Early Isotropization of the Quark Gluon Plasma

Darmstadt, 19th May 2014

Thomas EPELBAUM

IPhT

Viscous Hydrodynamics

I) Macroscopic theory
II) Few parameters: P_L, P_T, ϵ, $\vec{u}$
III) Need input:

1) Equation of state $f(P_L, P_T) = \epsilon$
2) Small anisotropy
3) Initialization: $\epsilon(\tau_0)$, $P_L(\tau_0)$? ...
4) viscous coefficients: shear viscosity η,...
5) Short isotropization time

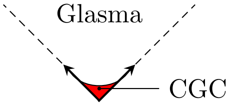
Viscous Hydrodynamics

I) Macroscopic theory
II) Few parameters: $P_L, P_T, \epsilon, \vec{u}$
III) Need input:

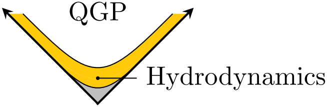
1) Equation of state $f(P_L, P_T) = \epsilon$
2) Small anisotropy
3) Initialization: $\epsilon(\tau_0)$, $P_L(\tau_0)$? ...
4) viscous coefficients: shear viscosity η,...
5) Short isotropization time

None of this is easy to get from QCD

Early transition: the problem



Isotropization?
Time scale?

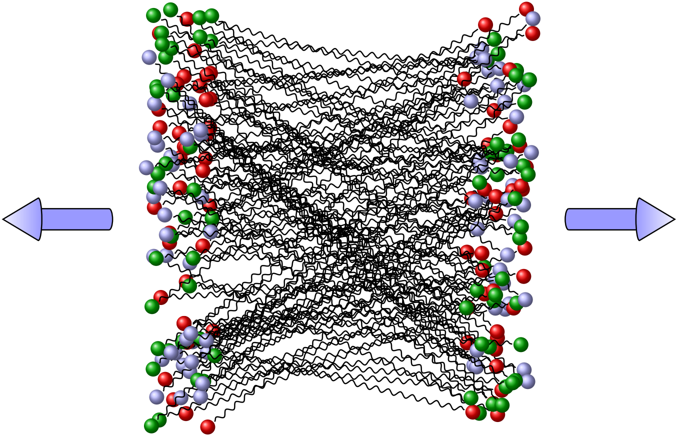


Huge anisotropy (negative P_L)

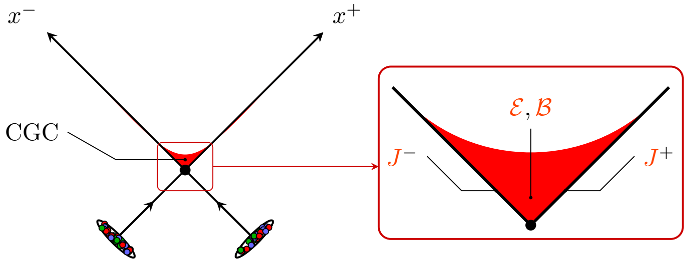
Small anisotropy

Long time puzzle: Does (fast) isotropization occur?

Weakly coupled method at high density:
$\alpha_s \ll 1$ **but** $f_{\text{gluon}} \sim \frac{1}{\alpha_s}$



Theoretical framework (Weakly coupled but strongly interacting)

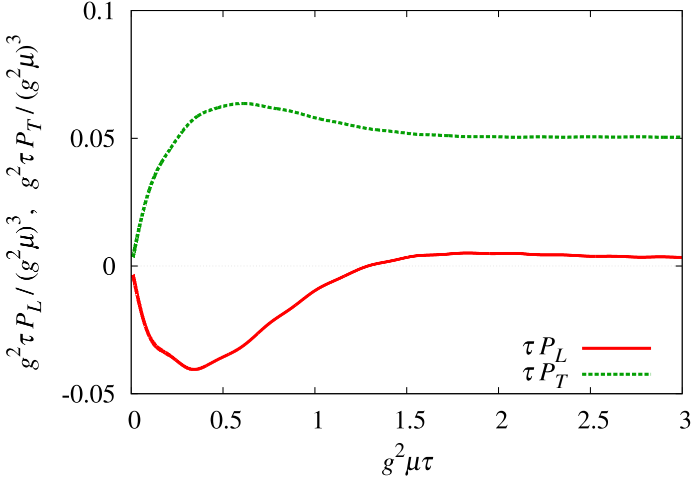


LO:

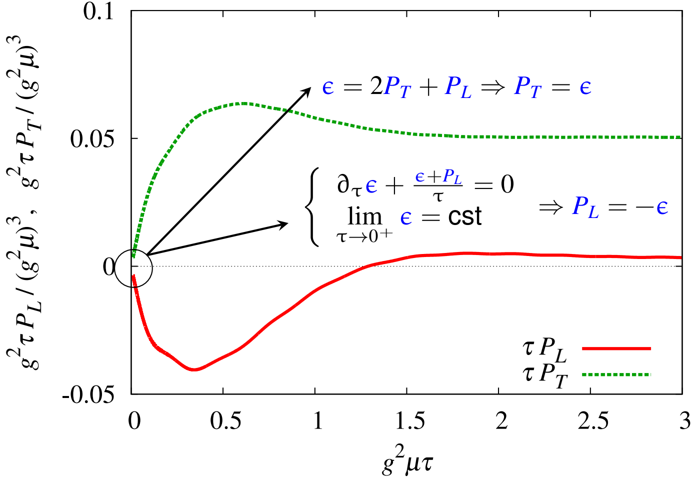
$$\epsilon = \frac{1}{2} \underbrace{\left(\vec{\mathcal{E}}^2 + \vec{\mathcal{B}}^2\right)}_{\text{Classical color fields}} \qquad \mathcal{D}_\mu \mathcal{F}^{\mu\nu} = \underbrace{J^\nu}_{\text{Color sources on the light cone}}$$

[KRASNITZ, VENUGOPALAN (1998)]

Strong anisotropy at all time



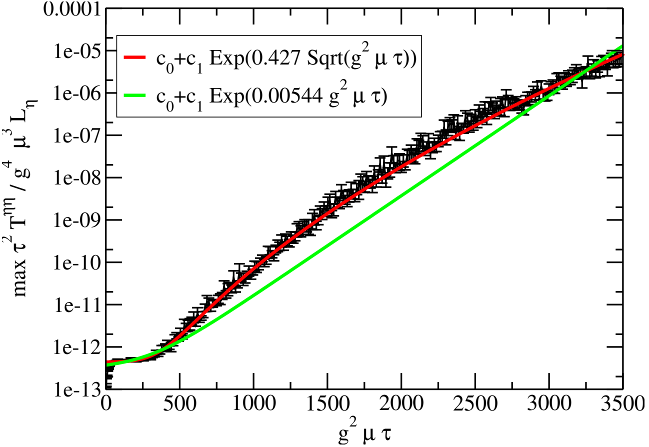
Strong anisotropy at all time



$$E^2(x) = \underbrace{\mathcal{E}^2(\boldsymbol{x}_\perp)}_{\text{LO}} + \underbrace{\frac{1}{2}\int_{\vec{k}} \left| e_{\vec{k}}(x) \right|^2}_{\text{NLO}} + \cdots$$

$e_{\vec{k}}(x)$ perturbation to $\mathcal{E}(x)$ created by a plane wave of momentum $\vec{k}$ in the remote past.

Obtained by solving the linearized equation of motions.

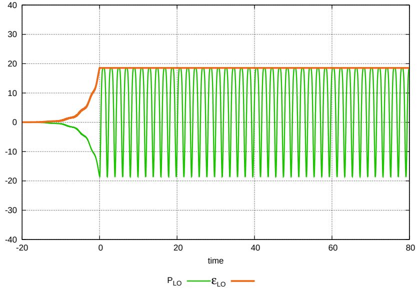


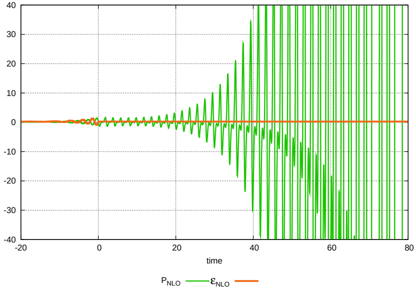
[ROMATSCHKE, VENUGOPALAN (2006)]

Small Fluctuations grow exponentially (Weibel instability)

- Because of instabilities, the NLO correction eventually becomes as large as the LO ⇒ Important effect, should be included

- NLO alone will grow forever ⇒ unphysical effect, should be taken care of





- Such growing contributions are present at all orders of the perturbative expansion

How to deal with them?

