Bottomonium melting from a stochastic potential

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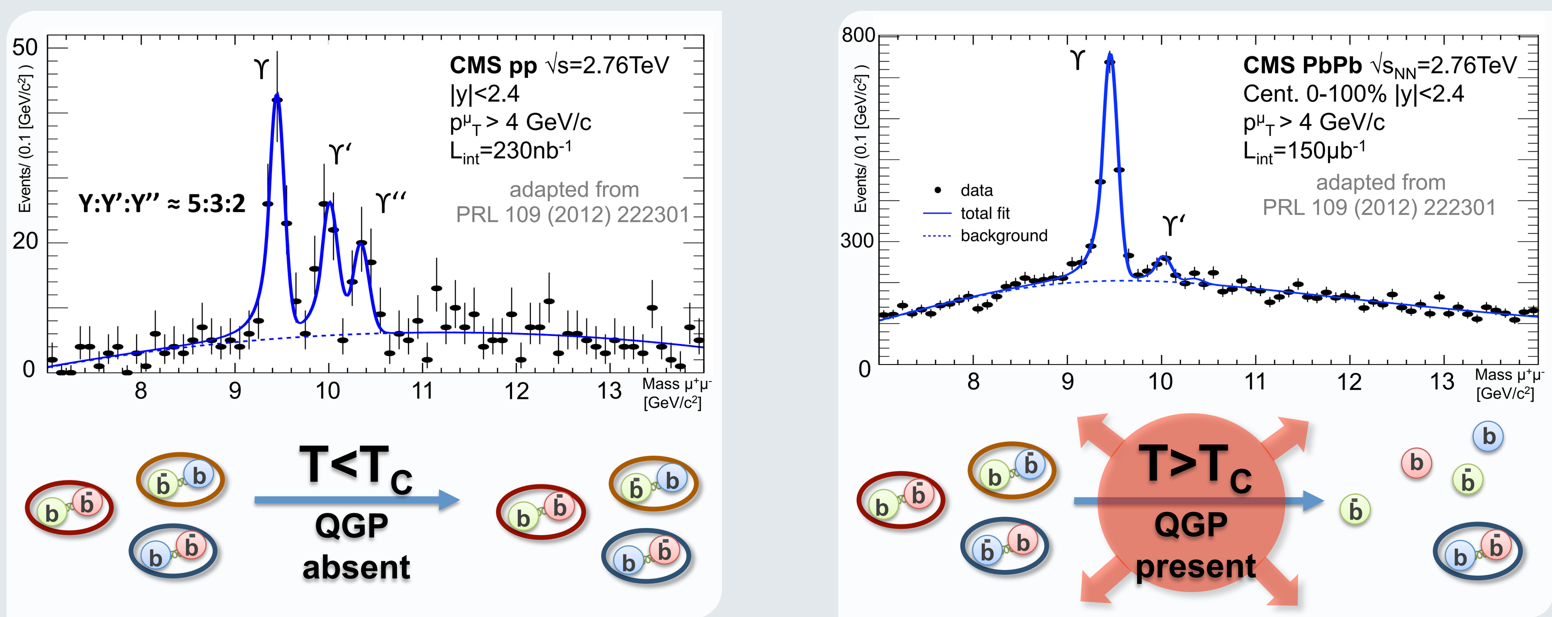
Physics Motivation

- **Bottomonium:** Formed in the early stages of a relativistic heavy-ion collision, $b\bar{b}$ mesons traverse the collision center as probes of the bulk properties.
- **Experimental finding**: Relative excited states suppression in the presence of a hot medium Figures below adapted from: [CMS collaboration] PRL 109 (2012) 222301



- **Goal**: Understand how the presence of a thermal medium contributes to relative excited states suppression.
- **Challenge:** Set up a consistent **real-time description** of Bottomonium in the QGP from first principles.

