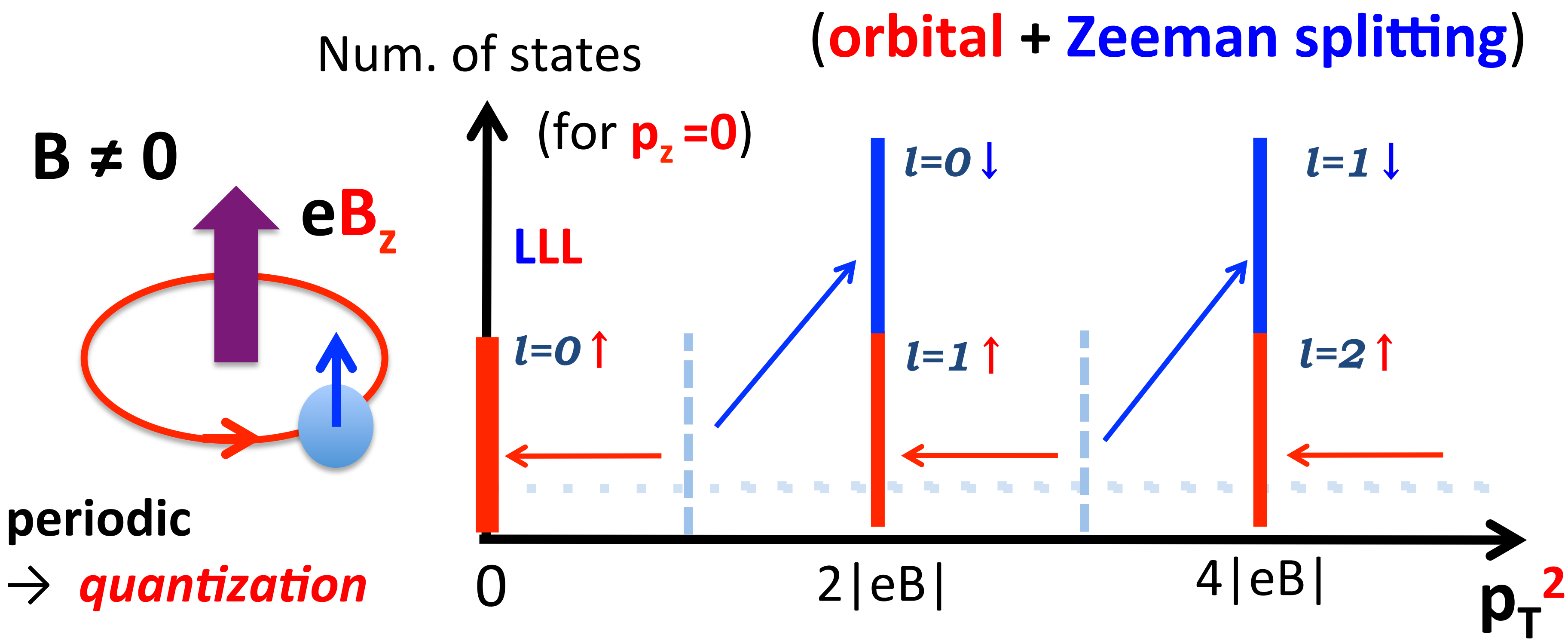


periodic
→ ***quantization***



Quantum mechanics in mag. fields

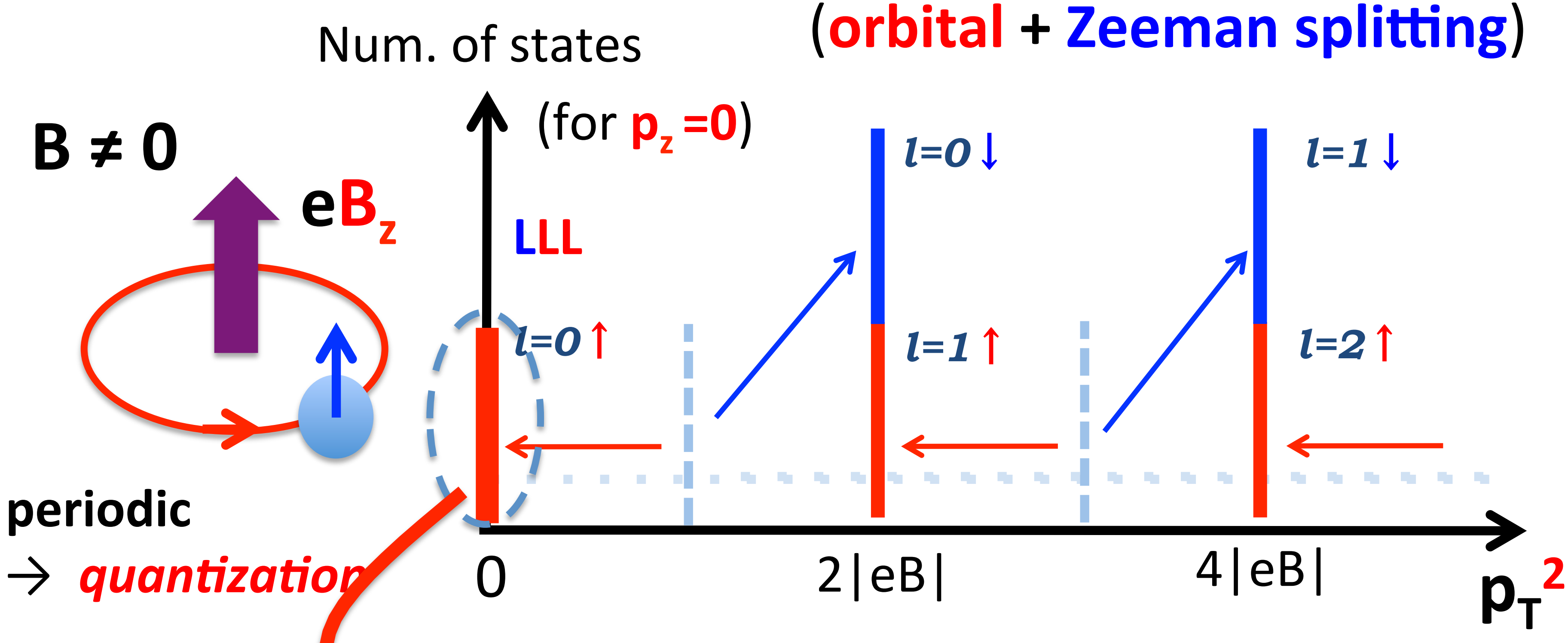
(spin 1/2, free particles)



Quantum mechanics in mag. fields

(spin 1/2, free particles)

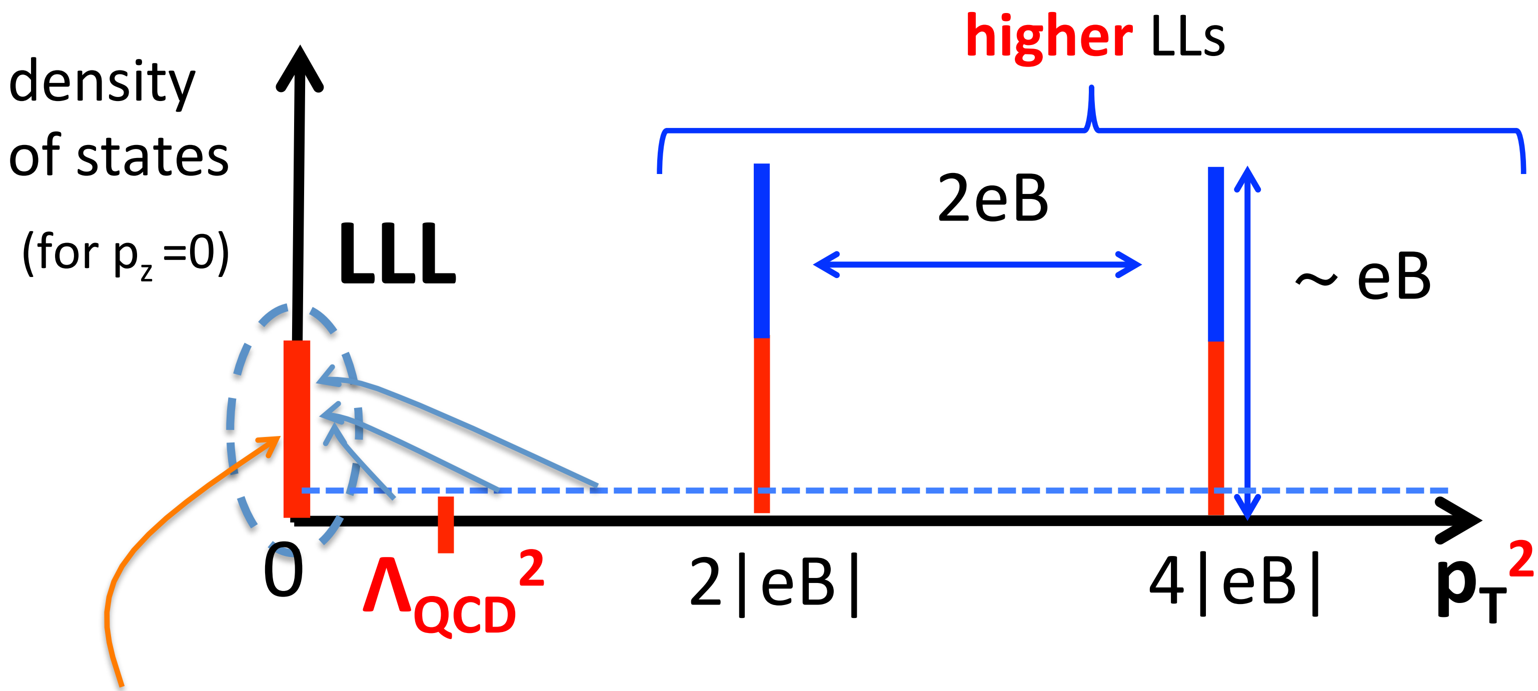
(orbital + Zeeman splitting)



- ***trans. Energy : zero & B-indep.***

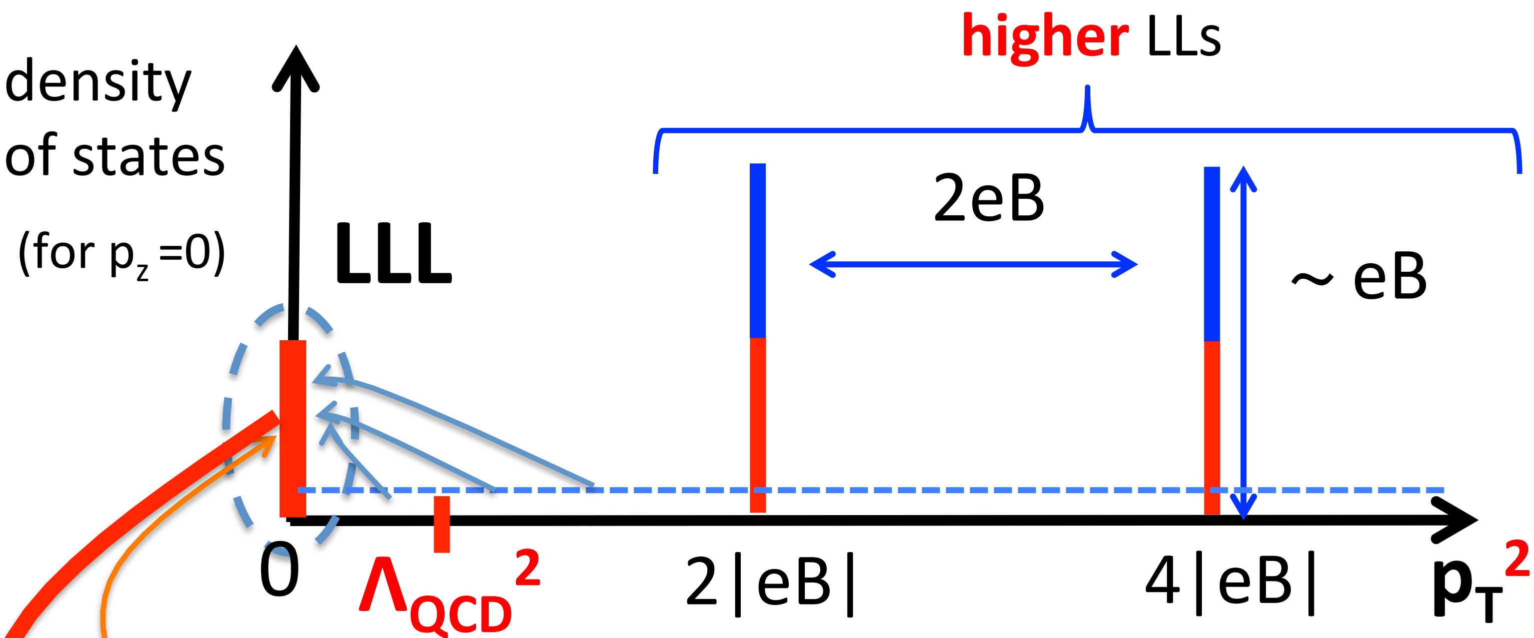
(at tree level)

“*Enhanced*” IR phase space for quarks



More quarks can stay at low energy than B=0 case.

“*Enhanced*” IR phase space for quarks



More quarks can stay at low energy than B=0 case.