- At the initial time $\tau = \tau_0$, take:

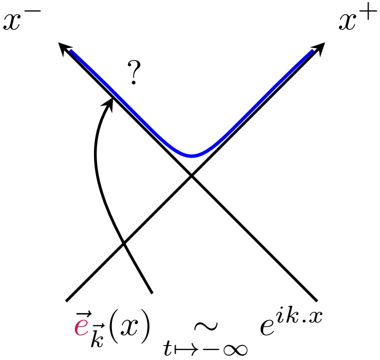
$$\vec{E}_0(\tau_0, \vec{x}) = \vec{\mathcal{E}}_0(\tau_0, \vec{x}) + \int_{\vec{k}} c_{\vec{k}} \vec{e}_{\vec{k}}(\tau_0, \vec{x})$$

  where $c_{\vec{k}}$ are random coefficients: $\left\langle c_{\vec{k}} c_{\vec{k}'} \right\rangle \sim \delta_{\vec{k}\vec{k}'}$

- Solve the **Classical** equation of motion $D_\mu F^{\mu\nu} = J^\nu$
- Compute $\left\langle \vec{E}^2(\tau, \vec{x}) \right\rangle$, where $\langle\rangle$ is the average on the $c_{\vec{k}}$ (Monte-Carlo)
- One can show that this resums all the fastest growing terms at each order, leading to a result that remains bounded when $\tau \to \infty$ [GELIS, LAPPI, VENUGOPALAN (2008)]

This gives: LO+NLO+Subset of higer orders

- Need to know $\vec{e}_{\vec{k}}(\tau_0, \vec{x})$ at the time τ_0 we start the numerical simulation
- For practical reasons, we must start in the forward light cone $(\tau_0 > 0)$



This can be done analytically [TE,GELIS 1307:1765]

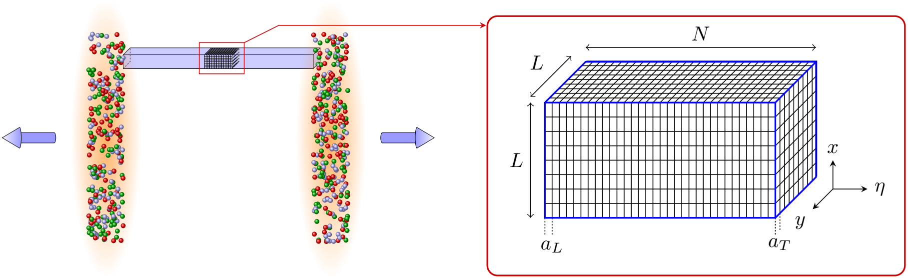
Result

$$e^{i}_{\nu\vec{k}_\perp}(\tau,\boldsymbol{x}_\perp,\eta) = i\nu\, e^{i\nu\eta}\left[F^{i,2}_{\nu\vec{k}_\perp}(\mathcal{U}_2,\tau,\boldsymbol{x}_\perp) - F^{i,1}_{\nu\vec{k}_\perp}(\mathcal{U}_1,\tau,\boldsymbol{x}_\perp)\right]$$
$$e^{\eta}_{\nu\vec{k}_\perp}(\tau,\boldsymbol{x}_\perp,\eta) = e^{i\nu\eta}\mathcal{D}^i\left[F^{i,2}_{\nu\vec{k}_\perp}(\mathcal{U}_2,\tau,\boldsymbol{x}_\perp) - F^{i,1}_{\nu\vec{k}_\perp}(\mathcal{U}_1,\tau,\boldsymbol{x}_\perp)\right]$$

- $\mathcal{U}_1$ depends on the color source J^+ of the first nucleus
- $\mathcal{U}_2$ depends on the color source J^- of the second nucleus
- Analytical checks performed on the solution
  - Gauss's law
  - linearized Yang-Mills EOM
  - Orthonormality of the mode functions

YM ON A LATTICE

Gauge potential $A^\mu \to$ link variables (exact gauge invariance on the lattice)

