Non-Equilibrium Phase Transitions in Field Theories
Applications to the Quark Meson Model

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H-QM Helmholtz Research School Quark Matter Studies

We investigate non-equilibrium dynamics in a linear sigma model to study the chiral phase transition in QCD.
In high-energy heavy-ion collisions, the quark-gluon-plasma is supposed to go from a hot, chiral-restored and deconfined phase to a cold, chiral-broken and confined phase. While there are many calculations in thermal and chemical equilibrium, we use a dynamical 3+1D linear sigma model with constituent quarks to examine the evolution of equilibrium and non-equilibrium scenarios.
In a first attempt we employ a mean-field ansatz to reproduce the thermodynamical properties of the model, in a later ansatz we use a new numeric method to allow inelastic and hard interactions between particles and fields.

DSLAM (Dynamically simulated linear sigma model)

We use a linear sigma model with constituent quarks and a non-vanishing mean-field $<\sigma>$:

$$\mathcal{L} = \bar{\psi}\left[i\partial\!\!\!/ - g\left(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau}\right)\right]\psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - U\left(\sigma,\vec{\pi}\right)$$

with two quark flavours $q = (u, d)$. The meson potential with explicit symmetry breaking is:

$$U\left(\sigma,\vec{\pi}\right) = \frac{\lambda^2}{4!}\left(\sigma^2 + \pi^2 - \nu^2\right)^2 - f_\pi m_\pi^2\sigma - U_0$$

Model Parameter

λ^2	$= 20$	coupling parameter
g	$\approx 3\ldots 6$	quark-sigma coupling
U_0	$= m_\pi^4/\left(4\lambda^2\right) - f_\pi^2 m_\pi^2$	ground state term
f_π	$= 93\ \mathrm{MeV}$	pion decay constant
m_π	$= 138\ \mathrm{MeV}$	pion mass
ν^2	$= f_\pi^2 - m_\pi^2/\lambda^2$	field shift term

Mean-field equations of motion for the meson fields in classical approximation are:

$$\partial_\mu\partial^\mu\ \sigma + \lambda^2\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)\sigma + g\langle\bar{\psi}\psi\rangle - f_\pi m_\pi^2 = 0$$

$$\partial_\mu\partial^\mu\ \vec{\pi} + \lambda^2\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)\vec{\pi} + g\langle\bar{\psi}\imath\gamma_5\psi\rangle = 0$$

with the one-loop scalar and pseudo-scalar quark density, acting as a source for the fields:

$$\langle\bar{\psi}\psi(\mathbf{r})\rangle = g\sigma(\mathbf{r})\int \mathrm{d}^3\mathbf{p}\,\frac{f(\mathbf{r},\mathbf{p}) + \bar{f}(\mathbf{r},\mathbf{p})}{E(\mathbf{r},\mathbf{p})}$$

Quarks and anti-quarks are propagated via Vlasov-Equation in test-particle approximation

$$\left[\partial_t + \frac{p}{E(t,\mathbf{r},\mathbf{p})}\cdot\nabla_\mathbf{r} - \nabla_\mathbf{r}E(t,\mathbf{r},\mathbf{p})\nabla_\mathbf{p}\right]\ f(t,\mathbf{r},\mathbf{p}) = 0,$$

with a time and space dependent mass term:

$$E(t,\mathbf{r},\mathbf{p}) = \sqrt{\mathbf{p}(t)^2 + M(\mathbf{r})^2}$$
$$M(t,\mathbf{r})^2 = g^2\left[\sigma(t,\mathbf{r})^2 + \vec{\pi}(t,\mathbf{r})^2\right]$$

Thermodynamics

The thermodynamical properties of the linear sigma model can be deduced from the dynamical simulation by initializing the model isotropically with the steady state solution

$$\left(\frac{\partial^2}{\partial t^2} - \nabla_{\vec{r}}^2\right)\sigma_0(t,\vec{r}) \equiv 0$$

This is done by solving the equations of motion self-consistently:

$$\left[g^2\int \mathrm{d}^3\mathbf{p}\frac{f(t,\mathbf{r},\mathbf{p}) + \bar{f}(t,\mathbf{r},\mathbf{p})}{E(t,\mathbf{r},\mathbf{p})} + \lambda^2\left(\sigma_0^2 - \nu^2\right) + \right]\sigma_0 - f_\pi m_\pi^2 = 0$$

with the thermal Fermi distribution $f(t,\mathbf{r},\mathbf{p})$ for quarks and $\bar{f}(t,\mathbf{r},\mathbf{p})$ for anti-quarks.