New regime **to probe** ***non-pert. domains*** **of QCD,**

specialized to ***quantum fluctuations of quarks***

Important formula

"Ritus bases"

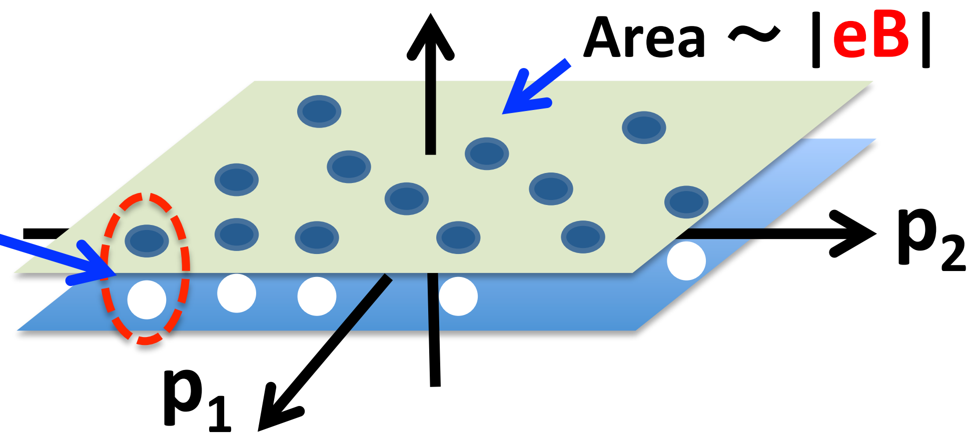
$$\langle\bar\psi(x)\psi(x)\rangle_{4\mathrm{D}} = \frac{|eB|}{2\pi}\int_{p_L}(-1)\,\mathrm{tr}\left[S^{2\mathrm{D}}_{\mathrm{LLL}}(p_L) + \sum_{n=1} S^{2\mathrm{D}}_{n\mathrm{LL}}(p_L)\right]$$

$$\boxed{\langle\bar\psi(x)\psi(x)\rangle_{4\mathrm{D}} = \frac{|eB|}{2\pi}\langle\bar\psi(x_L)\psi(x_L)\rangle_{2\mathrm{D}}}$$

(*degeneracy factor*)

Intuitively,

$\langle\bar\psi\psi\rangle_{2\mathrm{D}}$



Theoretical Problems:

Lattice vs Models

(*NJL or QED type models, ...*)

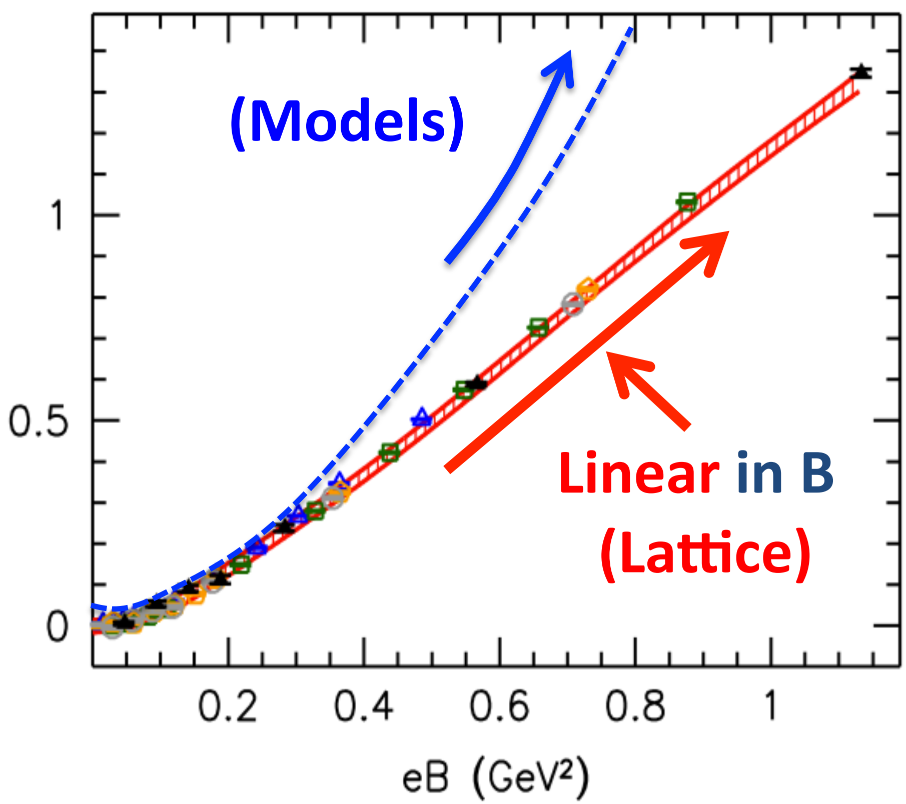
Problems: Lattice vs Models

(Bali et al, 11)

Problem 1)

$$\Delta\langle\bar{\psi}\psi\rangle_B/\langle\bar{\psi}\psi\rangle_{B=0}$$

(T = 0)



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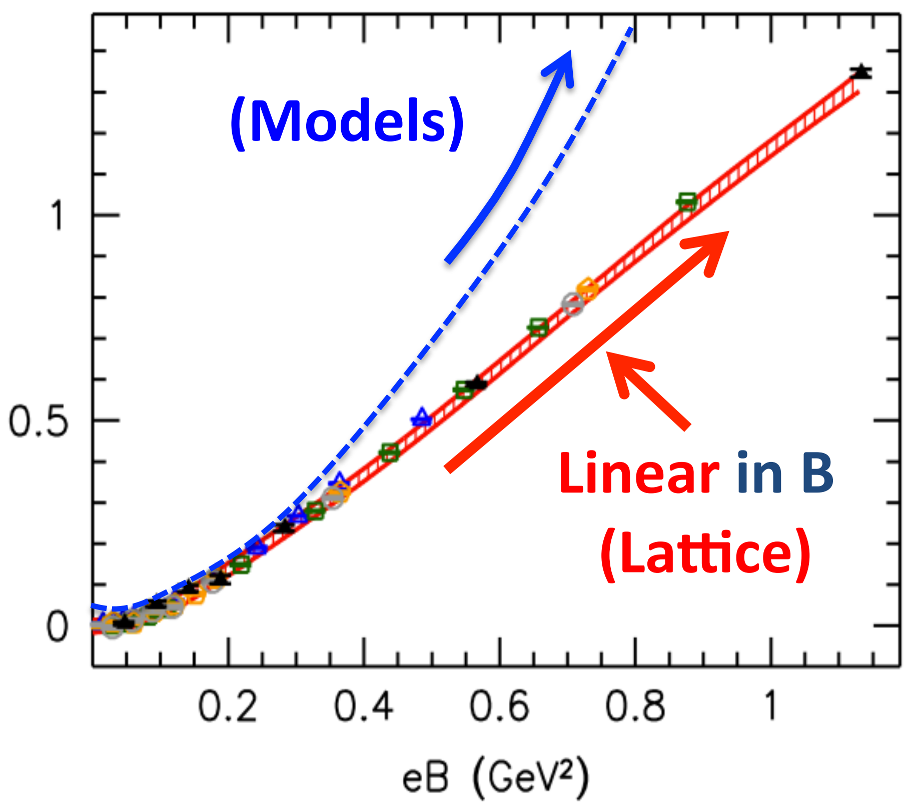
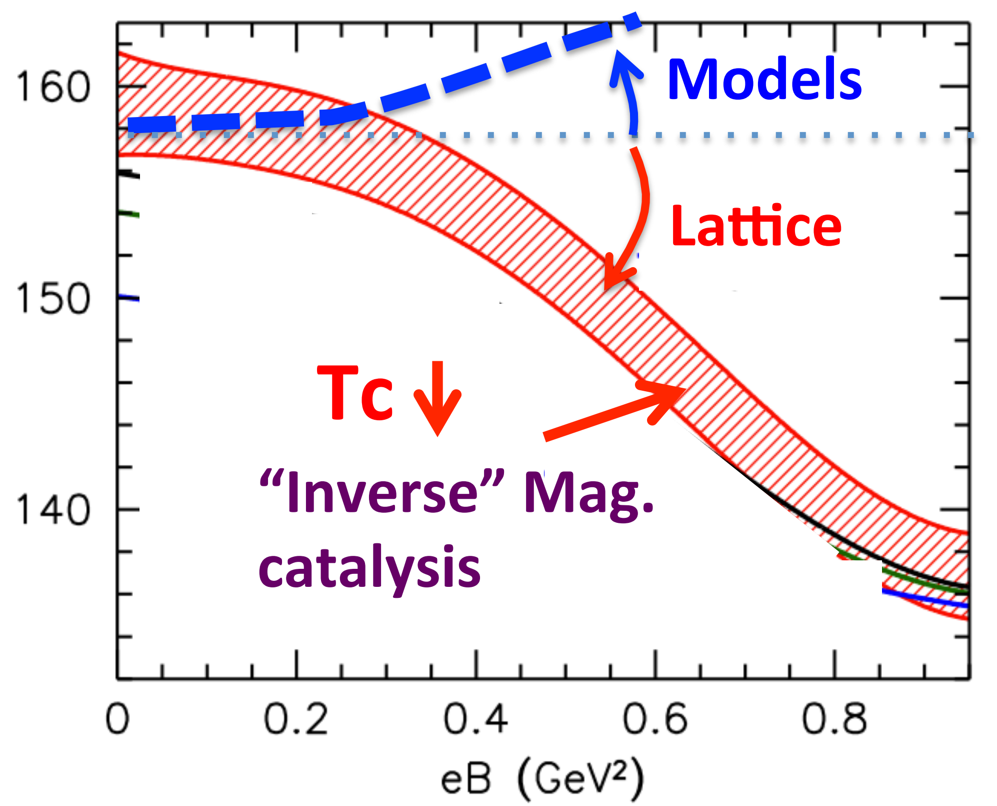


Problem 2)

T_{chiral} $(\sim T_{deconf.})$



Origin of problems (models)

(The NJL, QED-like treatments, ...)

$$M_q \sim |eB|^{1/2}$$ **(models)**

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Problem 1)

$$\langle \bar{\psi}\psi \rangle_{\mathrm{2D}} \sim |eB|^{1/2}$$

$\times$ **|eB|**

$$\langle \bar{\psi}\psi \rangle_{\mathrm{4D}} \sim |eB|^{3/2}$$

$\neq$ **lattice data** $\propto$ **|eB|**

Origin of problems (models)

(The NJL, QED-like treatments, ...)

$$M_q \sim |eB|^{1/2}$$ (models)

Problem 1)

$$\langle \bar{\psi}\psi \rangle_{2D} \sim |eB|^{1/2}$$

× |eB|

$$\langle \bar{\psi}\psi \rangle_{4D} \sim |eB|^{3/2}$$

≠ lattice data ∝ |eB|

Problem 2)

Thermal fluct. of quarks will not be activated until

$$T \sim M_q \sim \underline{|eB|^{1/2}}$$

→ Tc grows as B ↑

≠ lattice data

Our Goal

We are going to show : for QCD

$$M_q \sim \Lambda_{\mathrm{QCD}} \quad \neq \quad |eB|^{1/2} \text{ (models)}$$

even at $|eB| >> \Lambda_{QCD}{}^2$!!

Our Goal

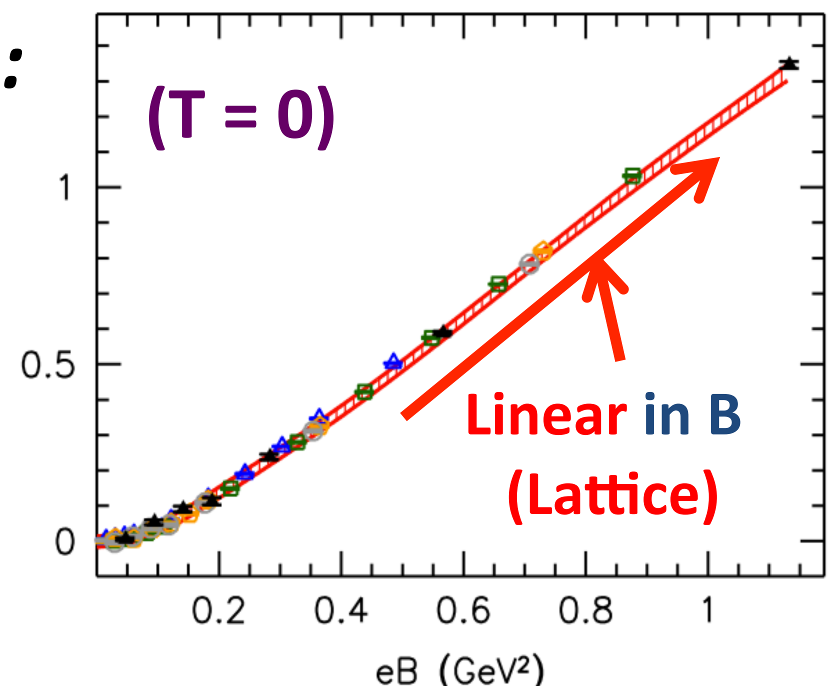
We are going to show : for QCD

$$M_q \sim \Lambda_{\rm QCD} \quad \neq \quad |eB|^{1/2} \ (\text{models})$$

even at $|eB| >> \Lambda_{QCD}{}^2$ ***!!***

If so, "problem 1" is solved :

$$\langle\bar{\psi}\psi\rangle_{\rm 4D} = \frac{|eB|}{2\pi}\, \underbrace{\langle\bar{\psi}\psi\rangle_{\rm 2D}}_{\sim\, \Lambda_{\rm QCD}}$$

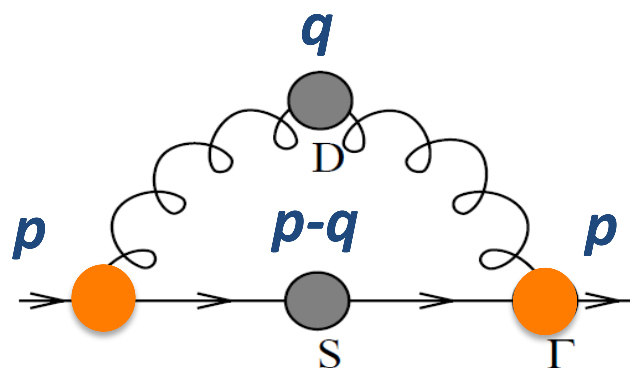


Structure of the Schwinger-Dyson eq.