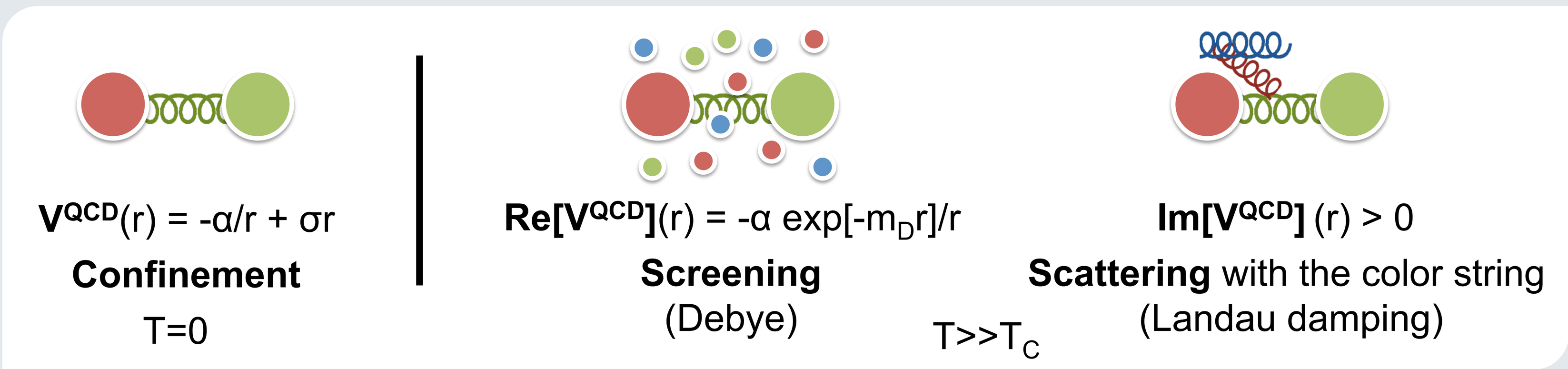
The QCD Heavy Quark Potential

- **Separation of scales** $m_b >> T$ and $m_b >> \Lambda_{QCD}$: Interaction between $Q\bar{Q}$ and medium can be captured in an in-medium potential within a **Schrödinger equation**
- **Derivation** of Schrödinger equation **from first principles QCD possible**, using effective field theory methods. The potential between static quarks ($m\to\infty$) is obtained from the real-time rectangular Wilson loop $W_\square$. see e.g.: Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423 Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017

$$i\partial_t\Big\langle\psi_{Q\bar{Q}}(\mathbf{r},t)\psi^*_{Q\bar{Q}}(\mathbf{r},0)\Big\rangle_{\rm med}=V^{\rm QCD}(\mathbf{r})\Big\langle\psi_{Q\bar{Q}}(\mathbf{r},t)\psi^*_{Q\bar{Q}}(\mathbf{r},0)\Big\rangle_{\rm med}$$

$$V^{\rm QCD}(\mathbf{r})=\lim_{t\to\infty}\frac{i\partial_t W_\square(\mathbf{r},t)}{W_\square(\mathbf{r},t)}\qquad W_\square(\mathbf{r},t)=\Big\langle{\rm Tr}\Big[\exp\Big(-ig\int_\square dz^\mu A_\mu(z)\Big)\Big]\Big\rangle$$

- **At finite temperature, V^{QCD}(r) is complex valued.** Laine et al. JHEP03 (2007) 054, (perturbative) A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001 (lattice) see also POSTER by Y. Burnier A-05
- A look at the relevant physics processes:



Beraudo et. al. NPA 806:312,2008

- **Caution: V^{QCD}(r) is not suitable for directly evolving the Bottomonium wavefunction.**
  - Since **static quarks cannot annihilate**: Im[V]≠0 would lead to a contradiction
  - **Instead**: correlation between initial state and current state decays (**Decoherence**) Y. Akamatsu, A.R. Phys.Rev. D85 (2012) 105011

Bottomonium as Open Quantum System

- Derivation of the Schrödinger equation ≡ tracing out the medium degrees of freedom

$$H=H_{Q\bar{Q}}\otimes\mathbb{1}_{\rm med}+\mathbb{1}_{Q\bar{Q}}\otimes H_{\rm med}+H_{\rm int}\qquad \sigma_{Q\bar{Q}}(t)={\rm Tr}_{\rm med}[\sigma(t)]$$

