Global Event Structure of Nuclear Collisions from Classical Gluon Fields

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Abstract and Summary

The classical gluon fields dominating high energy nuclear collisions at very early time can be calculated analytically.

Their energy momentum tensor contains information about the global event structure: (energy density profile, directed, radial and elliptic flow, angular momentum) that will be passed on to a quark gluon plasma phase at later times.

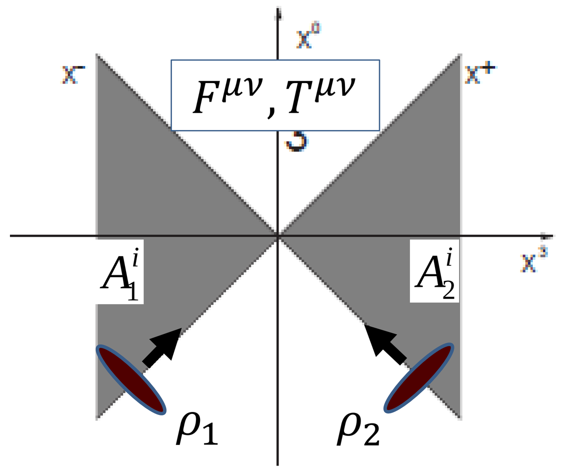
We obtain analytic classical solutions and demonstrate some of their interesting properties. The evolution will be followed by a fluid dynamic stage.

Classical Gluon Dynamics

Single nucleus at high energy: soft and semi-hard gluon modes ($\Lambda \lesssim q \lesssim Q_s$) are described by their classical fields A^μ, generated by large-x sources (color charge densities ρ) on the light cone through the Yang Mills equation [1]

$$D_\mu F^{\mu\nu}(A) = J^\nu(\rho) \quad \overset{solve}{\Longrightarrow} \quad A^\mu(\rho)$$

Two colliding nuclei: Fields and energy momentum tensor in the forward light cone as a function of the fields $A_1^\mu(\varrho_1), A_2^\mu(\varrho_2)$ in both nuclei [2]:



$$F^{\mu\nu}\left(A_1^\mu, A_2^\mu\right) \qquad T^{\mu\nu}\left(A_1^\mu, A_2^\mu\right)$$

Analytic solutions of Yang-Mills for times $\tau \lesssim 1/Q_s$: Small time expansion (near-field) [3,4,5] + resummation of leading UV-divergences [5].

Analytical Solutions for Glasma Fields

Initial longitudinal chromo-electric and magnetic fields at $\tau = 0$

$$E_0 = ig\delta^{kl}\left[A_1^k, A_2^l\right] \qquad B_0 = ig\epsilon^{kl}\left[A_1^k, A_2^l\right]$$

Initial transverse fields generated for $\tau > 0$:

$$E^i = -\frac{\tau}{2}\left[\sinh\eta \; D^i E_0 + \cosh\eta \; \epsilon^{ij} D^j B_0\right]$$

$$B^i = -\frac{\tau}{2}\left[\sinh\eta \; D^i B_0 - \cosh\eta \; \epsilon^{ij} D^j E_0\right]$$

Rapidity-odd, curl-free from Gauss' Law.
Rapidity-even, divergence-free from Ampere's and Faraday's Law.

Rapidity-odd fields occur naturally [4]!

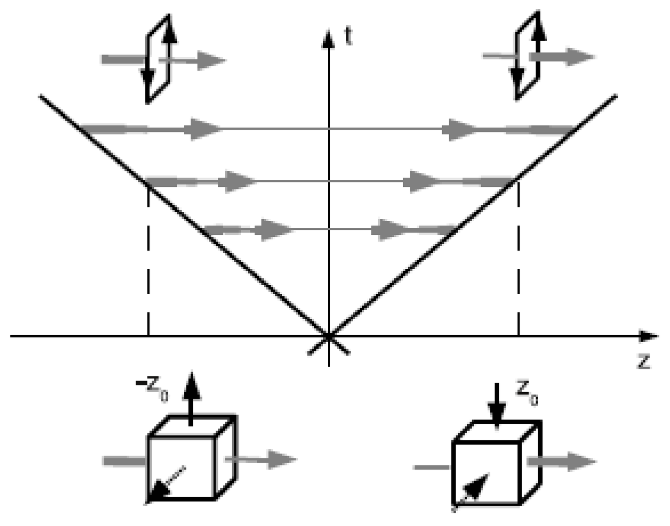


Figure 1: Two observers at $z = z_0$ and $z = -z_0$ test Ampère's and Faraday's Laws with areas a^2 in the transverse plane and Gauss' Law with a cube of volume a^3. The transverse fields from Ampère's and Faraday's Laws (black solid arrows) are the same in both cases, while the transverse fields from Gauss' Law (black dashed arrows) are observed with opposite signs. Initial longitudinal fields are indicated by solid grey arrows, thickness reflects field strength.

