Early Isotropization of the Quark Gluon Plasma

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Viscous Hydrodynamics

I) Macroscopic theory
II) Few parameters: $P_L, P_T, \epsilon, \vec{u}$
III) Need input:

1) Equation of state $f(P_L, P_T) = \epsilon$
2) Small anisotropy
3) Initialization: $\epsilon(\tau_0), P_L(\tau_0)$? ...
4) Viscous coefficients: shear viscosity $\eta$, ...
5) Short isotropization time
Viscous Hydrodynamics

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None of this is easy to get from QCD
HEAVY ION COLLISIONS: THE GENERAL PICTURE

Early transition: the problem

Isotropization?
Time scale?

Huge anisotropy
(negative $P_L$)

Small anisotropy

Long time puzzle: Does (fast) isotropization occur?
HOW TO STUDY THE TRANSITION?

Weakly coupled method at high density:

\[ \alpha_s \ll 1 \text{ but } f_{\text{gluon}} \sim \frac{1}{\alpha_s} \]
The Color Glass Condensate [McLerran, Venugopalan (1993)]

Theoretical framework (Weakly coupled but strongly interacting)

LO:
\[ \epsilon = \frac{1}{2} \left( \vec{E}^2 + \vec{B}^2 \right) \]

Classical color fields

\[ \mathcal{D}_\mu \mathcal{F}^{\mu\nu} = J^\nu \]

Color sources on the light cone

[Krasnitz, Venugopalan (1998)]
Strong anisotropy at all time

\[ g^2 \tau P_L / (g^2 \mu)^3, \quad g^2 \tau P_T / (g^2 \mu)^3 \]

\[ g^2 \mu \tau \]

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Strong anisotropy at all time

\[ \epsilon = 2P_T + P_L \Rightarrow P_T = \epsilon \]

\[ \lim_{\tau \to 0^+} \epsilon = \text{cst} \Rightarrow P_L = -\epsilon \]
\[ E^2(x) = \mathcal{E}^2(x_\perp) + \frac{1}{2} \int e_{\vec{k}}(x)|^2 + \cdots \]

\[ e_{\vec{k}}(x) \] perturbation to \( \mathcal{E}(x) \) created by a plane wave of momentum \( \vec{k} \) in the remote past.

Obtained by solving the linearized equation of motions.
THE COLOR GLASS CONDENSATE AT NLO

[ROMATSCHKE, VENUGOPALAN (2006)]

Small Fluctuations grow exponentially (Weibel instability)
Because of instabilities, the NLO correction eventually becomes as large as the LO ⇒ Important effect, should be included

NLO alone will grow forever ⇒ unphysical effect, should be taken care of

Such growing contributions are present at all orders of the perturbative expansion

How to deal with them?
THE CLASSICAL-STATISTICAL METHOD

- At the initial time $\tau = \tau_0$, take:

$$\vec{E}_0(\tau_0, \vec{x}) = \vec{E}_0(\tau_0, \vec{x}) + \int_k c^*_{\vec{k}} \vec{e}_{\vec{k}}(\tau_0, \vec{x})$$

where $c_{\vec{k}}$ are random coefficients: $\langle c^*_{\vec{k}} c_{\vec{k}'} \rangle \sim \delta_{\vec{k} \vec{k}'}$

- Solve the **Classical** equation of motion $D_\mu F^{\mu \nu} = J^\nu$

- Compute $\langle \vec{E}^2(\tau, \vec{x}) \rangle$, where $\langle \rangle$ is the average on the $c_{\vec{k}}$ (Monte-Carlo)

- One can show that this resums all the fastest growing terms at each order, leading to a result that remains bounded when $\tau \to \infty$

  [GELIS, LAPPI, VENUGOPALAN (2008)]

This gives: LO+NLO+Subset of higher orders
• Need to know $\vec{e}_k(\tau_0, \vec{x})$ at the time $\tau_0$ we start the numerical simulation
• For practical reasons, we must start in the forward light cone ($\tau_0 > 0$)