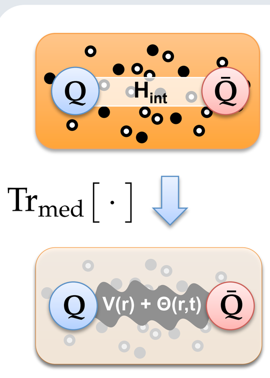
density matrix σ(t): $\frac{d}{dt}\sigma(t)=-i[H,\sigma(t)]$ overall | subsystem $\frac{d}{dt}\sigma_{Q\bar{Q}}(t)=\mathcal{L}\sigma_{Q\bar{Q}}(t)$

- Time evolution $\mathcal{L}$ of the subsystem density matrix σ_{QQ} can be implemented (unraveled) into stochastic dynamics for the $\bar{Q}Q$ wavefunction. H.-P. Breuer, F. Petruccione, Theory of Open Quantum Systems
- **Intuitive idea**: In the QGP, medium **partons scatter with the color string** in between the $Q\bar{Q}$. I.e. at each step in time Δt the Debye screened and **real-valued potential V_{QQ} is kicked stochastically**: Y. Akamatsu, A.R. Phys.Rev. D85 (2012) 105011 (caveat: neglects backreaction)

$$\psi_{Q\bar{Q}}(\mathbf{r},t)=\mathcal{T}\exp\Big[-i\int_0^t\Big\{-\frac{\nabla^2}{m_Q}+V_{Q\bar{Q}}(\mathbf{r})+\Theta(\mathbf{r},t)\Big\}\Big]\psi_{Q\bar{Q}}(\mathbf{r},0)$$

$$\langle\Theta(\mathbf{r},t)\rangle=0,\quad\langle\Theta(\mathbf{r},t)\Theta(\mathbf{r}',t')\rangle=\Gamma(\mathbf{r},\mathbf{r}')\delta_{tt'}/\Delta t$$

$$i\frac{d}{dt}\psi_{Q\bar{Q}}(\mathbf{r},t)=\Big(-\frac{\nabla^2}{m_Q}+V_{Q\bar{Q}}(\mathbf{r})+\Theta(\mathbf{r},t)-i\frac{\Delta t}{2}\Theta^2(\mathbf{r},t)\Big)\psi_{Q\bar{Q}}(\mathbf{r},t)$$



For a formal derivation, see Y. Akamatsu, Phys.Rev. D87 (2013) 4, 045016; arXiv:1403.5783 and POSTER by Y. Akamatsu F-02

- **Stochastic evolution** of ψ_{QQ} is **unitary** and the energy of the system remains real
- **After thermal average**: **imaginary part in in V^{QCD} emerges from the noise correlations Γ**

$$i\frac{d}{dt}\langle\psi_{Q\bar{Q}}(\mathbf{r},t)\psi^*_{Q\bar{Q}}(\mathbf{r},0)\rangle_\Theta=\Big(-\frac{\nabla^2}{m_Q}+V_{Q\bar{Q}}(\mathbf{r})-\frac{i}{2}\Gamma(\mathbf{r},\mathbf{r})\Big)\langle\psi_{Q\bar{Q}}(\mathbf{r},t)\psi^*_{Q\bar{Q}}(\mathbf{r},0)\rangle_\Theta$$

hence we identify: **V_{QQ}(r)=Re[V^{QCD}(r)] and Γ(r,r)=Im[V^{QCD}(r)]**

- **Of phenomenological relevance**: What is the probability to find a Bottomonium vacuum state ϕ_n at time t: **admixture c_{nn}** $H^{vac}\phi_n=E_n\phi_n$

$$c_{nn}=\int d^3r d^3r'\ \phi_n^*(\mathbf{r})\,\langle\psi_{Q\bar{Q}}(\mathbf{r},t)\psi^*_{Q\bar{Q}}(\mathbf{r}',t)\rangle_\Theta\,\phi_n(\mathbf{r}')$$

- c_{nn} related to density matrix σ(r,r')=<ψ(r)ψ*(r')>$_\Theta$ and thus dependent on off-diagonal Γ
- Evolving ψ_{QQ} with Im[V_{QQ}]=Im[V^{QCD}] incorrect: <ψ_{QQ}(r)><ψ_{QQ}*(r')> ≠ <ψ_{QQ}(r)ψ_{QQ}*(r')>