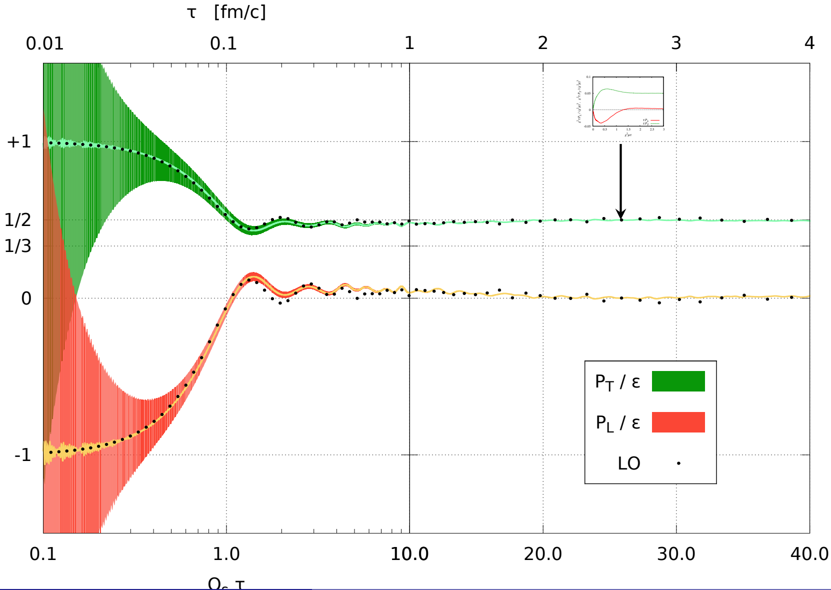


Numerical parameters

- Transverse lattice size $L = 64$, transverse lattice spacing $Q_s a_T = 1$
- Longitudinal lattice size $N = 128$, longitudinal lattice spacing $a_L = 0.016$
- Number of configurations for the Monte-Carlo $N_{\text{conf}} = 200$ to 2000
- Initial time $Q_s \tau_0 = 0.01$

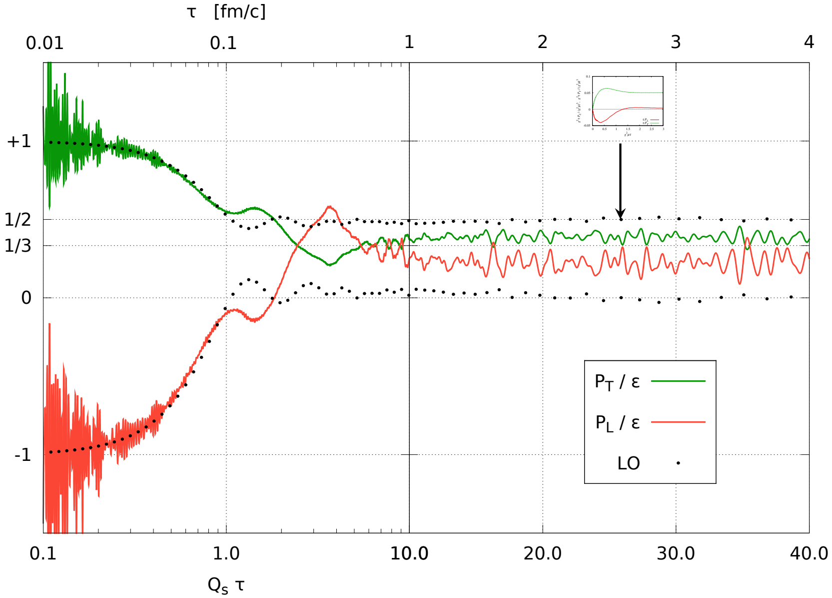
NUMERICAL RESULTS [TE,GELIS (2013)]