(*for LLL*)

1) No explicit B-dep. for the LLL

2) No p_T-dep. → " factorization "



$$M(p_L) \sim \int_{q_L} S^{\rm 2D}_{\rm LLL}(p_L - q_L; M) \boxed{\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} \, D^{\rm 4D}_{\rm NP}(q_L, q_\perp)}$$

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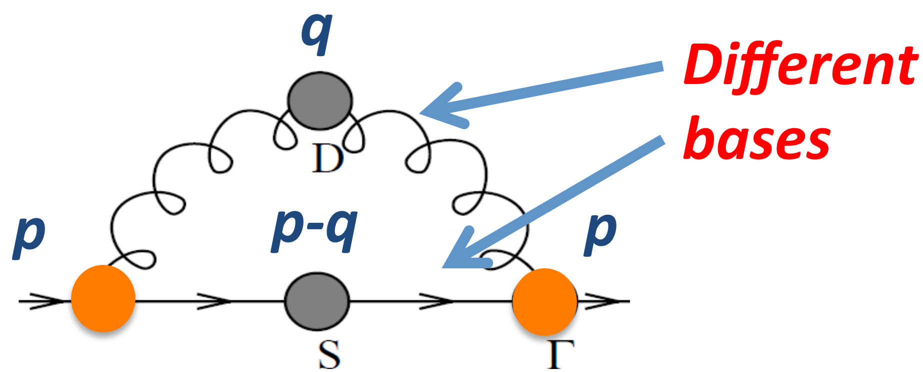

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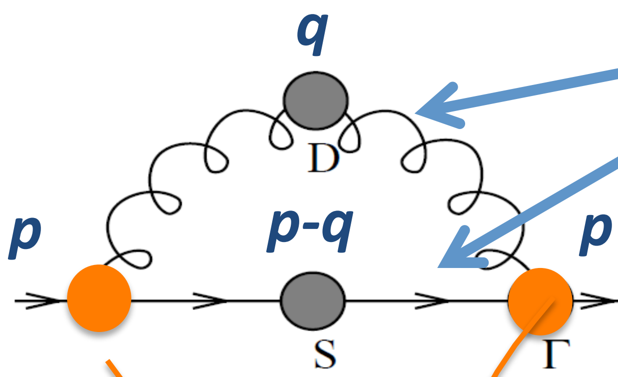

Different bases

" form factor "

$$M(p_L) \sim \int_{q_L} S^{\rm 2D}_{\rm LLL}(p_L - q_L; M) \boxed{\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} D^{\rm 4D}_{\rm NP}(q_L, q_\perp)}$$

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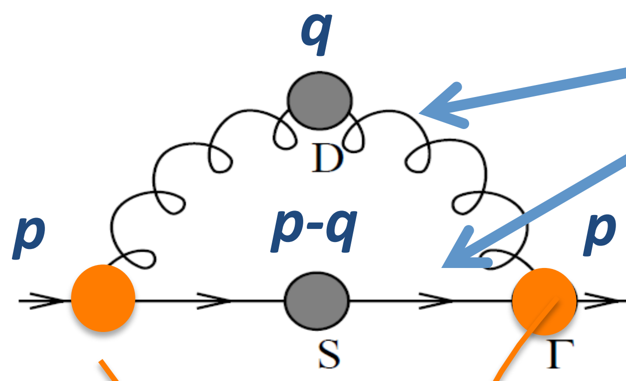

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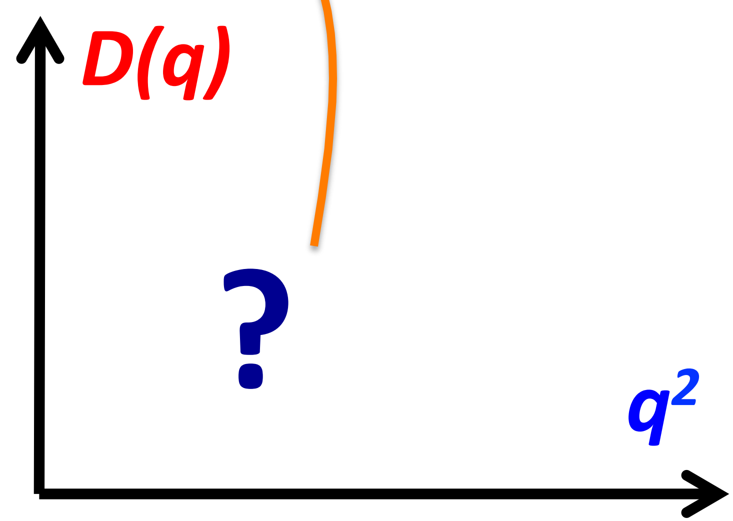
Key observation : All the *B-dep.* will come out from 2D " *smeared* " force !!

Comparison of forces, 1

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q_L, q_\perp)$$

Origin of all B-dep.



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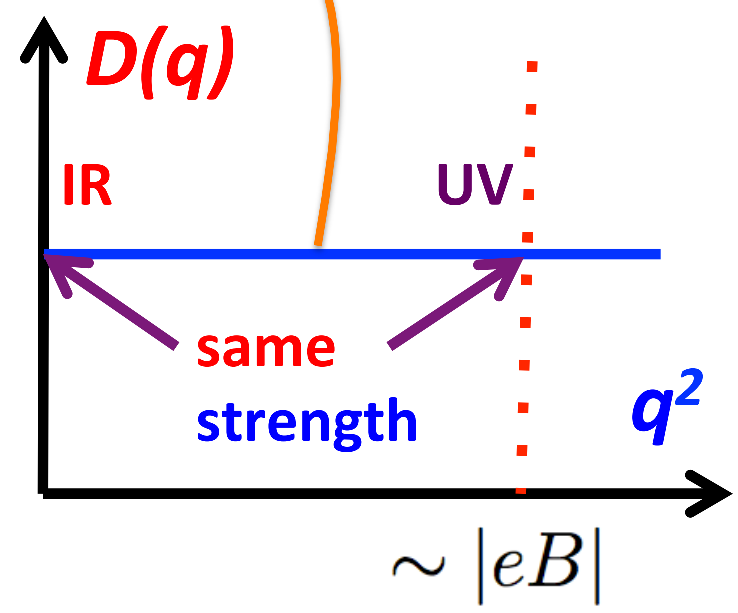
1) Contact int. (NJL, etc.)



Comparison of forces, 1

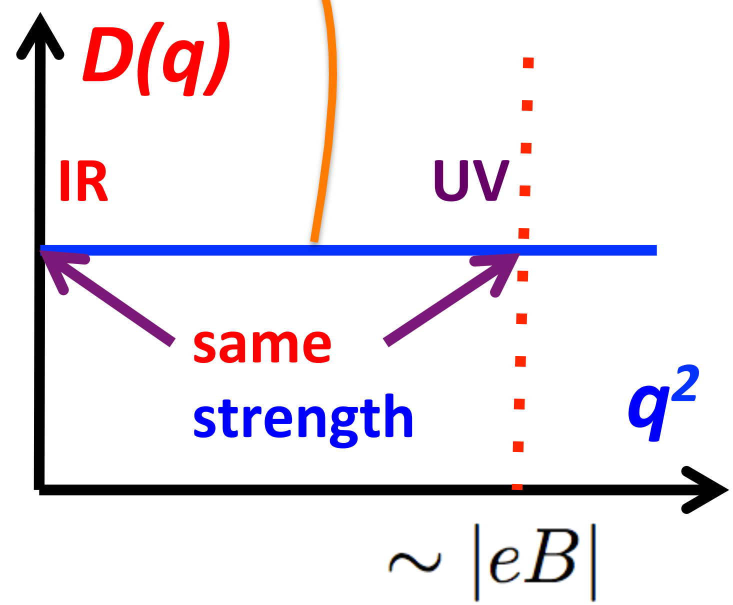
$$\int_{q_\perp} \underline{e^{-\frac{q_\perp^2}{2|eB|}}}\, D_{\mathrm{NP}}^{\mathrm{4D}}(q_L, q_\perp)$$

Origin of all B-dep.

1) Contact int. (NJL, etc.)

$$\boxed{\sim \underline{|eB|} \times \mathrm{const.}}$$

2D Force is strongly B-dep.



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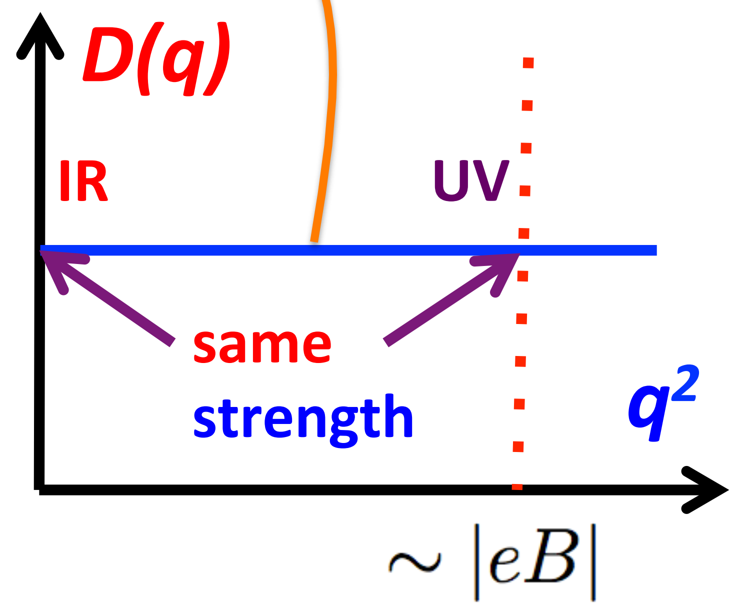
$$M \sim |eB|^{1/2}$$

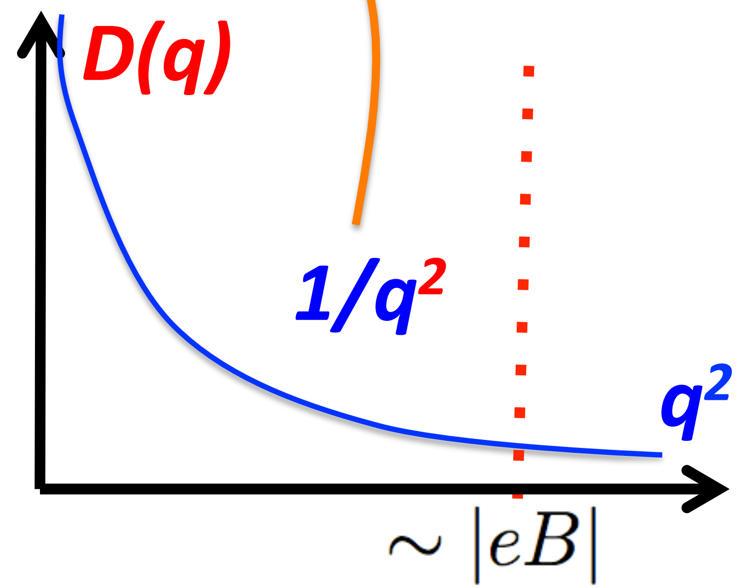


Comparison of forces, 2

$$\int_{q_\perp} \underline{e^{-\frac{q_\perp^2}{2|eB|}}}\, D_{\mathrm{NP}}^{\mathrm{4D}}(q_L, q_\perp)$$

Origin of all B-dep.

2) QED case (1/q² force)



Comparison of forces, 2

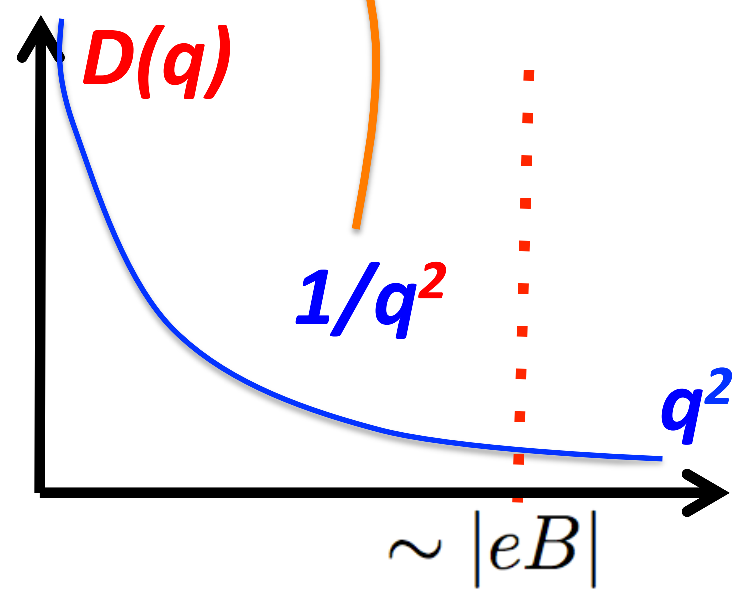
$$\int_{q_\perp} \underline{e^{-\frac{q_\perp^2}{2|eB|}}}\, D^{4D}_{NP}(q_L, q_\perp)$$

Origin of all B-dep.

2) QED case ($1/q^2$ force)

$$\boxed{\sim \ln \frac{q_L^2}{\underline{|eB|}}}$$

2D Force is still marginally B-dep.



Comparison of forces, 2

$$\int_{q_\perp} \underline{e^{-\frac{q_\perp^2}{2|eB|}}}\, D^{4D}_{NP}(q_L, q_\perp)$$

Origin of all B-dep.