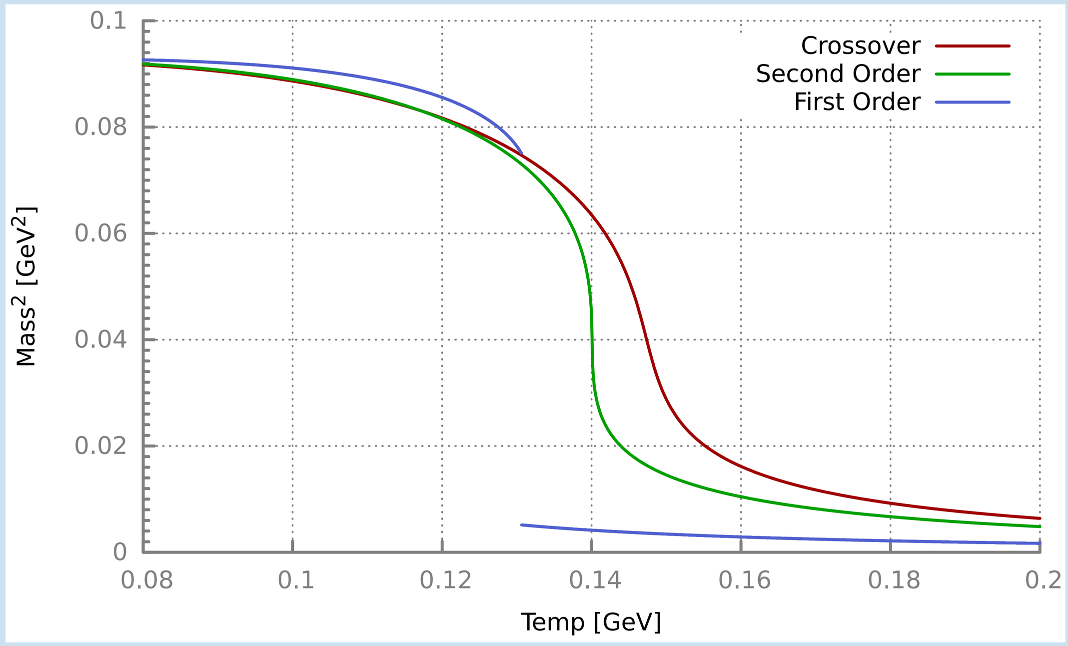

Figure 1: Phase transition of the order parameter $\sigma(T)$. Depending on the coupling strength g, different order of the phase transition are observed: $g = 3.3$ cross-over, $g = 3.63$ second order and $g = 5.5$ first order transition.