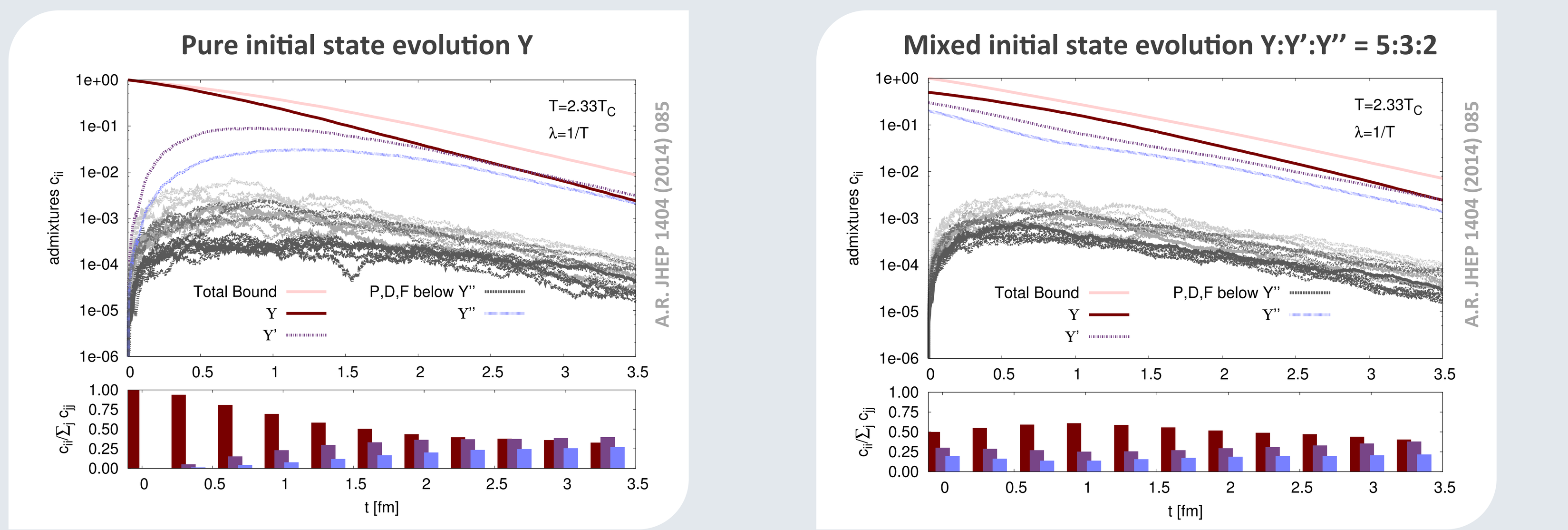
Bottomonium in a Static QGP at T=396MeV

- **Stochastic evolution**: 256^3 lattice with L=7.68fm Δx=0.03fm Δt=7.5x10^{-4}fm 10 samples
  - **Initial vacuum states** from **Cornell potential + string breaking**: supports e.g. 3 bound S-wave, 2 bound P-wave states For more details see: A.R. JHEP 1404 (2014) 085
  - **Static QGP** at high temperature: **perturbative QCD potential** m_D=1GeV g=2.14

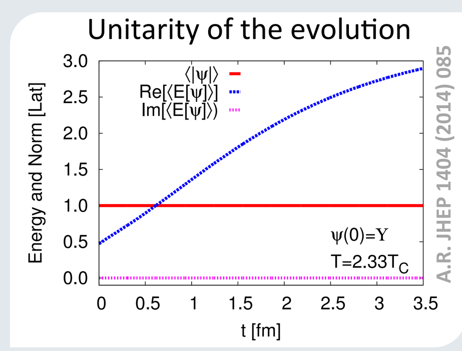
$$V_{Q\bar{Q}}(r)=-\frac{3g^2}{4\pi}\frac{e^{-m_D|r|}}{|r|}\quad\Gamma(\mathbf{r},\mathbf{r})=-\frac{3g^2T}{4\pi}\vartheta(m_D|r|),\quad\vartheta(x)=2\int_0^\infty dz\frac{z}{(z^2+1)^2}\Big[1-\frac{\sin[zx]}{zx}\Big]$$