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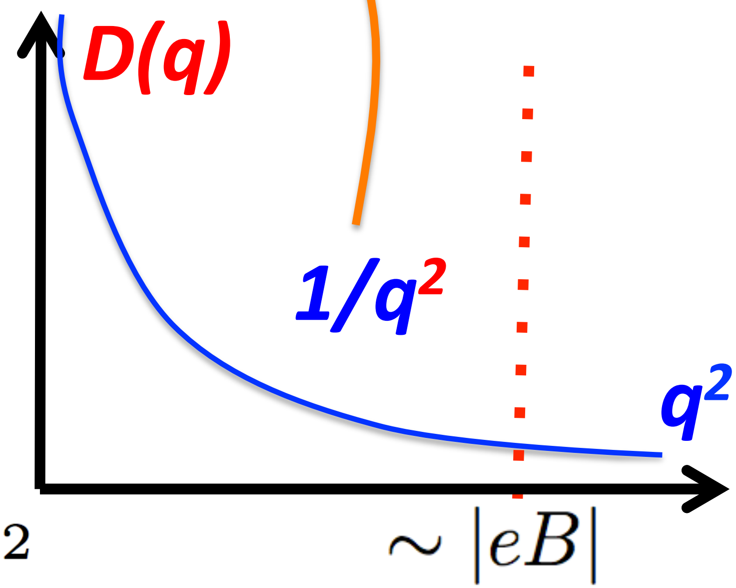


$$M \sim |eB|^{1/2}\, \underline{e^{-O(1)/\alpha^{1/2}}}$$

(exponentially small)

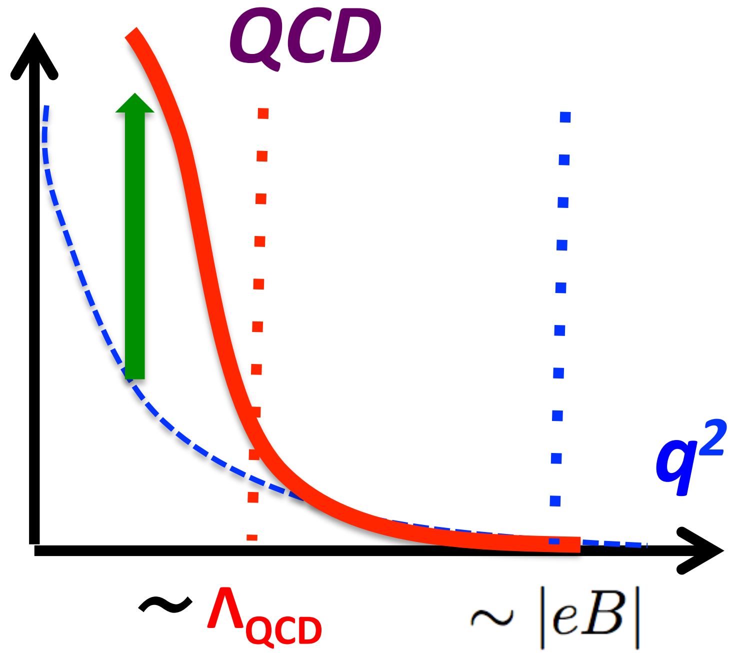
Comparison of forces, 3

Suppose: QCD force has stronger "*IR enhancement*"

$$\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} D^{4\mathrm{D}}(q_L, q_\perp)$$

For small q_{perp} $\sim \Lambda_{QCD}$:

we can set : $\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} \sim 1$



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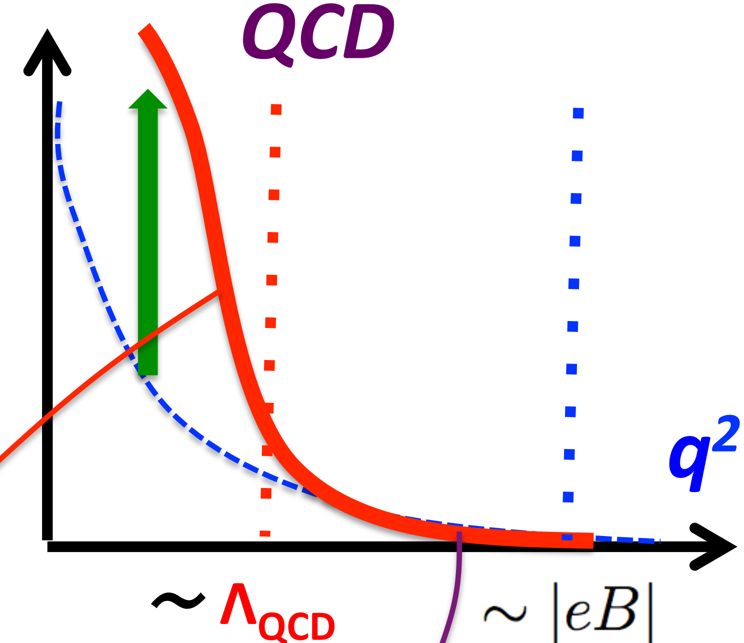
$$\sim \int_0^{\underline{\sim\Lambda_{\mathrm{QCD}}^2}} \mathrm{d}q_\perp^2 D^{4\mathrm{D}}(q_L, q_\perp)$$



+ *small B-dep. corrections*

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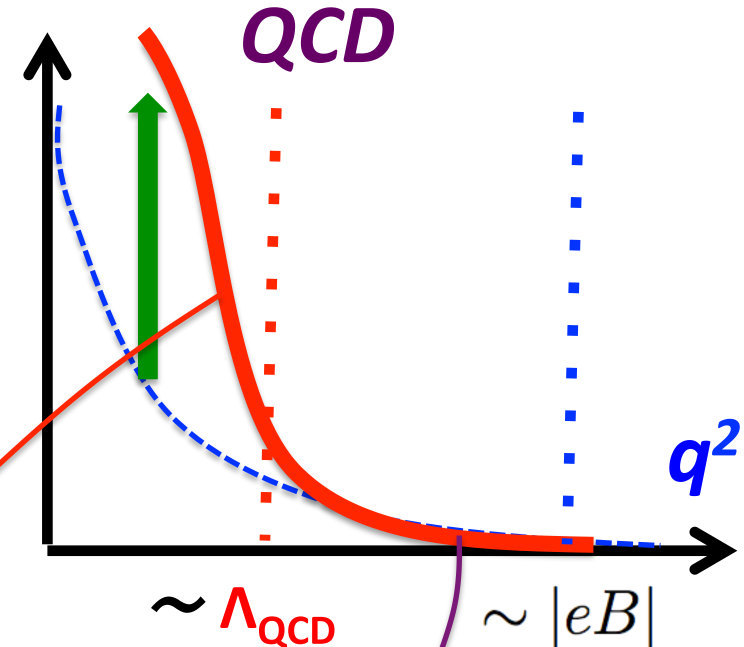

$$\sim \int_0^{\sim\Lambda_{\mathrm{QCD}}^2} \mathrm{d}q_\perp^2 D^{4\mathrm{D}}(q_L, q_\perp)$$

+ *small B-dep. corrections*

The dominant part ➡ $M \sim \Lambda_{\mathrm{QCD}}$ ***"nearly B-indep."***

"Thermal fluct." of quarks

At $T \sim \Lambda_{\mathrm{QCD}}$) *allowed phase space* *Boltzmann factor*

$B = 0$) $$\sim \Lambda_{\mathrm{QCD}}^3 \times \mathrm{e}^{-\frac{E}{\Lambda_{\mathrm{QCD}}}}$$ $E \sim \Lambda_{\mathrm{QCD}}$

"Thermal fluct." of quarks

At $T \sim \Lambda_{QCD}$)

allowed phase space

Boltzmann factor

$B = 0$) $\sim \Lambda_{\mathrm{QCD}}^3 \times \mathrm{e}^{-\frac{E}{\Lambda_{\mathrm{QCD}}}}$ $E \sim \Lambda_{\mathrm{QCD}}$

B ↑

$M \sim |eB|^{1/2}$ (model)

Boltzmann factor

→ $\mathrm{e}^{-\frac{|eB|^{1/2}}{\Lambda_{\mathrm{QCD}}}}$ << 1

Reduced thermal fluct.

Tc ↑

"Thermal fluct." of quarks

At $T \sim \Lambda_{QCD}$)　*allowed phase space*　*Boltzmann factor*

$B = 0$)　$$\sim \Lambda_{\mathrm{QCD}}^3 \times e^{-\frac{E}{\Lambda_{\mathrm{QCD}}}}$$　$E \sim \Lambda_{\mathrm{QCD}}$

B ↑

$M \sim |eB|^{1/2}$ (model)

$M \sim \Lambda_{QCD}$ (QCD)

Boltzmann factor

→ $$e^{-\frac{|eB|^{1/2}}{\Lambda_{\mathrm{QCD}}}} \ll 1$$

Reduced thermal fluct.

Tc ↑

Boltzmann factor

→ ***No big change.***

"Thermal fluct." of quarks

At $T \sim \Lambda_{QCD}$) allowed phase space Boltzmann factor

B = 0) $\sim \Lambda_{\mathrm{QCD}}^3 \times \mathrm{e}^{-\frac{E}{\Lambda_{\mathrm{QCD}}}}$ $E \sim \Lambda_{\mathrm{QCD}}$

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$M \sim |eB|^{1/2}$ (model)

$M \sim \Lambda_{QCD}$ (QCD)

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→ No big change.

Allowed phase space

→ $|eB| \times \Lambda_{QCD}$ (enhanced !)

"Thermal fluct." of quarks

At $T \sim \Lambda_{QCD}$) allowed phase space Boltzmann factor

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$M \sim \Lambda_{QCD}$ (QCD)

Boltzmann factor

→ $\mathrm{e}^{-\frac{|eB|^{1/2}}{\Lambda_{\mathrm{QCD}}}}$ << 1

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Boltzmann factor

→ No big change.

Allowed phase space

→ $|eB| \times \Lambda_{QCD}$ (enhanced !)

"Enhanced" thermal fluct.

Tc ↓

Summary

1 **)** ***Enhanced IR phase space*** **for quarks**

→ ***New regime*** ***for Non-pert. QCD***

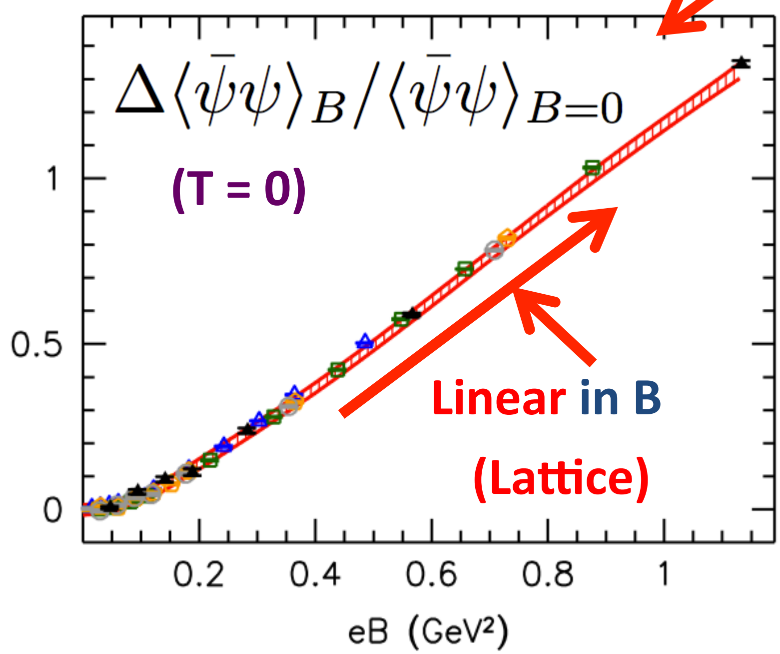
2 **)** ***Resolutions*** **of theoretical paradox**

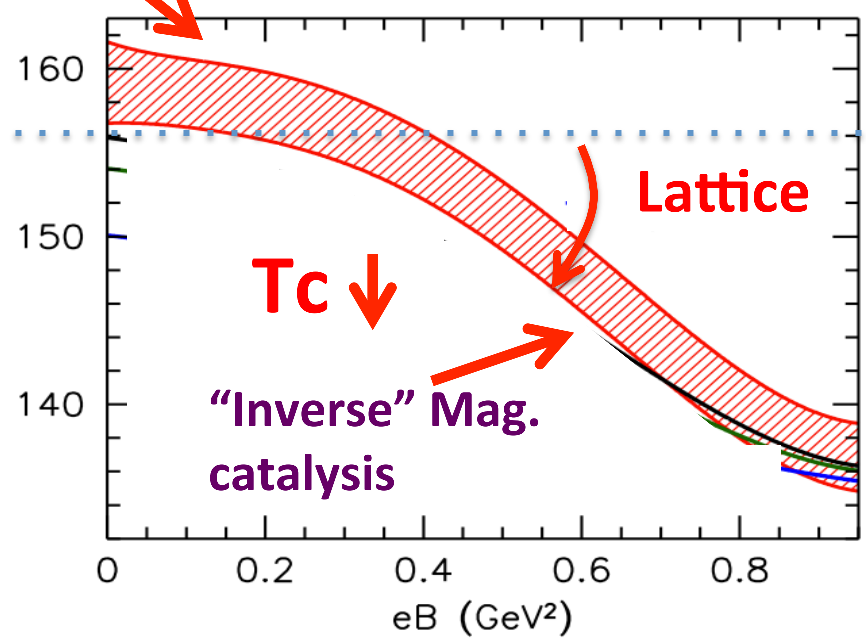
$$\langle\bar{\psi}\psi\rangle_{\rm 4D} = \frac{|eB|}{2\pi}\langle\bar{\psi}\psi\rangle_{\rm 2D}$$

$$M_q \sim \Lambda_{\rm QCD}$$

"Enhanced" thermal fluct.

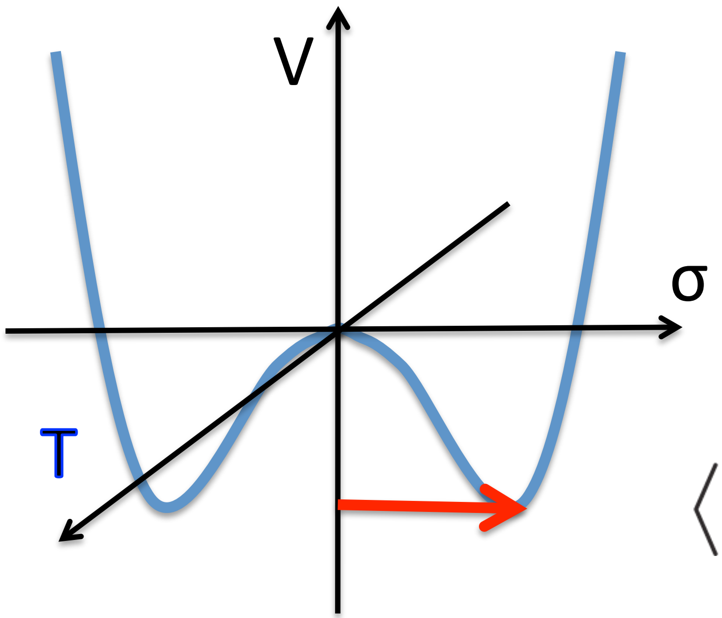
(enhanced IR phase space)



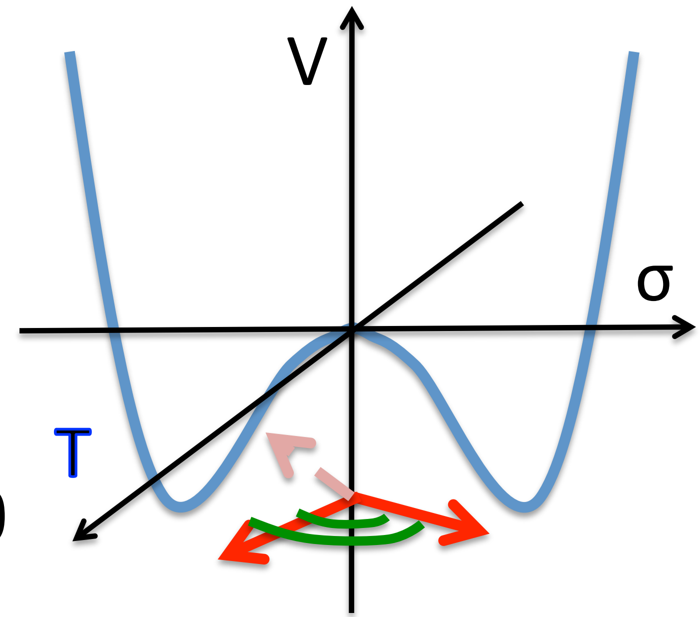


Backup

Phase fluctuations



$\langle \rho \rangle \neq 0$



$\langle e^{i\theta} \rangle \neq 0$ (SSB)

$\langle e^{i\theta} \rangle = 0$ (No SSB)

IR divergence in (1+1)D phase dynamics

- Phase fluctuations belong to:

Excitations (physical pion spectra)	ground state properties (No pion spectra)

The reasons to study ***mag.*** QCD

1, Theories can be confronted with the lattice results.