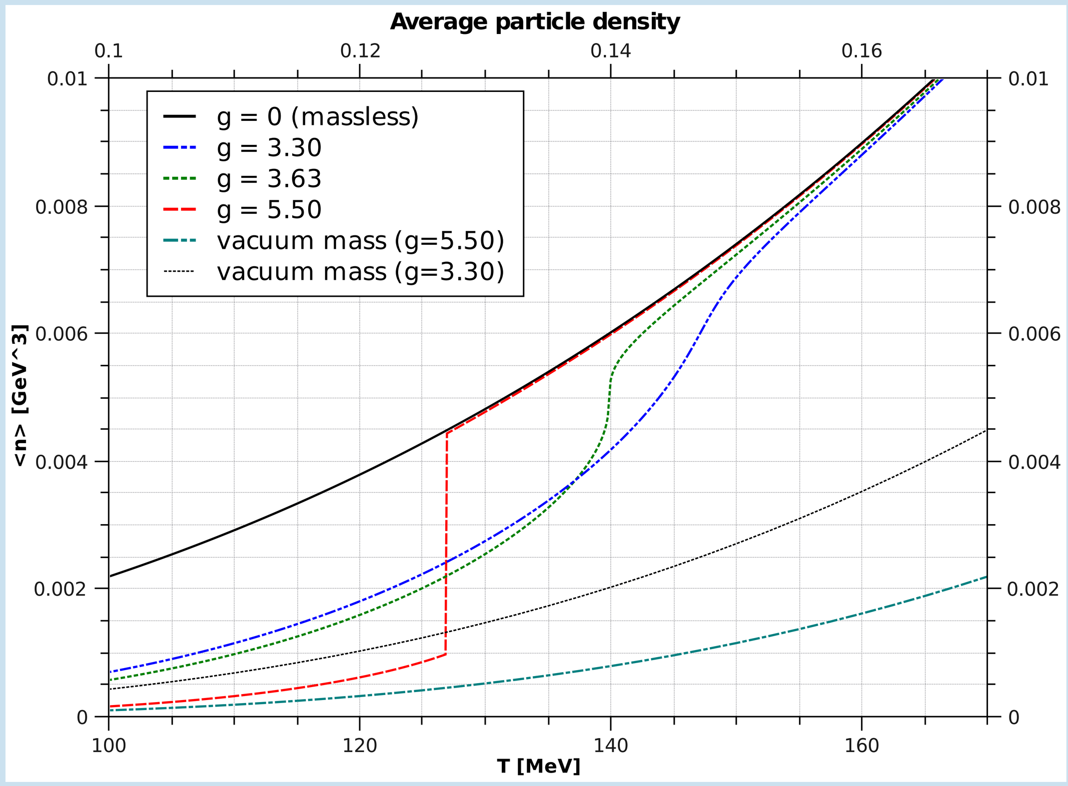

Figure 2: The average particle density is directly related to the meson fields and is therefore affected by the phase transition. For a first order phase transition, an instant jump in the temperature dependence is seen.