Simulation of Initial Transverse Fields

Randomly seeded longitudinal fields (shading in the plot to the right):

⇒ Divergence-free (Faraday & Ampere) transverse fields at midrapidity.

⇒ Linear combination of divergence- and curl-free fields away from midrapidity

(Plot: abelian case)

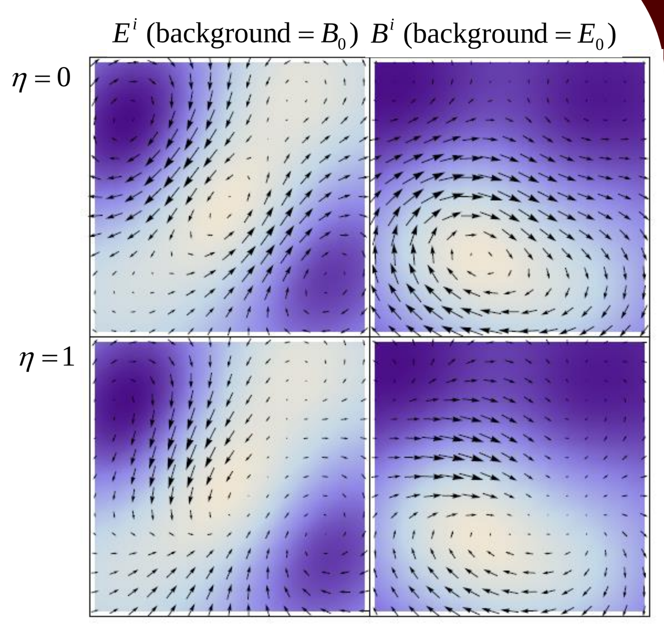


Figure 2: Transverse electric fields (left panels) and magnetic fields (right panels) at $\eta = 0$ (upper panels) and $\eta = 1$ (lower panels) in an abelian example for a random distribution of fields A_1^i, A_2^i. The initial longitudinal fields B_0 (left panels) and E_0 (right panels) are indicated through the density of the background (lighter color = larger values). At $\eta = 0$ the fields are divergence-free and clearly following Ampére's and Faraday's Laws, respectively.

Energy Momentum Tensor

General structure:

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon & \vec{S}_T & S_L \\ \vec{S}_T & \vec{P}_T & \vec{S}_T^\dagger \\ S_L & \vec{S}_T^\dagger & P_L \end{pmatrix}$$

ε = energy density in the lab frame.
$\vec{S}_T$, S_L = trans., long. components of the Poynting vector (= flow of gluon energy)

Analytic form (select examples)

$$\varepsilon(\tau,\eta,x,y) = \varepsilon_0 - \frac{\tau^2}{4}\left[\nabla^i\alpha^i + \delta + \frac{1}{2}\sinh(2\eta)\left(\nabla^i\beta^i - \theta\right) - \frac{1}{2}\cosh(2\eta)\,\delta\right] + O(\tau^4)$$

$$S_T^i(\tau,\eta,x,y) = \frac{\tau}{2}\left[\cosh(\eta)\alpha^i(x,y) + \sinh(\eta)\,\beta^i(x,y)\right] + O(\tau^3)$$

Coefficients $\varepsilon_0(x,y), \delta(x,y), \alpha^i(x,y), \beta^i(x,y)$, etc. calculable

Rapidity-odd and -even contributions to energy flow:
$\alpha^i = -\nabla^i\varepsilon_0$ follows gradients in energy density (hydro-like)
$\beta^i = E_0 D^i E_0 - E_0 D^i E_0$ does not follow gradients (antenna-like)

Event Averages

McLerran-Venugopalan (MV) model:
Gaussian distribution of charges, $\langle\rho_i\rangle = 0$, $\langle\rho_i^2\rangle \propto \mu_i$ (average charge distributions).

Event-by-event calculation: Coefficients $\varepsilon_0, \alpha^i, \beta^i, \delta, \ldots$ from a 2D-MC sampling of the ρ_i (similar to [6]).