(***No sign problem, systematic studies***)

2, Simple qualitative problems are still available (see below)

- **They discriminate models from QCD .**

3, Suitable for studies of non-thermal fluct. of quarks :

(***Quantum***, T=0)

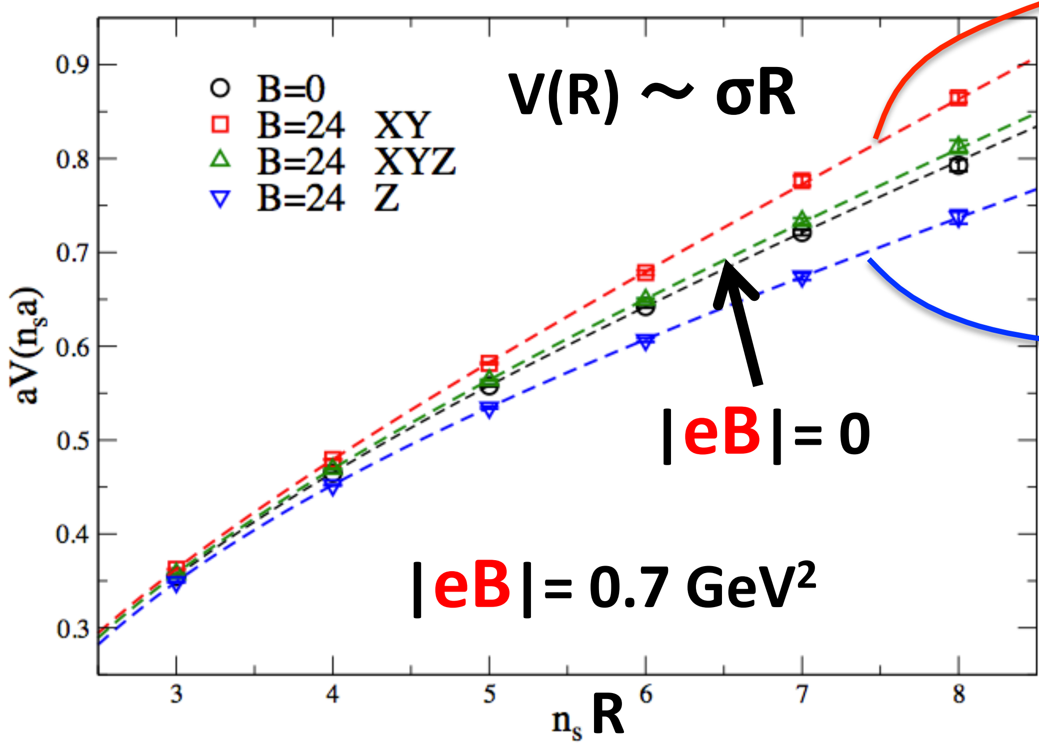
→ Extremely important for studies of cold quark matter.

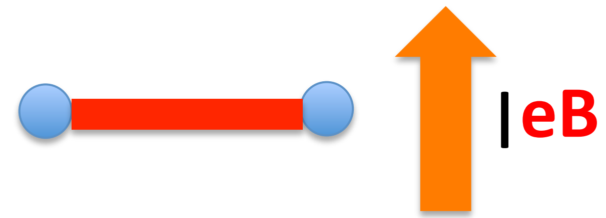
- **Test of 1/Nc : Back-reaction to the gluon sector,**
- **Quantum phase transition, ...**

Models vs Lattice, 3: *String tension*

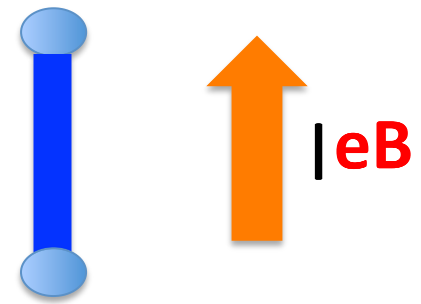
Heavy quark potential (T=0)

Lattice, (2+1) phys. pion (Bonati et al, 2014)





σ → 10 % enhancement



σ → 10 % “ reduction ”

hard to explain *if* $M \sim |eB|^{1/2}$ (models)

(because *back-reaction* is suppressed)

Field theory bases : quark part

" Ritus bases for non-int. fermions in B "

1) Choose the gauge for EM fields : e.g.) $A_2^{\mathrm{em}} = Bx_1$

2) Apply "*spin projection*" :

(σ_z : spin)

$$\psi_\pm \equiv \mathcal{P}_\pm \psi \qquad \mathcal{P}_\pm = \frac{1 \pm \mathrm{i}\gamma_1\gamma_2\,\mathrm{sgn}(e_f B)}{2}$$

3) Expand by proper spatial wavefunctions :

$$\psi_\pm(x) = \sum_{l=0} \int \frac{\mathrm{d}^2 p_L \mathrm{d}p_2}{(2\pi)^3}\, \psi^\pm_{l,p_2}(p_L)\, \underline{H_l\left(x_1 - \frac{p_2}{B}\right)}\, \mathrm{e}^{-\mathrm{i}p_2 x_2}\, \mathrm{e}^{-\mathrm{i}p_L x_L}$$

$$p_L \equiv (p_0, p_z)$$

Harmonic oscillator w.f. with

$$m\omega = |eB|$$

Field theory bases : quark part

The action for the LLL (n=0): $\chi = \psi_+^{l=0}$

$$\mathcal{S}_{\rm LLL} = \int_{p_L, p_2} \bar{\chi}_{p_2}(p_L) \underline{(-\mathrm{i}\not{p}_L + m)} \chi_{p_2}(p_L)$$

(No B-dep. !)

for the n-th LLs : $\psi_n = \psi_+^{l=n} + \psi_-^{l=n-1}$

$$\mathcal{S}_{\rm nLL} = \int_{p_L, p_2} \bar{\psi}_{n,p_2}(p_L) \Big(-\mathrm{i}\not{p}_L + \underline{\mathrm{i}\, \mathrm{sgn}(eB) \sqrt{2n|eB|}\gamma_2} + m \Big) \psi_{n,p_2}(p_L)$$

The propagators :

diagonal

$$\langle \psi_{n,p_2}(p_L) \bar{\psi}_{n',p_2'}(p_L') \rangle = \underline{S_n^{\rm 2D}(p_L)} \times \delta_{nn'} \delta(p_2 - p_2') \delta^2(p_L - p_L')$$

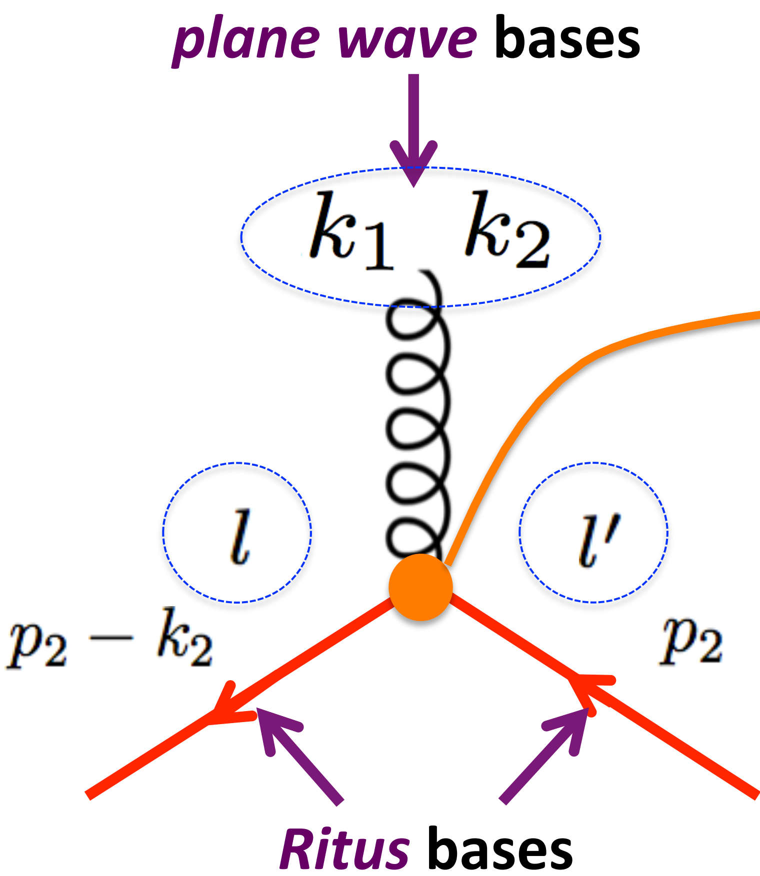
(1+1)-dimensional for each index "n"

(***depend only on*** $\mathbf{p_L}$)

Couplings b.t.w. different LLs

$$S_{\rm int} = \int_x \bar{\psi}(x)\gamma_\mu t_a \psi(x)\, A^a_\mu(x)$$

4D Gluons couple to different LLs.



" form factor " $\Delta l = |l - l'|$

$$I_{l,l'}(\vec{k}_\perp) \propto \left(\frac{k_\perp^2}{2|eB|}\right)^{\frac{\Delta l}{2}} \mathrm{e}^{-k_\perp^2/4|eB|}$$

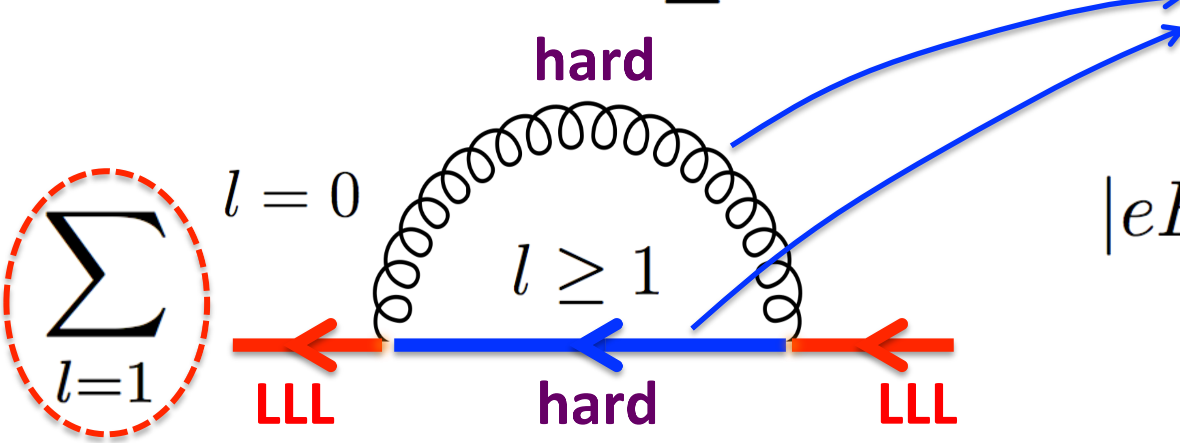
For *Δl ≠ 0* processes :

small overlap with soft gluons

(Only *Δl = 0* process are dangerous)

LLL mass gap : 3-distinct contributions

1) Coupling with $l \geq 1$



"Perturbative"

under control for

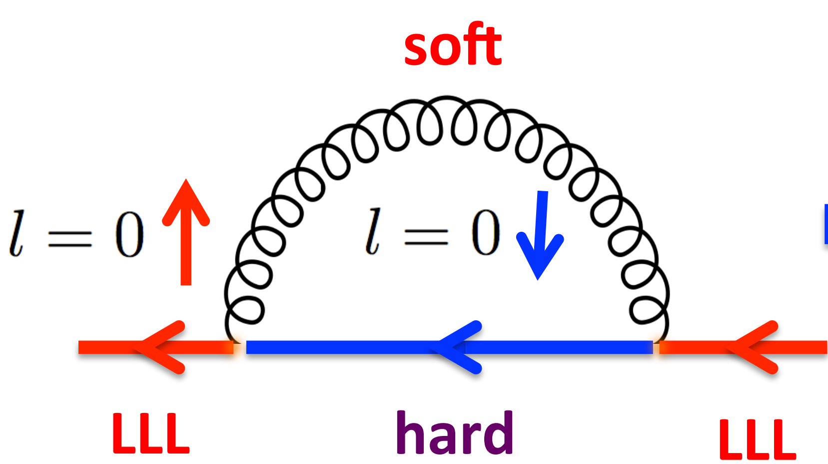
$$|eB| \geq (0.1-0.3)\,\mathrm{GeV}^2$$

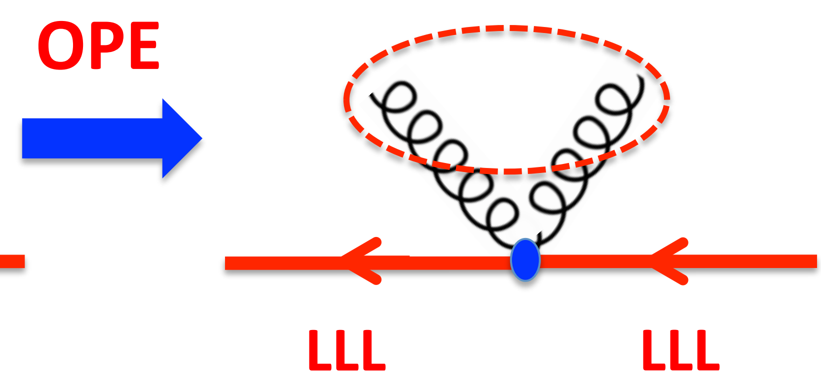
& very small B-dep.