This can be done analytically [TE, GELIS 1307:1765]
The NLO spectrum

Result

\[ e^{i \nu \vec{k}_\perp} (\tau, x_\perp, \eta) = i \nu e^{i \nu \eta} \left[ F^{i,2 \nu \vec{k}_\perp} (\mathcal{U}_2, \tau, x_\perp) - F^{i,1 \nu \vec{k}_\perp} (\mathcal{U}_1, \tau, x_\perp) \right] \]

\[ e^{\eta \nu \vec{k}_\perp} (\tau, x_\perp, \eta) = e^{\nu \eta} \mathcal{D}^i \left[ F^{i,2 \nu \vec{k}_\perp} (\mathcal{U}_2, \tau, x_\perp) - F^{i,1 \nu \vec{k}_\perp} (\mathcal{U}_1, \tau, x_\perp) \right] \]

- \( \mathcal{U}_1 \) depends on the color source \( J^+ \) of the first nucleus
- \( \mathcal{U}_2 \) depends on the color source \( J^- \) of the second nucleus
- Analytical checks performed on the solution
  - Gauss’s law
  - Linearized Yang-Mills EOM
  - Orthonormality of the mode functions
Gauge potential $A^\mu \rightarrow$ link variables (exact gauge invariance on the lattice)

Numerical parameters

- Transverse lattice size $L = 64$, transverse lattice spacing $Q_s a_T = 1$
- Longitudinal lattice size $N = 128$, longitudinal lattice spacing $a_L = 0.016$
- Number of configurations for the Monte-Carlo $N_{\text{conf}} = 200$ to $2000$
- Initial time $Q_s \tau_0 = 0.01$
Numerical results [TE, Gelis (2013)]

\[ \alpha_s = 8 \times 10^{-4} \ (g = 0.1) \]
NUMERICAL RESULTS [TE, GELIS (2013)]

\[ \alpha_s = 2 \times 10^{-2} \ (g = 0.5) \]

\[ \tau \text{ [fm/c]} \]

\[ Q_s \tau \]

\[ P_T / \epsilon \quad \text{green} \]

\[ P_L / \epsilon \quad \text{red} \]

\[ \text{LO} \quad \cdot \]
Numerical results [TE, Gelis (2013)]

\[ \alpha_s = 2 \times 10^{-2} \ (g = 0.5) \]
ANOMALOUSLY SMALL VISCOSITY

Assuming simple first order viscous hydrodynamics

\[ \epsilon \approx \epsilon_0 \tau^{-\frac{4}{3}} - 2\eta_0 \tau^{-2} \]

we can compute the dimensionless ratio \((\eta = \eta_0 \tau^{-1})\)

\[ \eta \epsilon^{-\frac{3}{4}} \lesssim 1 \]

In contrast, perturbation theory at LO gives \(\eta \epsilon^{-\frac{3}{4}} \sim 300\).

If the system is nearly thermal

\[ \epsilon^{\frac{3}{4}} \sim s \implies \frac{\eta}{s} \text{ close to } \frac{1}{4\pi} \]
Conclusion

- Correct NLO spectrum from first principles

- Fixed anisotropy for $g = 0.5$ at $\tau \sim 1\text{fm}/c$

- Assuming first order viscous hydrodynamics: $\eta e^{-3/4} \lesssim 1$

- Compatible with viscous hydrodynamical expansion

- No need for strong coupling to get hydrodynamization
\[ \langle A \rangle \sim 0, \; \langle E \rangle \sim 0 \]
\[ \langle A^2 \rangle - \langle A \rangle^2 \sim \frac{Q_s^2}{g^2} \]
\[ \langle E^2 \rangle - \langle E \rangle^2 \sim \frac{Q_s^4}{g^2} \]