$\alpha_s = 8\,10^{-4}$ $(g = 0.1)$



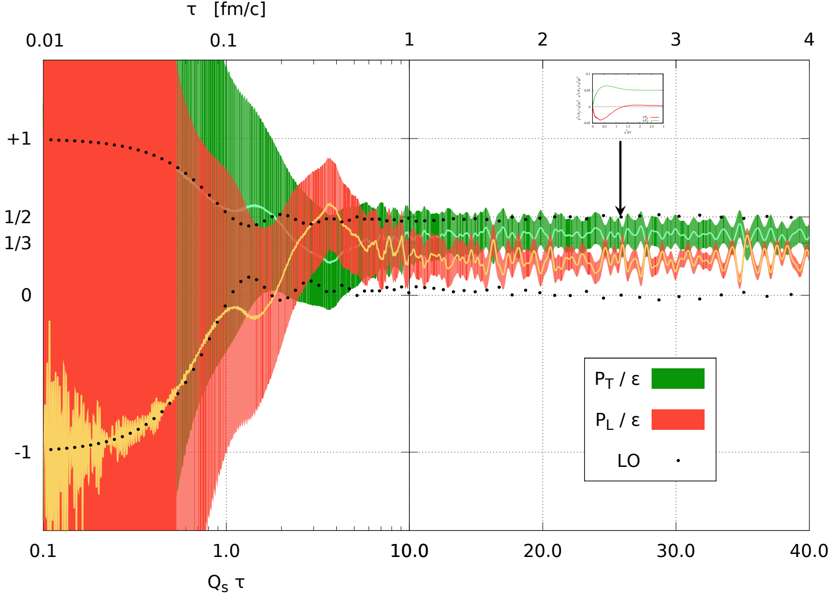
NUMERICAL RESULTS [TE,GELIS (2013)]

$\alpha_s = 2\,10^{-2}$ $(g = 0.5)$



NUMERICAL RESULTS [TE,GELIS (2013)]

$\alpha_s = 2\,10^{-2}\ (g = 0.5)$



Assuming simple first order viscous hydrodynamics

$$\epsilon \approx \underbrace{\epsilon_0 \tau^{-\frac{4}{3}}}_{\text{Ideal hydro}} - \underbrace{2\eta_0 \tau^{-2}}_{\text{first order correction}}$$

we can compute the dimensionless ratio ($\eta = \eta_0 \tau^{-1}$)

$$\eta \epsilon^{-\frac{3}{4}} \lesssim 1$$

In contrast, perturbation theory at LO gives $\eta \epsilon^{-\frac{3}{4}} \sim 300$.

If the system is nearly thermal

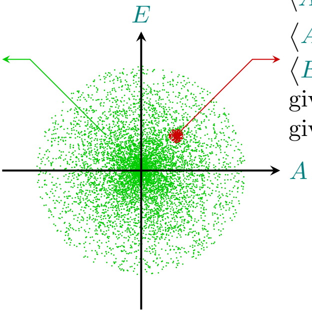
$$\epsilon^{\frac{3}{4}} \sim s \Longrightarrow \frac{\eta}{s} \text{ close to } \frac{1}{4\pi}$$

CONCLUSION

- Correct NLO spectrum from first principles
- Fixed anisotropy for $g = 0.5$ at $\tau \sim 1 fm/c$
- Assuming first order viscous hydrodynamics: $\eta\epsilon^{-\frac{3}{4}} \lesssim 1$
- Compatible with viscous hydrodynamical expansion
- No need for strong coupling to get hydrodynamization

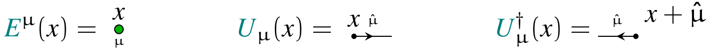
BACKUP: COMPLETELY INCOHERENT INITIAL FIELDS VS CGC

$\langle A \rangle \sim 0$, $\langle E \rangle \sim 0$
$\left\langle A^2 \right\rangle - \langle A \rangle^2 \sim \frac{Q_s^2}{g^2}$
$\left\langle E^2 \right\rangle - \langle E \rangle^2 \sim \frac{Q_s^4}{g^2}$
May give correct answer at LO
Not correct at NLO

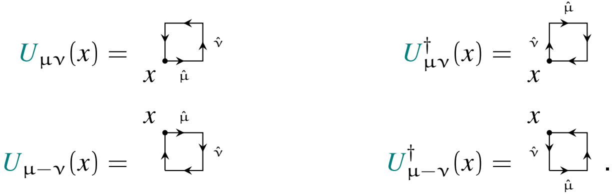


$\langle A \rangle \sim \frac{Q_s}{g}$, $\langle E \rangle \sim \frac{Q_s^2}{g}$
$\left\langle A^2 \right\rangle - \langle A \rangle^2 \sim Q_s^2$
$\left\langle E^2 \right\rangle - \langle E \rangle^2 \sim Q_s^4$
give correct answer at LO
give correct answer at NLO