T.K., Nan Su (2013)

2) Coupling with 1st LL but $l=0 \downarrow$

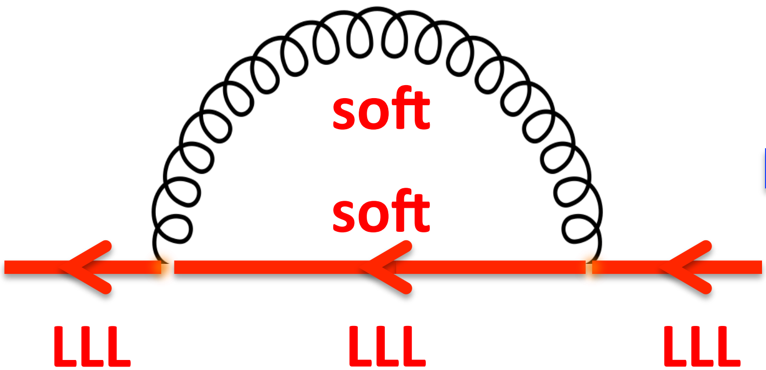
$$\sim \frac{m\langle G^2\rangle}{|eB|^2} \ll \Lambda_{\mathrm{QCD}}$$





LLL mass gap : 3-distinct contributions

3) Couplings within LLLs



→ Everything must be treated "Non-perturbatively"

Natural framework → Schwinger-Dyson eq.

with

Non-perturbative " *force* "

e.g.) full gluon propagator × full vertex for quenched QCD

Example) a *toy* model study

" *Linear rising* " *potential* for color charges

$$D_{\mu\nu} = C_F \times g_{\mu 0} g_{\nu 0} \times \frac{\sigma}{(\vec{p}^2)^2}$$

string tension

- Motivated by Coulomb gauge studies.

(ref: Gribov, Zwanziger)

- The model has " *IR enhancement* ".

- *Confining*, in the sense that

"No $q\bar{q}$ continuum in the meson spectra."

- *Oversimplifications* : *No 1/p^2 tail, No color mag. int.*, etc.

- *We will solve eqs. within " rainbow ladder "*

Schwinger-Dyson eq. for the LLL

e.g.) scalar part

$$M(p_L) = \int_{q_L} \gamma_0 \, S^{2D}_{LLL}(p_L - q_L; M)\, \gamma_0 \otimes \boxed{\int_{q_\perp} e^{-\frac{q_\perp^2}{2|eB|}} \, D^{4D}_{00}(q)}$$

for large B

$$\int_0^\infty dq_\perp^2 \, \frac{\sigma \, e^{-\frac{q_\perp^2}{2|eB|}}}{(q_\perp^2 + q_z^2)^2} \Rightarrow \frac{\sigma}{q_z^2} - \cancel{\frac{\sigma}{q_z^2 + \underline{2|eB|}}}$$

(confining in 2D)

The *B-dependence* dropped out, and we get

$$M(p_L) \simeq \int_{q_L} \gamma_0 \, S^{2D}_{LLL}(p_L - q_L; M)\, \gamma_0 \times \frac{\sigma}{q_z^2}$$

SD-eq. for *'t Hooft model* (QCD_2) in A_z =0 gauge

(the *Bethe-Salpeter* eq. can be also reduced to QCD_2)

Few comments on unquenched QCD

Now imagine back-reaction from quark to gluon sector :

"Thermal" quark fluctuations)

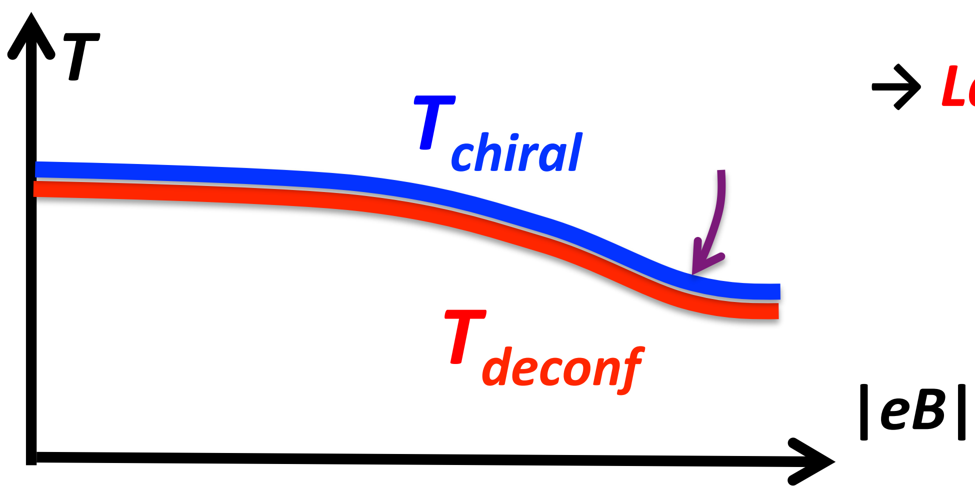
phase space　　Boltzmann factor

B = 0) $\sim \int \mathrm{d}^3\vec{p}\ \mathrm{e}^{-\frac{E}{T}} \sim \underline{\Lambda_{\mathrm{QCD}}^3} \times \mathrm{e}^{-\frac{\Lambda_{\mathrm{QCD}}}{T}}$

enhanced　　**No big change**

B ≠ 0) $\sim \underline{|eB|\,\Lambda_{\mathrm{QCD}}} \times \mathrm{e}^{-\frac{\Lambda_{\mathrm{QCD}}}{T}}$

→ ***Larger back-reaction*** **at larger B**



Bethe-Salpeter eq. for the LLLs

Consider ***meson currents*** for which

both quark & anti-quark can couple to the ***LLL states***.

(Some currents **CAN NOT**, see next slide.)

Dim. reduction can be carried out in the same way :

*Both **total** & **relative** momenta are **indep. of trans. momenta**.*

- *Quark & anti-quark **align** in the z-direction.*

Classifications of Mesons (2-flavor case)

Expanding quark fields by the Landau levels: $\psi^f = \psi^f_{\mathrm{LLL}} + \sum_{n=1} \psi^f_n$

we can pick out currents $\bar{\psi}\Gamma\psi$ for which

both quark & anti-quark can decay to the **LLL.**

List of light mesons:

neutral $(u\bar{u}\,, d\bar{d}) \otimes (1\,, \gamma_5\,, \gamma_L\,, \gamma_L\gamma_5\,, \sigma_{LL'}\,, \sigma_{\perp\perp'})$

charged $(u\bar{d}\,, d\bar{u}) \otimes (\gamma_\perp\,, \gamma_\perp\gamma_5\,, \sigma_{L\perp})$

e.g.)

- **Neutral** pion (charged pions do NOT).
- **Neutral**, **longitudinal** part of **vector** mesons.
- **Charged, transverse** part of **vector** mesons.

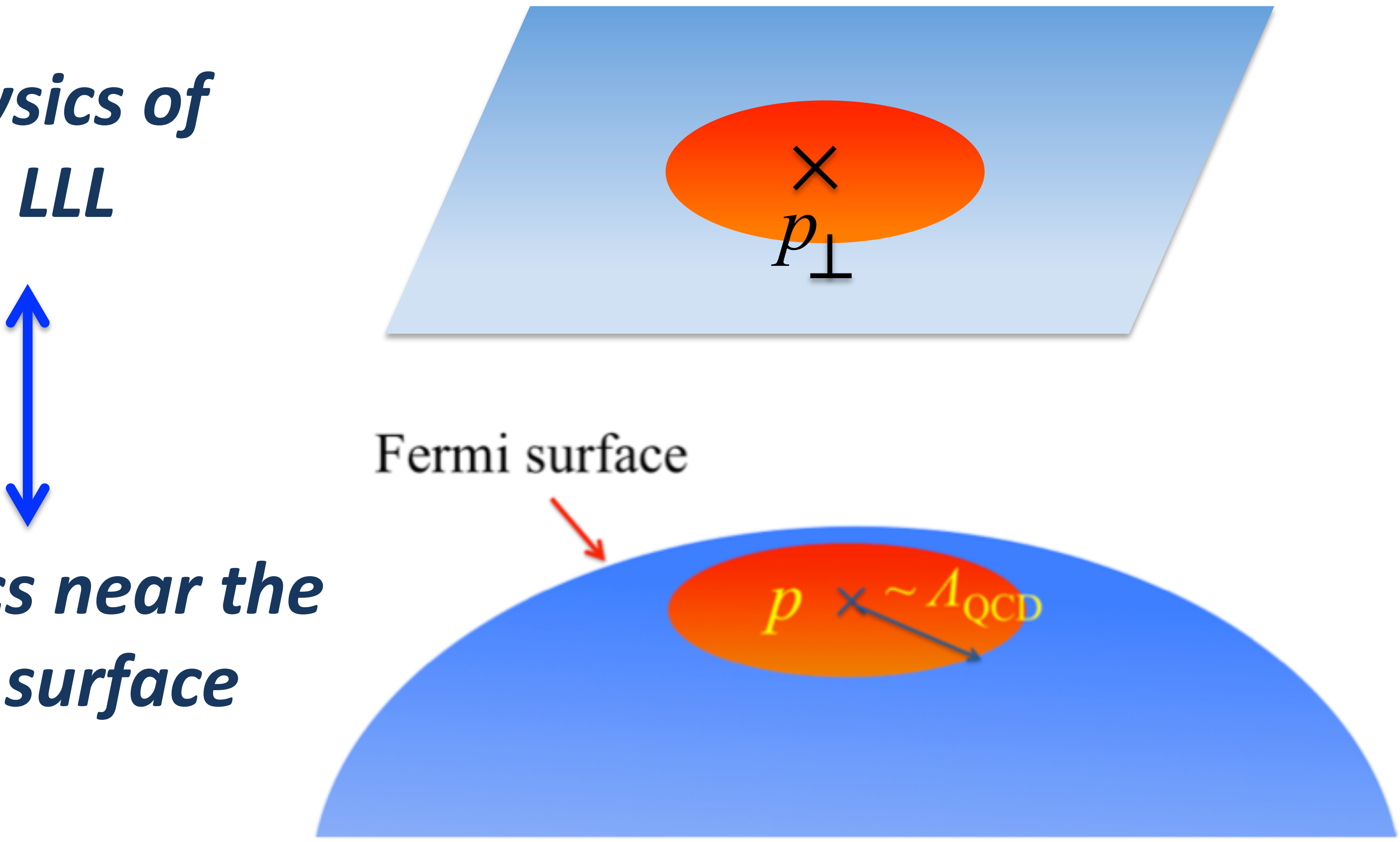
....

(***seems to be consistent with known lattice results.***)

Implications for dense QCD ?

Physics of the LLL

Physics near the Fermi surface



Similar modulo Fermi surface curvature

What's new? : History

1) *ChsB in mag. fields (concept) : 1989 -*

Klevansky-Lemmer (89), Suganuma-Tatsumi (90),

Gusynin-Miransky-Shovkovy (94-), ... **(for NJL, QED,...)**

(***Not specific to QCD***, "***universal*** **aspects" of fermions at B**)

2) *QCD in mag. fields (paradigm shift) : 2007 -*

Kharzeev-McLerran-Warringa (07), Fukushima-Kharzeev-Warringa (08),..

(***QCD topology & Its phenomenological applications***)

3) *Lattice studies on ChSB & Deconf. : 2008 -*

Buividovich et al. (2008) **(quenched)**

D'Elia-Muckherjee-Sanflippo (2010) **(full, heavy pion)**

Bali et al. (2012) **(full, physical pion)**

The reasons to study mag. QCD

1, Theories can be confronted with the lattice results.

(***No sign problem, systematic studies***)

2, Simple qualitative problems are still available.

- **They discriminate models from QCD (see next slides).**

3, Suitable for studies of non-thermal fluct. of quarks :

(***Quantum***, T=0)

→ Extremely important for studies of cold quark matter.

The reasons to study mag. QCD

1, Theories can be confronted with the lattice results.

(*No sign problem, systematic studies, test of approximations*)

2, Useful for studies of *non-thermal* fluct. of quarks :

(***Quantum***, T=0)

→ **Important for studies of *cold* quark matter :**

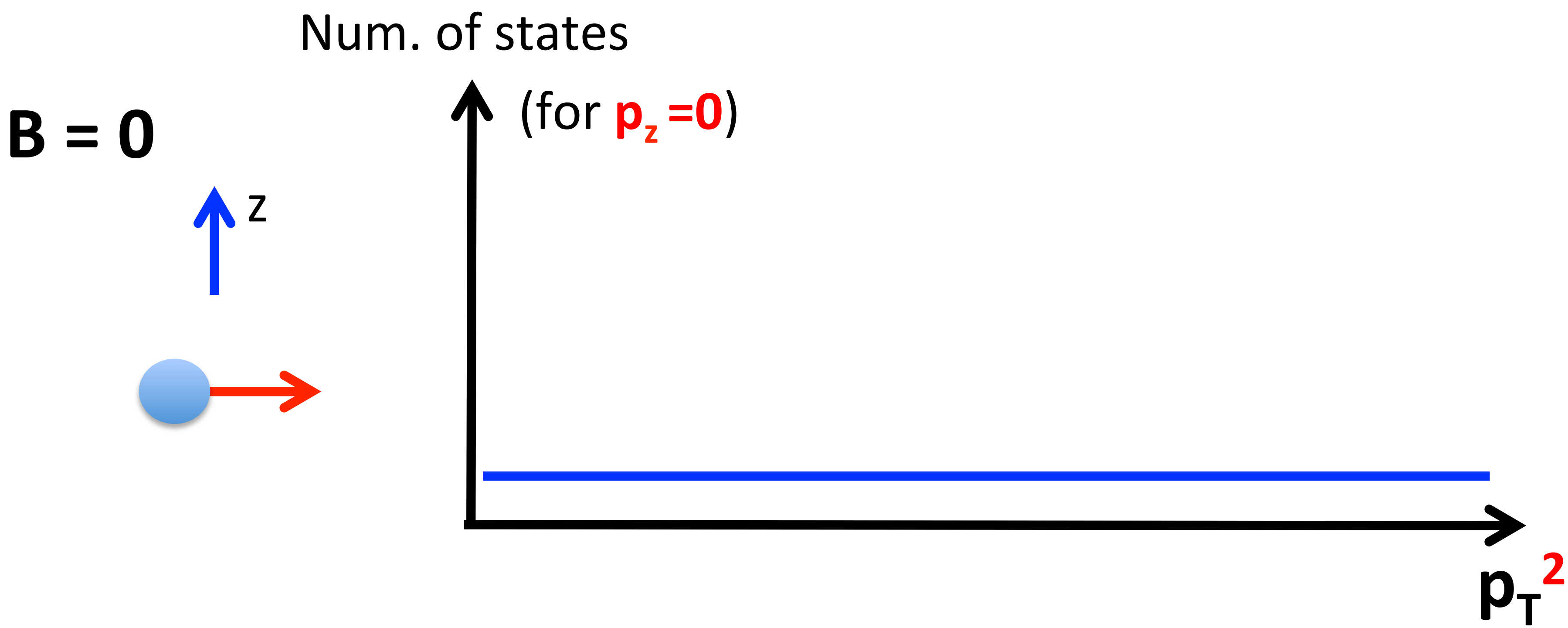
- **Test of 1/Nc, quantum phase transitions, ...**

3, Simple, qualitative problems are still available (theory).