May give correct answer at LO
Not correct at NLO

\[ \langle A \rangle \sim \frac{Q_s}{g}, \; \langle E \rangle \sim \frac{Q_s^2}{g} \]
\[ \langle A^2 \rangle - \langle A \rangle^2 \sim Q_s^2 \]
\[ \langle E^2 \rangle - \langle E \rangle^2 \sim Q_s^4 \]
give correct answer at LO
give correct answer at NLO
EOM on a lattice

Writing

\[ E^\mu(x) = x_\mu \]
\[ U_\mu(x) = x_\mu \]
\[ U^{\dagger}_\mu(x) = \mu x + \hat{\mu} \]

and

\[ U_{\mu\nu}(x) = \]
\[ U^{\dagger}_{\mu\nu}(x) = \]
\[ U_{\mu-\nu}(x) = \]
\[ U^{\dagger}_{\mu-\nu}(x) = \]

We can therefore rewrite the EOM as

\[ \partial_\tau x^i = \frac{-i}{2ga_1a_j} \sum_j \left[ \left( x^j \right) - \left( x^j \right) + \left( x^j \right) - \left( x^j \right) \right] \]
\[ \partial_\tau x^j = -i g a_I x^j \]
RENORMALIZATION PROCEDURE

\[ \langle E^2_{L, \text{div}} \rangle \sim Q_s^2 k^2_{\perp, \text{max}} + k^4_{\perp, \text{max}} + k^4_{\perp, \text{max}} \ln^2 \frac{\nu_{\text{max}}}{\tau} + \ldots \]

3 last diagrams can be subtracted with a simulation where

\[ A^a_\mu (x) = 0 + a^a_\mu (x) \]

\[ E^2_L \ "\text{fine}\" \ (B^2_L \ \text{too}) \]
\[ \langle E_{T, \text{div}}^2 \rangle \sim Q_s^2 \frac{\nu_{\text{max}}^2}{\tau^2} + k_{\perp, \text{max}}^2 \frac{\nu_{\text{max}}^2}{\tau^2} + k_{\perp, \text{max}}^4 \ln^2 \frac{\nu_{\text{max}}}{\tau} + \ldots \]

3 last diagrams can be subtracted with a simulation where

\[ A^a_\mu (x) = 0 + a^a_\mu (x) \]

How to deal with the first 2? \( \rightarrow \) fit for the time being.

Otherwise \( E_T^2 \) and \( B_T^2 \) behaves as \( \tau^{-2} \) at early time.
Renormalization Procedure

\[ \epsilon = E_T^2 + B_T^2 + E_L^2 + B_L^2 \]

\[ P_T = E_L^2 + B_L^2 \]

\[ P_L = E_T^2 + B_T^2 - E_L^2 - B_L^2 \]
RENORMALIZATION PROCEDURE

\[
\langle P_T \rangle_{\text{phys.}} = \langle P_T \rangle_{\text{backgd. + fluct.}} - \langle P_T \rangle_{\text{fluct. only}} \\
\langle \epsilon, P_L \rangle_{\text{phys.}} = \langle \epsilon, P_L \rangle_{\text{backgd. + fluct.}} - \langle \epsilon, P_L \rangle_{\text{fluct. only}} + A \tau^{-2}.
\]

\(\tau^{-}\) term only one to satisfy Bjorken law and EOS:

\[
\partial_\tau \tau^{-\alpha} + 2\tau^{-\alpha-1} = 0
\]
How come that problematic divergent diagrams behaves as mass terms?

In the continuum limit, they don’t exist local gauge invariant operators of dimension two.

On the lattice though, they could be terms like

$$g^2 \frac{\nu_{\text{max}}^2}{k_{\perp, \text{max}}^2 \tau^2} \text{Tr} F^2,$$

where

$$F_{\mu \nu}(x) \sim \begin{array}{c}
\hat{\nu} \\
\hat{\mu}
\end{array} \quad \begin{array}{c}
\hat{\mu} \\
\hat{\nu}
\end{array} \sim \begin{array}{c}
\hat{\nu} \\
\hat{\mu}
\end{array}$$