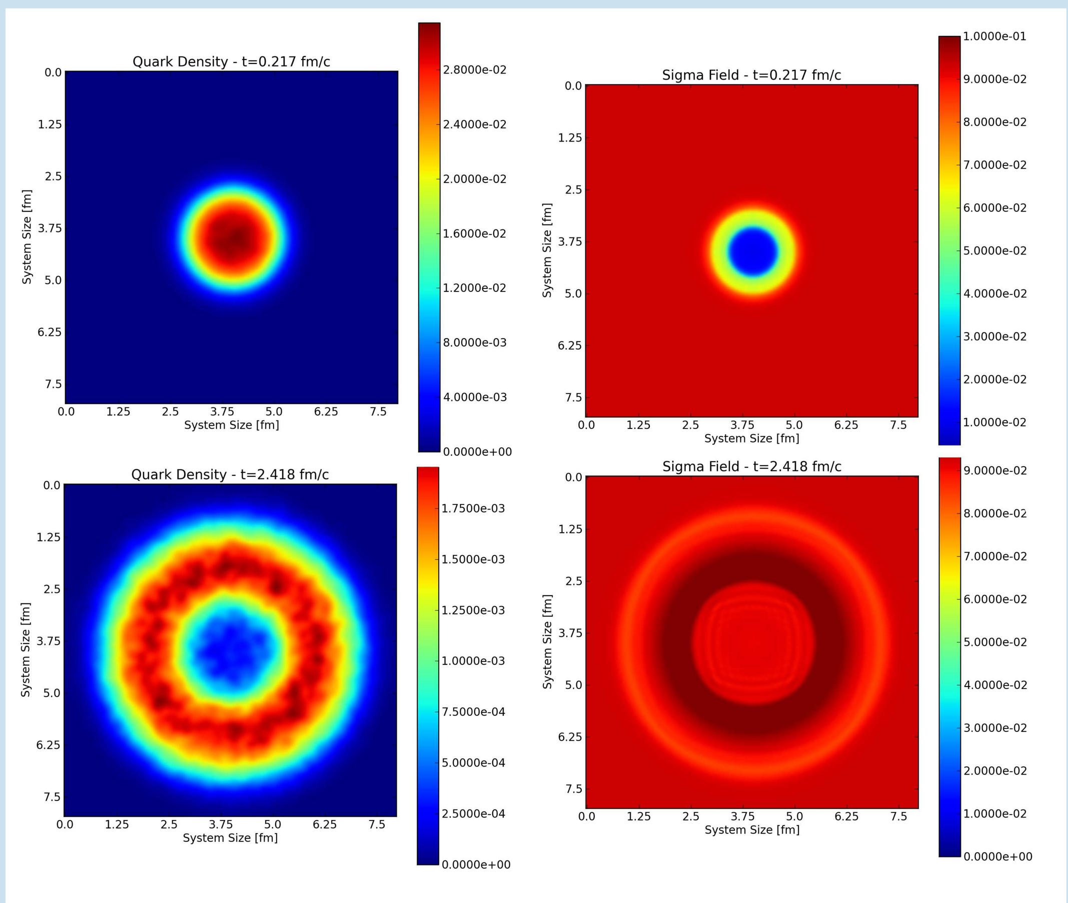

Figure 3: Transport simulation of the expansion of an hot and dense droplet. The droplet is initialized with a Woods-Saxon like thermal distribution. The expansion of the droplet leads to shell-like structures.

Wave-Particle and Inelastic Interactions

By employing the Vlasov-equation, the total particle number in the system is conserved. Particle number conservation is found in many comparable studies, which is a reasonable approximation if chemical processes are slow in comparison to spatial expansion processes. However, a deviation from chemical equilibrium can lead to dramatic effects in the behavior of the phase transition (see Figure 4).
For an effective thermal and chemical equilibration of both fields and particles, we integrated the offshell Yukawa process

$$\bar{\psi}\psi \leftrightarrow \sigma$$

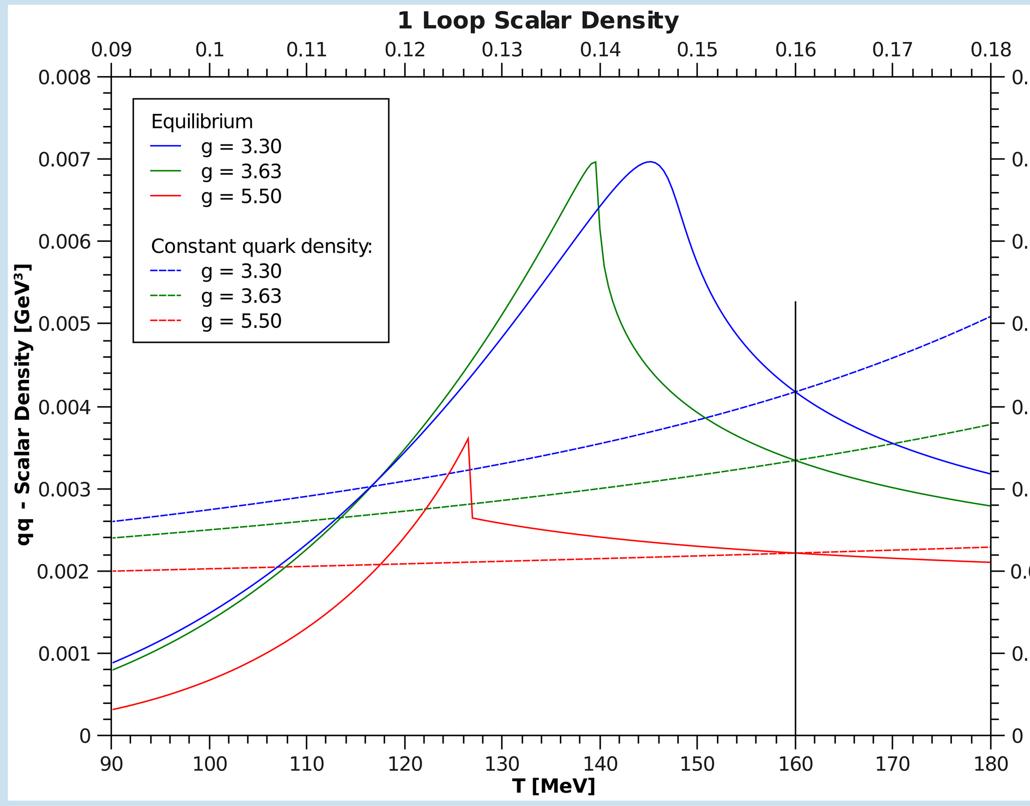

Figure 4: Linear sigma model in the thermal limit. If chemical processes suppressed (conservation of the particle density), the chiral phase transition vanishes.