- **They discriminate models from QCD (see next slides).**

Quantum mechanics in mag. fields

(spinless, free particles)

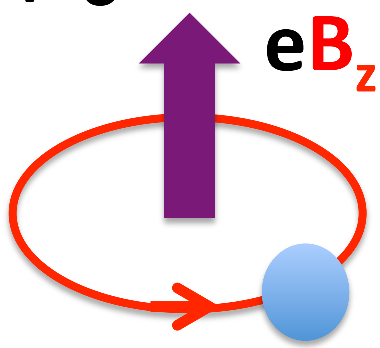


Quantum mechanics in mag. fields

(spinless, free particles)

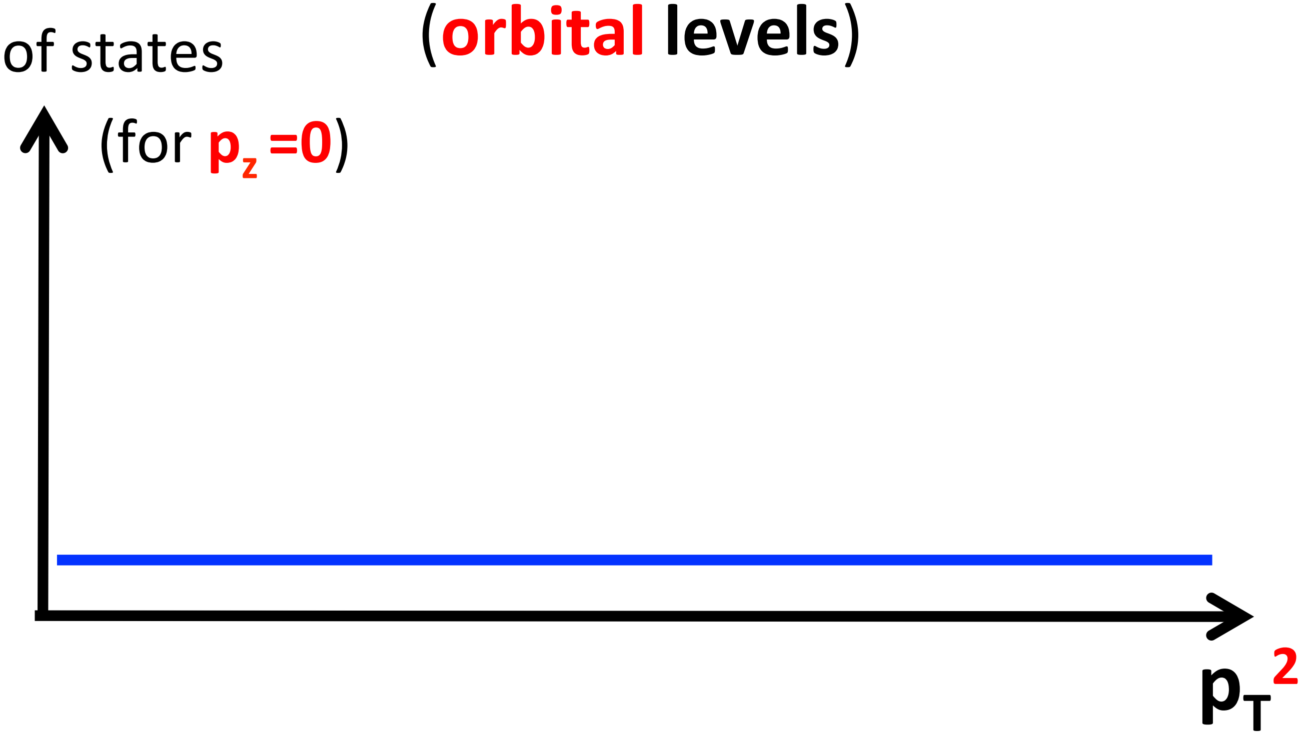
(orbital levels)

B ≠ 0



periodic

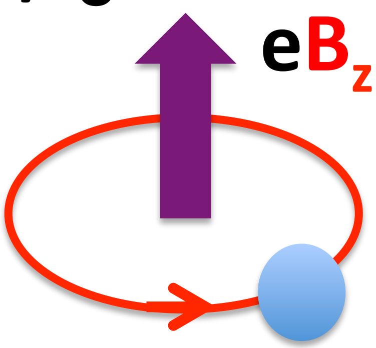
→ *quantization*



Quantum mechanics in mag. fields

(spinless, free particles)

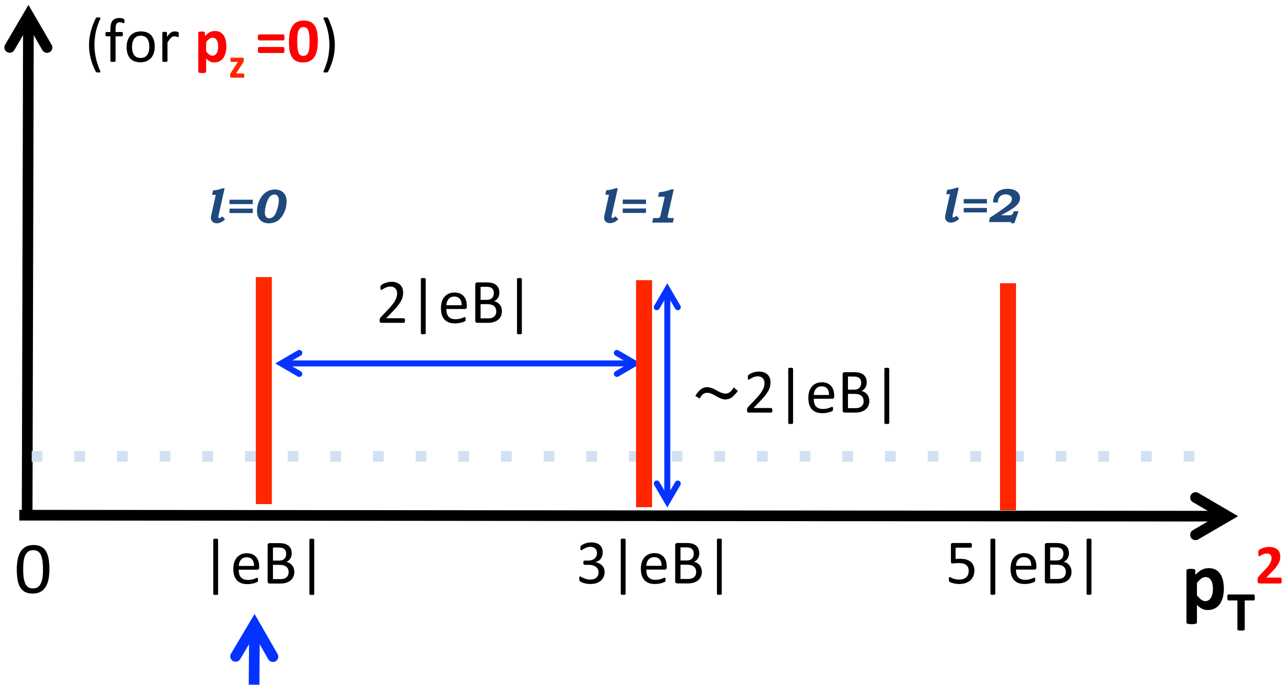
B ≠ 0



periodic
→ ***quantization***

Num. of states

(orbital levels)

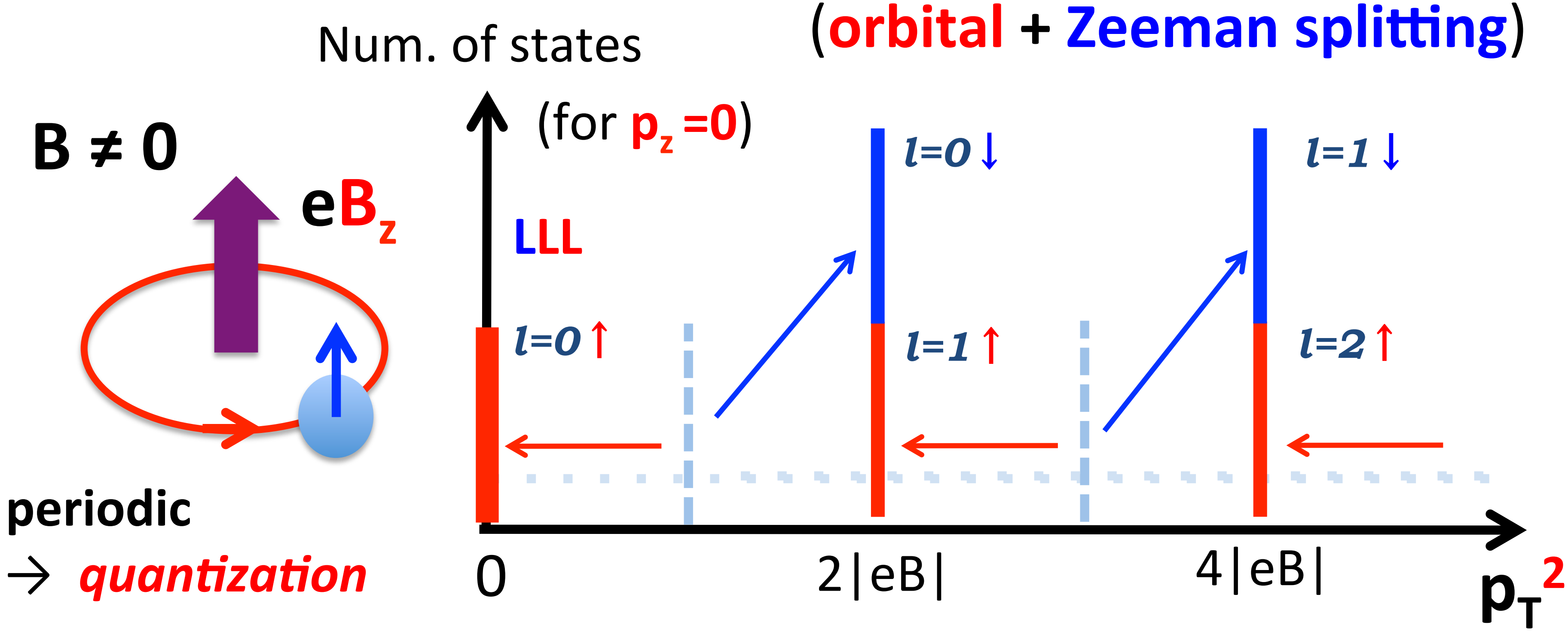


zero point energy

Quantum mechanics in mag. fields

(spin 1/2, free particles)

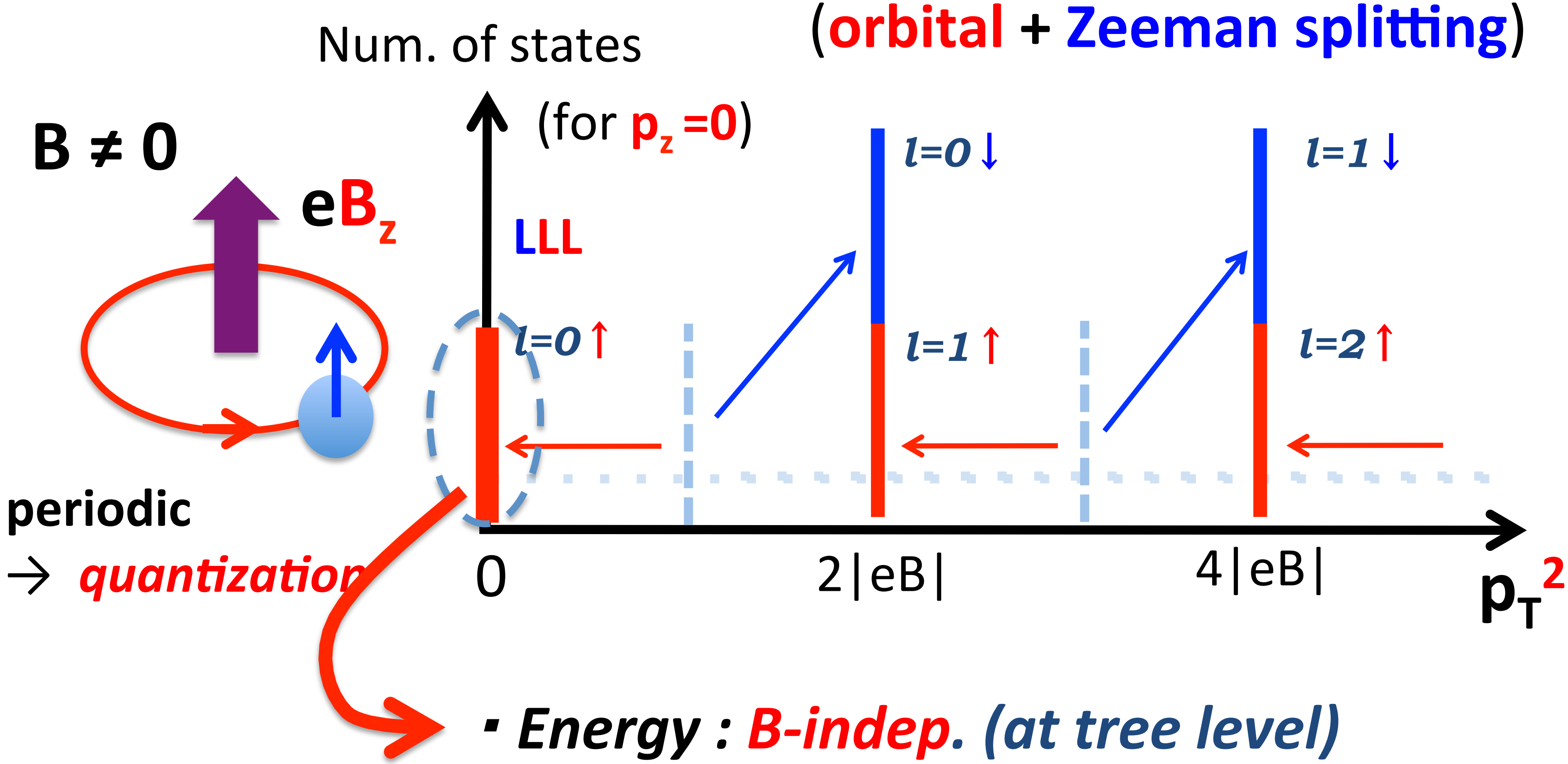
(orbital + Zeeman splitting)