Laine et al. JHEP03 (2007) 054

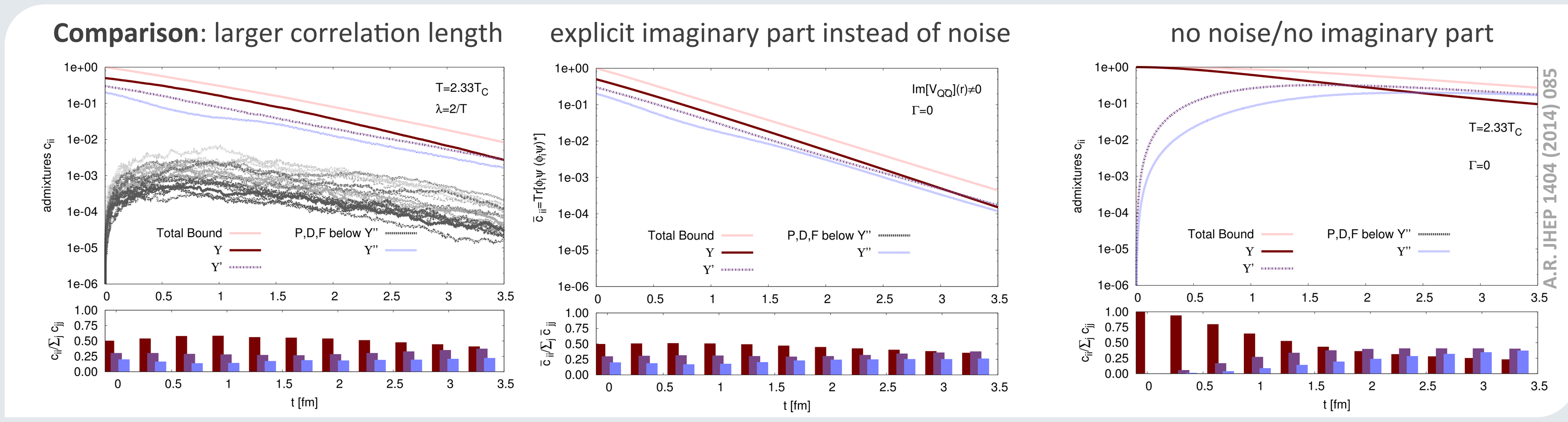
  - **Ansatz for Γ(r,r')** = (Γ(r,r)Γ(r',r'))^{½} exp[-(r-r')^2/2λ^2] λ=1/T
- **Numerical results**: absolute admixtures (top) relative admixtures $c_{ii}/(c_Y+c_{Y'}+c_{Y''})$ (bottom)



- Vacuum state inserted in medium: receives energy from kicks
- Mixing due to H^{med}≠H^{vac} populates neighboring (same L only)
- Stochastic (de)excitations lead to strong absolute suppression
- p+p like initial conditions: t<1.5fm excited states suppression (since T=const.): t>1.5fm excited states regeneration



- Influence of Γ(r,r') on admixtures: λ=1/T shows strongest ground state survival



- Explicit imaginary part: relative excited states suppression is much less pronounced

Conclusion and Outlook

- The Open Quantum Systems based **stochastic potential approach** allows to consistently **incorporate the effect of the imaginary part** of the QCD heavy quark potential in the **real-time evolution** of the two-body $Q\bar{Q}$ wavefunction.
- **State mixing** and **stochastic (de-)excitations** contribute to intricate melting patterns. Mixed initial conditions (inspired by the p+p dimuon spectra) show **early time excited states suppression.** Due to the constant temperature we find late time replenishment.
- **Quantitative difference** found between stochastic potential approach and simply adding an imaginary part to the potential. **Relative excited states suppression more pronounced in the open-quantum systems picture.**
- Need to find lattice observable from which the off-diagonal elements Γ(**r**,**r'**) can be extracted non-perturbatively
- More realistic settings need to be considered: variation of temperature from hydrodynamic simulations, finite velocity of the initial state, etc...
- Investigate the evolution of a system starting from the NRQCD based initial conditions deployed in JHEP 1303 (2013) 091 . (work in progress with J. Casalderrey-Solana)
- Need to **incorporate** the effects of the **drag force** to extend the validity of the simulation to **later times,** as proposed by Akamatsu in PRD (2013) 4, 045016 and arXiv: 1403.5783 . Seek connection to thermal spectra from lattice QCD.

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