The non-trivial interaction between a classical field and a classical particle is implemented by using the energy and momentum conservation in a small interaction volume. This ansatz allows to find a solution for the wave equation which allows to transfer discrete amounts of particle energy and momentum to and from the field.

$$E(\phi_i(\mathbf{r}) + \delta(\mathbf{r})) - E(\phi_i(\mathbf{r})) = \Delta E$$
$$\vec{P}(\phi_i(\mathbf{r}) + \delta(\mathbf{r})) - \vec{P}(\phi_i(\mathbf{r})) = \Delta\vec{P}$$

These interactions induce a thermal noise in and damping in the chiral fields. The distribution of energy transfers can be modelled from physical processes and samples with Monte-Carlo techniques. This allows full control over the time and strength of energy transfer and dissipative processes on the fields.

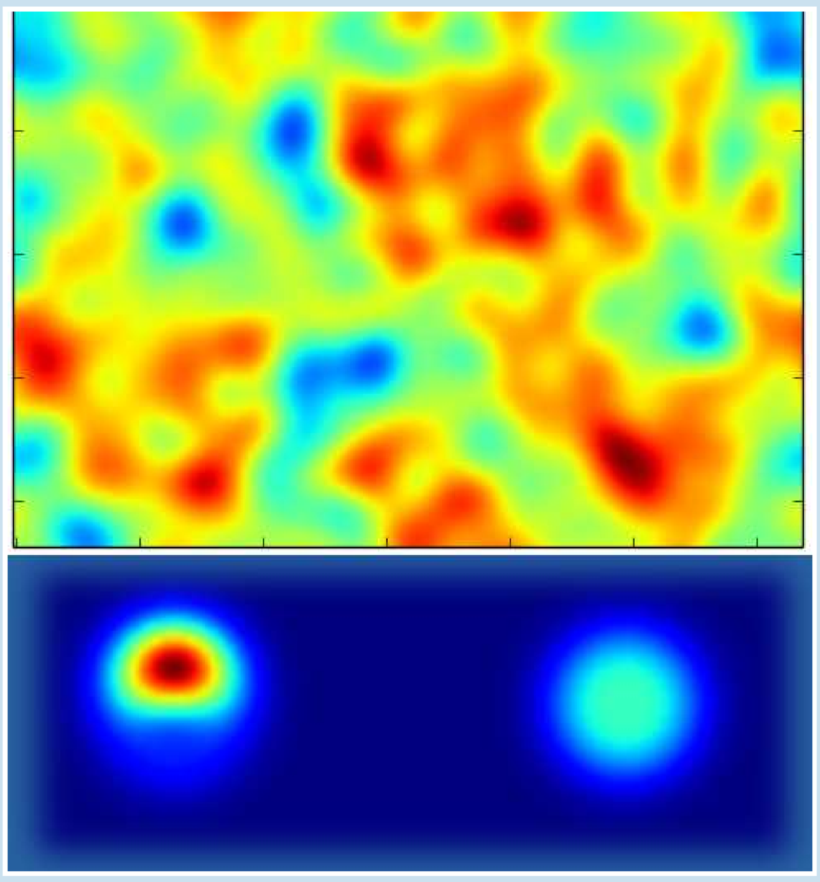

Figure 5: Top: Thermal fluctuations of the sigma field induced by particle annihilation and creation processes. The fields show gaussian white noise without the need to employ external noise terms.
Bottom: energy and momentum transfer by two quark annihilation in an interaction volume. On the left side, a netto momentum in forward direction is transfered, on the right side only energy is transfered.

Outlook

Inelastic processes are vital in a dynamical transport simulation of the linear sigma model, otherwise the phase transition can not be described in an evolving system. We have implemented these processes by quark-sigma particle creation and annihilation processes which respect detailed balance and equilibrium rates. Further investigations will be done in simulations of quenched scenarios with focus on reheating and and critical slowing down. By scanning over a temperature range, the behavior of thermal fluctuations at and near a critical point is analysed.

HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

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