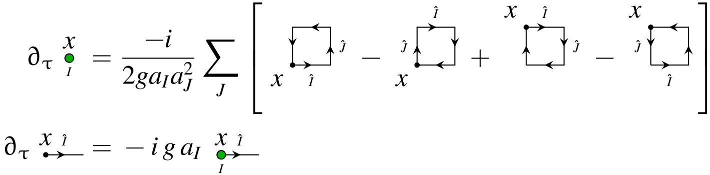
Writing



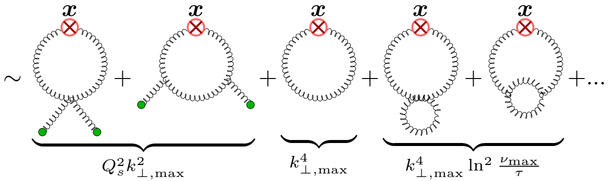
and



We can therefore rewrite the EOM as



$$\left\langle E^2_{L,\text{div}} \right\rangle \sim Q_s^2 k^2_{\perp,\text{max}} + k^4_{\perp,\text{max}} + k^4_{\perp,\text{max}} \ln^2 \frac{\nu_\text{max}}{\tau} + \ldots$$

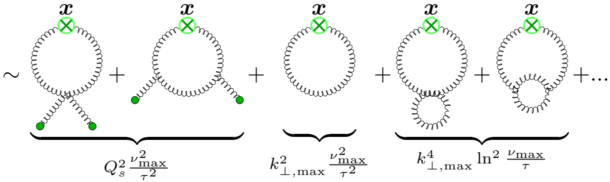


3 last diagrams can be substracted with a simulation where

$$A^a_\mu(x) = 0 + a^a_\mu(x)$$

E^2_L "fine" (B^2_L too)

$$\left\langle E^2_{T,\mathrm{div}} \right\rangle \sim Q_s^2 \frac{\nu^2_{\max}}{\tau^2} + k^2_{\perp,\max} \frac{\nu^2_{\max}}{\tau^2} + k^4_{\perp,\max} \ln^2 \frac{\nu_{\max}}{\tau} + \dots$$



3 last diagrams can be substracted with a simulation where

$$A^a_\mu(x) = 0 + a^a_\mu(x)$$

How to deal with the first 2? $\rightarrow$ fit for the time being.

Otherwise E_T^2 and B_T^2 behaves as τ^{-2} at early time.

Renormalization procedure

$$\epsilon = E_T^2 + B_T^2 + \underbrace{E_L^2}_{\text{fine}} + \underbrace{B_L^2}_{\text{fine}}$$

$$P_T = \underbrace{E_L^2}_{\text{fine}} + \underbrace{B_L^2}_{\text{fine}}$$

$$P_L = E_T^2 + B_T^2 - \underbrace{E_L^2}_{\text{fine}} - \underbrace{B_L^2}_{\text{fine}}$$

$$\langle P_T \rangle_{\text{phys.}} = \langle P_T \rangle_{\substack{\text{backgd.}\\ +\ \text{fluct.}}} - \langle P_T \rangle_{\substack{\text{fluct.}\\ \text{only}}}$$

$$\langle \epsilon, P_L \rangle_{\text{phys.}} = \underbrace{\langle \epsilon, P_L \rangle_{\substack{\text{backgd.}\\ +\ \text{fluct.}}}}_{\text{computed}} - \underbrace{\langle \epsilon, P_L \rangle_{\substack{\text{fluct.}\\ \text{only}}}}_{\text{computed}} + \underbrace{A\,\tau^{-2}}_{\text{fitted}} \;.$$

τ^{-} term only one to satisfy Bjorken law and EOS:

$$\partial_\tau \tau^{-\alpha} + 2\tau^{-\alpha-1} = 0$$

How come that problematic divergent diagrams behaves as mass terms?

In the continuum limit, they don't exist local gauge invariant operators of dimension two.

On the lattice though, they coud be terms like

$$g^2 \frac{\nu_{\max}^2}{k_{\perp,\max}^2 \tau^2} \mathrm{Tr} F^2,$$

where

$$F_{\mu\nu}(x) \sim$$