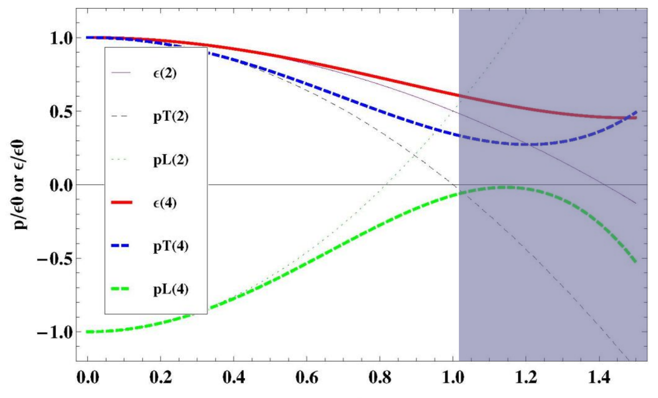
Analytic calculation of event averages: generalize MV framework to allow gradients in the average charge distributions [5].

Here: results for event-averaged energy momentum tensor
(selected components, lowest orders in time only)

$$\varepsilon_0 \propto \alpha_s^3 \mu_1 \mu_2 \qquad \frac{\alpha^i}{\varepsilon_0} = -\frac{\nabla^i(\mu_1\mu_2)}{\mu_1\mu_2} \qquad \frac{\beta^i}{\varepsilon_0} = -\frac{\mu_2\nabla^i\mu_1 - \mu_1\nabla^i\mu_2}{\mu_1\mu_2}$$

Results

Energy density, longitudinal and transverse pressure as a function of τ: behavior qualitatively similar to the recent numerical calculation in [7], up to $\tau \sim Q_s^{-1}$.

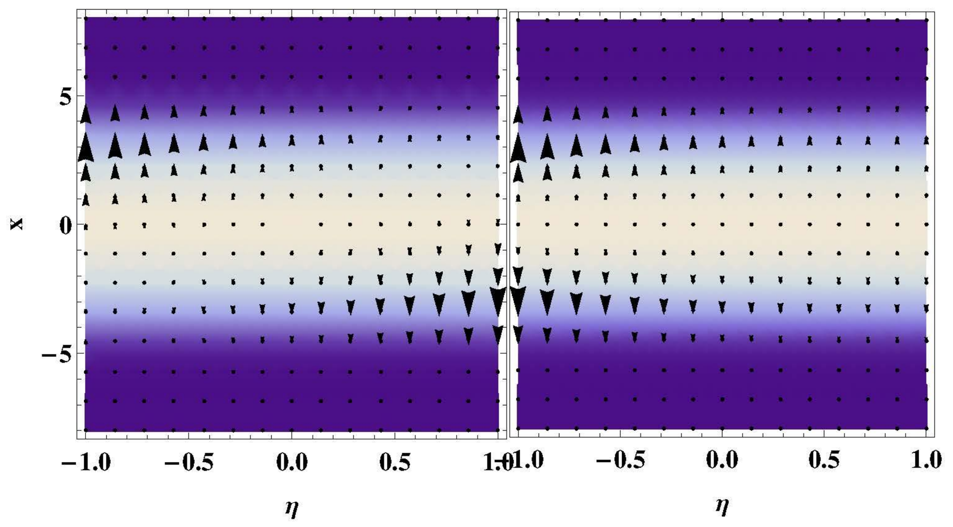


(2) = 2nd order, (4) = 4th order in time.

Transverse flow $\vec{S}_T$ as function of x and η at y=0:

Pb+Pb @ $b = 10$ fm: rapidity-odd flow = directed flow of energy

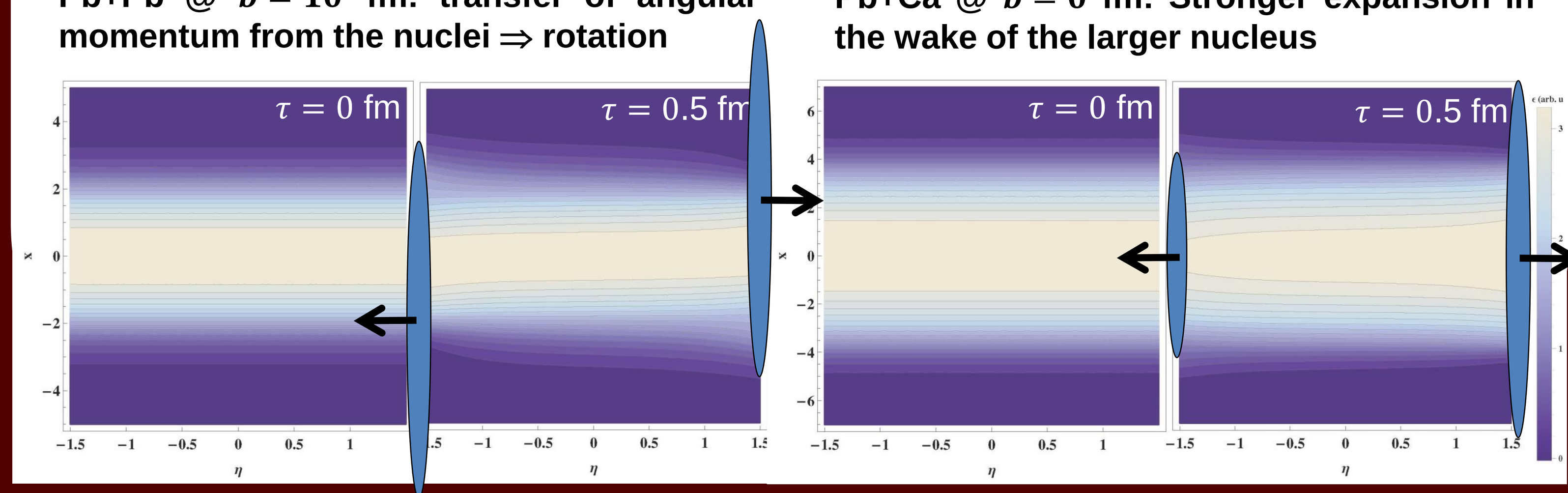
Pb+Ca @ $b = 0$ fm: rapidity-asymmetric expansion



Energy flow generates the time evolution of the energy density ε:

Pb+Pb @ $b = 10$ fm: transfer of angular momentum from the nuclei ⇒ rotation

Pb+Ca @ $b = 0$ fm: Stronger expansion in the wake of the larger nucleus



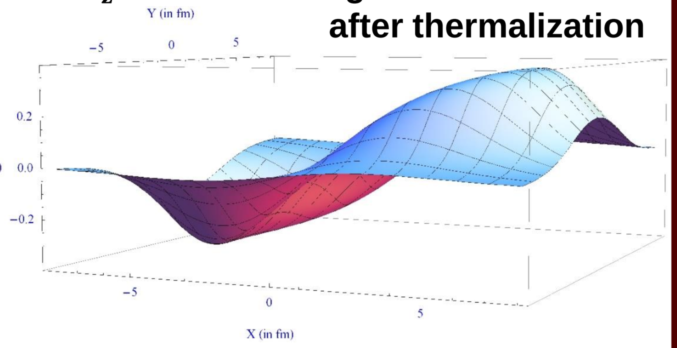
Discussion and Outlook

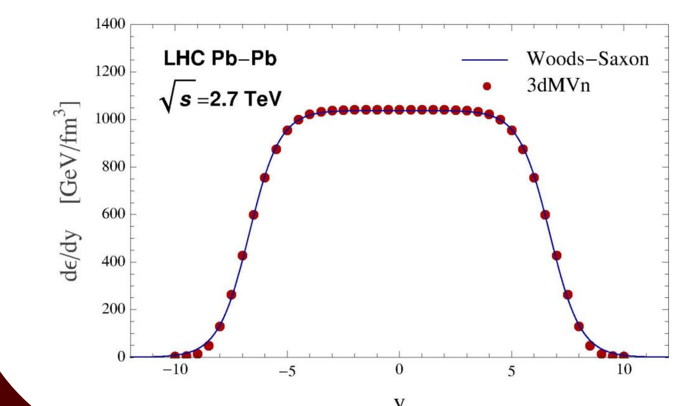
Interesting features in the time evolution of early flow and energy density: angular momentum, directed flow, asymmetric flow in A+B collision systems, …

Are there unique signatures of color glass? Antenna effects?

Outlook 1: further evolution in fluid dynamics assuming rapid thermalization [8]. Effects readily translate to hydrodynamic fields.

Typical shape of the nodal plane $v_z = 0$ of the longitudinal flow field after thermalization





Energy density ε as function of rapidity

Outlook 2: beyond boost-invariance; realistic behavior at large rapidities [9].

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