We use a linear sigma model with constituent quarks and a non-vanishing mean-field $<\sigma>$:

$$\mathcal{L} = \psi^\dagger [\partial^\mu g_{\mu\nu} \partial^\nu - m^2] \psi + \frac{1}{2} (\partial_{\mu} \sigma \partial^\mu \sigma - \mu^2 \sigma^2 - V(\sigma))$$

with two quark flavours $q = (u,d)$. The meson potential with explicit symmetry breaking is:

$$V(\sigma, \bar{\sigma}) = \frac{\lambda}{2} (\sigma^2 + \bar{\sigma}^2 - v^2)^2 - f_0 m_0^2 (\sigma - \bar{\sigma})$$

$\lambda$, $f_0$, $m_0$, $v$, $\mu^2$ are model parameters.

DSLAM (Dynamically simulated linear sigma model)

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$$\mathcal{L} = \psi^\dagger [\partial^\mu g_{\mu\nu} \partial^\nu - m^2] \psi + \frac{1}{2} (\partial_{\mu} \sigma \partial^\mu \sigma - \mu^2 \sigma^2 - V(\sigma, \bar{\sigma}))$$

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Thermodynamics

The thermodynamical properties of the linear sigma model can be deduced from the dynamical simulation by initializing the model isotropically with the steady state solution

$$\left(\frac{\partial}{\partial r} - \nabla \right) \sigma(t, \vec{r}) = 0$$

This is done by solving the equations of motion self-consistently:

$$\int d^3p f(t, \vec{r}, \vec{p}) + \frac{f(t, \vec{r}, -\vec{p})}{E(t, \vec{r}, \vec{p})} = 0$$

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with the thermal Fermi distribution $f(t, \vec{r}, \vec{p})$ for quarks and $\bar{f}(t, \vec{r}, \vec{p})$ for anti-quarks.

Wave-Particle and Inelastic Interactions

By employing the Vlasov-equation, the total particle number in the system is conserved. Particle number conservation is found in many comparable studies, which is a reasonable approximation if chemical processes are slow in comparison to spatial expansion processes. However, a deviation from chemical equilibrium can lead to dramatic effects in the behavior of the phase transition (see Figure 4).

For an effective thermal and chemical equilibration of both fields and particles, we integrated the offshell Yukawa process

$$\psi \leftrightarrow \sigma$$

Outlook

Inelastic processes are vital in a dynamical transport simulation of the linear sigma model, otherwise the phase transition cannot be described in an evolving system. We have implemented these processes by quark-sigma particle creation and annihilation processes which respect detailed balance and equilibrium rates. Further investigations will be done in simulations of quenched scenarios with focus on reheating and critical slowing down. By scanning over a temperature range, the behavior of thermal fluctuations at and near a critical point is analysed.

Figure 1: Phase transition of the order parameter $\sigma(T)$. Depending on the coupling strength $g$, different order of the phase transition are observed: $g = 3.3$ cross-over, $g = 3.63$ second order and $g = 5.5$ first order transition.

Figure 2: The average particle density is directly related to the meson fields and is therefore affected by the phase transition. For a first order phase transition, an instant jump in the temperature dependence is seen.

Figure 3: Transport simulation of the expansion of a hot and dense droplet. The droplet is initialized with a Woods-Saxon like thermal droplet. The average particle density is directly related to the meson fields and is therefore affected by the phase transition. For a first order phase transition, an instant jump in the temperature dependence is seen.

Figure 4: Linear sigma model in the thermal limit. If chemical processes suppressed (conservation of the particle density), the chiral phase transition vanishes.

These interactions induce a thermal noise and damping in the chiral fields. The distribution of energy transfers can be modelled from physical processes and samples with Monte-Carlo techniques. This allows full control over the time and strength of energy transfer and dissipative processes on the fields.

Figure 5: Top: Thermal fluctuations of the sigma field induced by particle annihilation and creation processes. The fields show gaussian white noise without the need to employ external noise terms. Bottom: energy and momentum transfer by two quark annihilation in an interaction volume. On the left side, a netto momentum in forward direction is transfered, on the right side only energy is transfered.