Quantum mechanics in mag. fields

(spin 1/2, free particles)

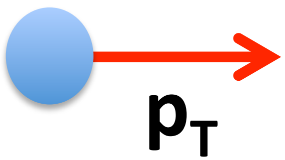
(orbital + Zeeman splitting)

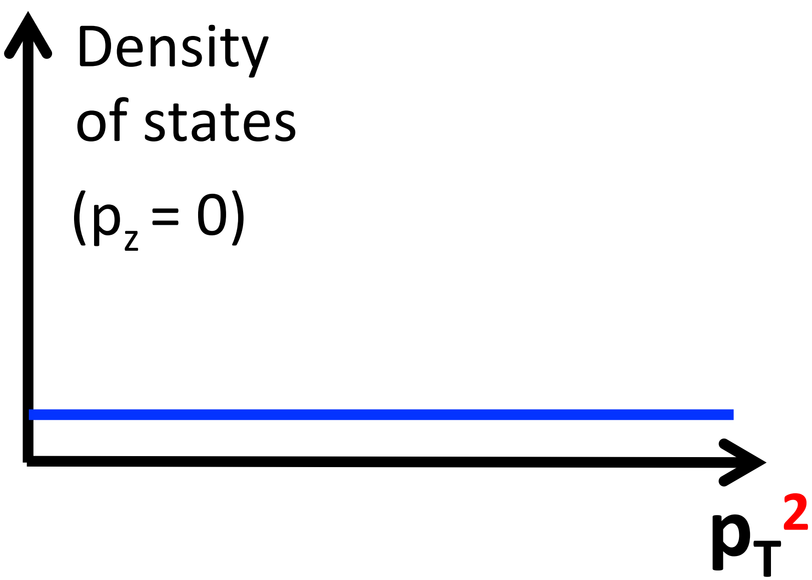


▪ ***Energy : B-indep. (at tree level)***

Landau Levels (LLs) for fermions

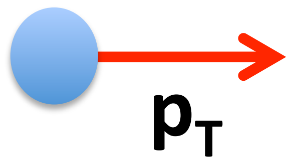
B = 0



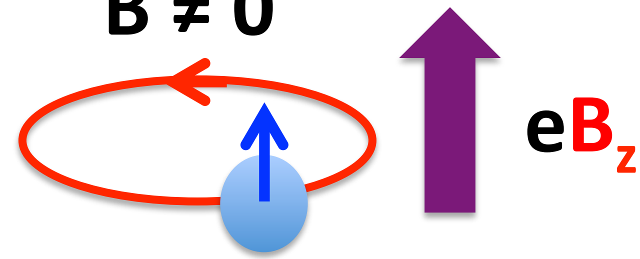


Landau Levels (LLs) for fermions

B = 0

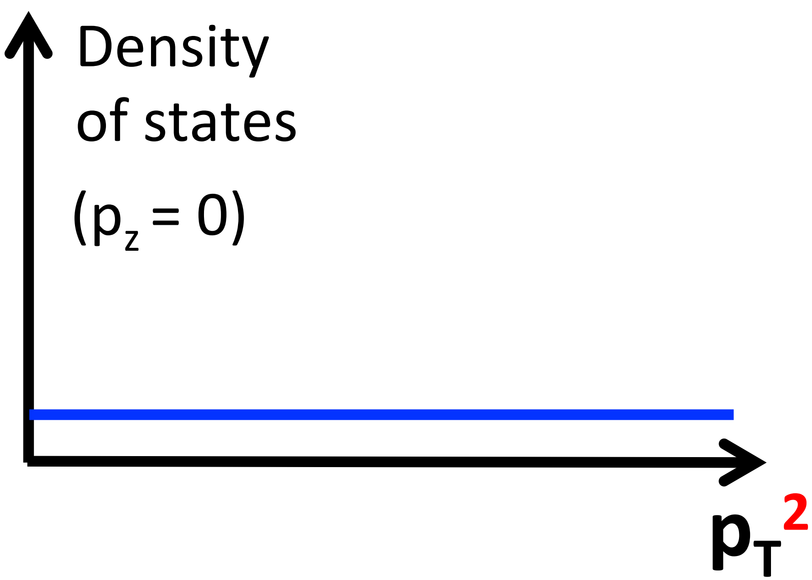


B ≠ 0



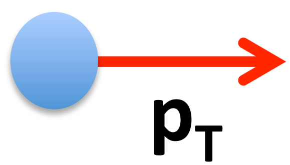
periodic → ***quantization*** **(orbital)**

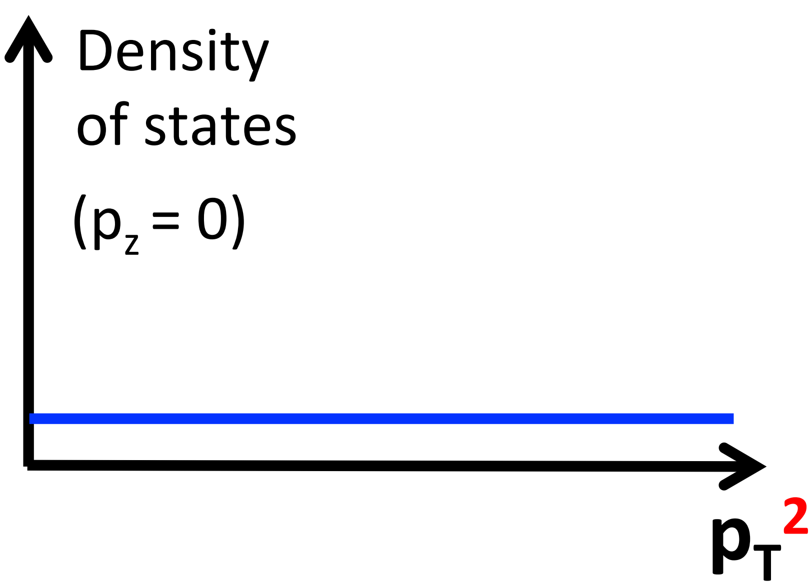
spin → ***Zeeman splitting***



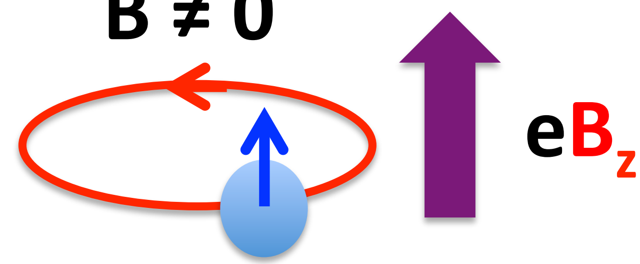
Landau Levels (LLs) for fermions

B = 0



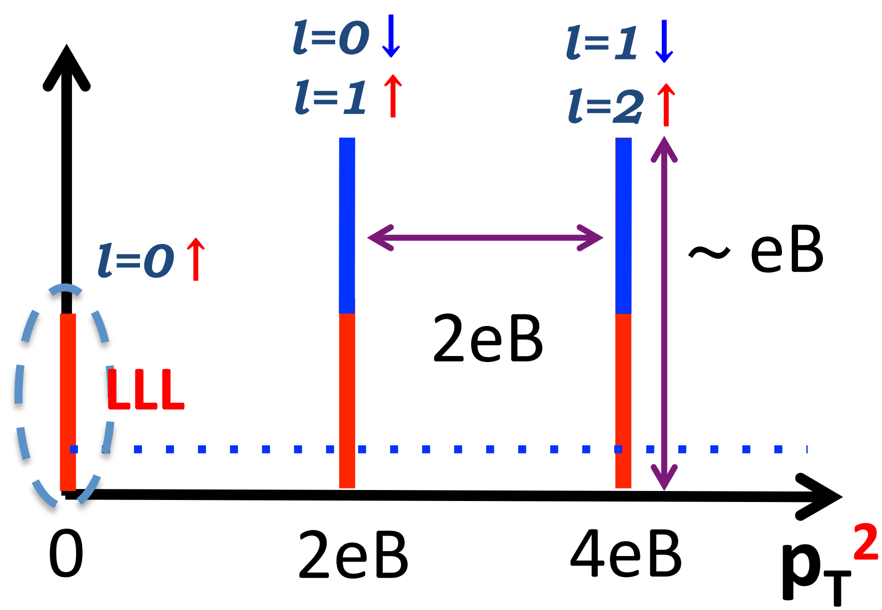


B ≠ 0

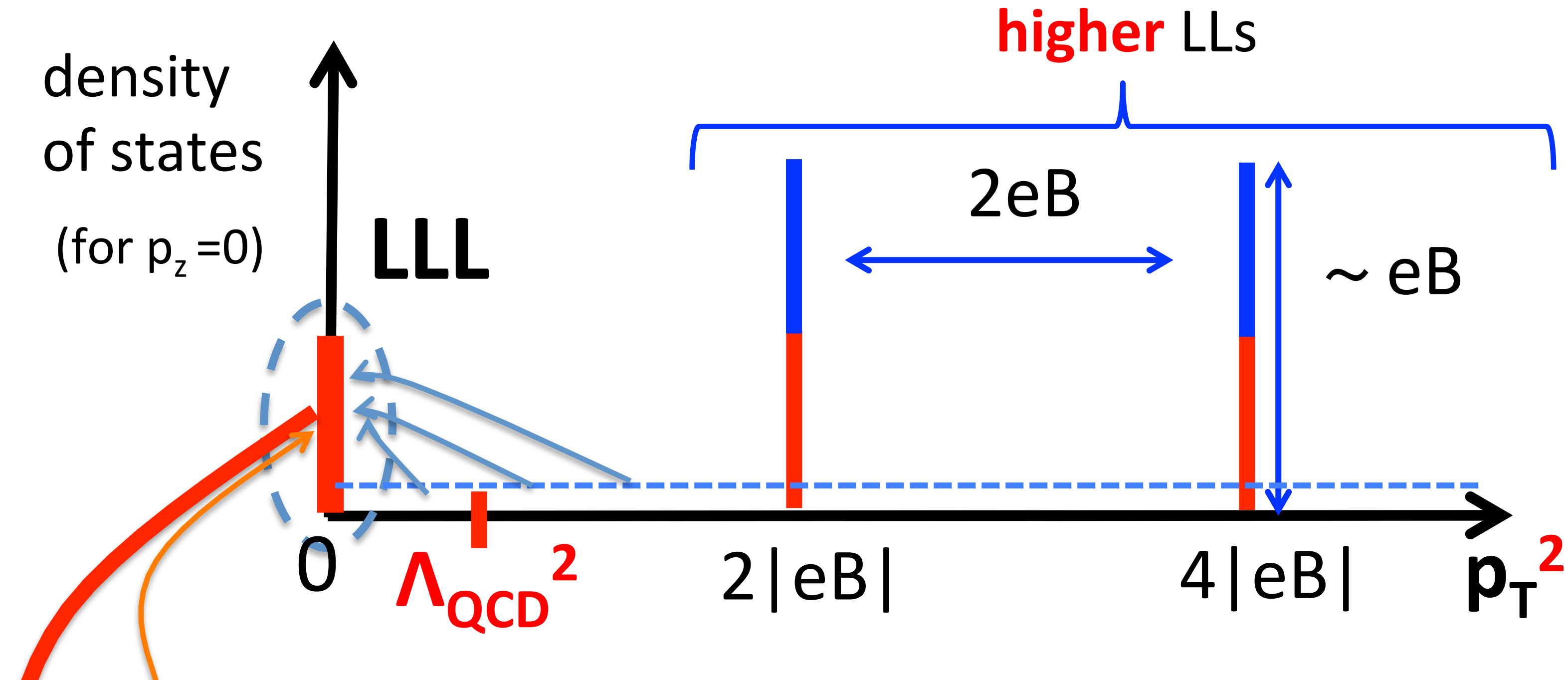


periodic → *quantization* (orbital)

spin → *Zeeman splitting*



"*Enhanced*" IR phase space for quarks



More quarks can stay at low energy than B=0 case.

- **Enhanced ChSB ~ *Magnetic Catalysis***
- **Larger impacts on gluon dynamics** (larger screening, ...)

e.g.) **Formula for *Chiral condensate***

"Ritus bases"

$$\langle\bar{\psi}(x)\psi(x)\rangle_{\mathrm{4D}} = \underbrace{\frac{|eB|}{2\pi}}_{\text{Degeneracy factor}} \underbrace{\int_{p_L}(-1)\,\mathrm{tr}\left[S^{\mathrm{2D}}_{\mathrm{LLL}}(\underline{p_L}) + \sum_{n=1} S^{\mathrm{2D}}_{n\mathrm{LL}}(\underline{p_L})\right]}_{\text{Dynamical part}}$$

Degeneracy factor

(for each LLs)

Dynamical part

$$\sim \langle\bar{\psi}(x_L)\psi(x_L)\rangle_{\mathrm{2D}}$$

"2D condensate"

Intuitively,

$$\langle\bar{\psi}\psi\rangle_{\mathrm{4D}}$$