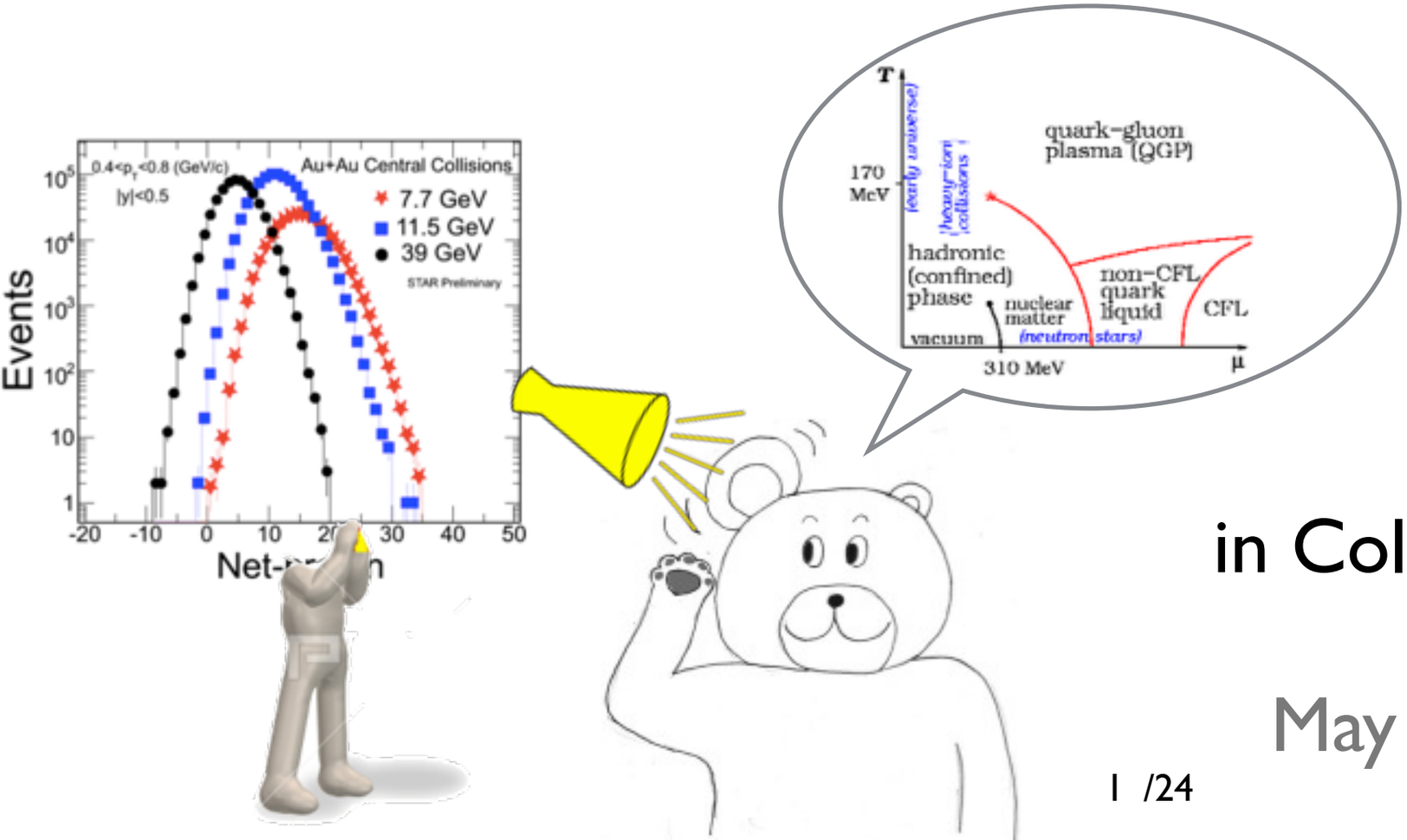
What are Multiplicity Distributions telling us on QCD Phase Diagram ?



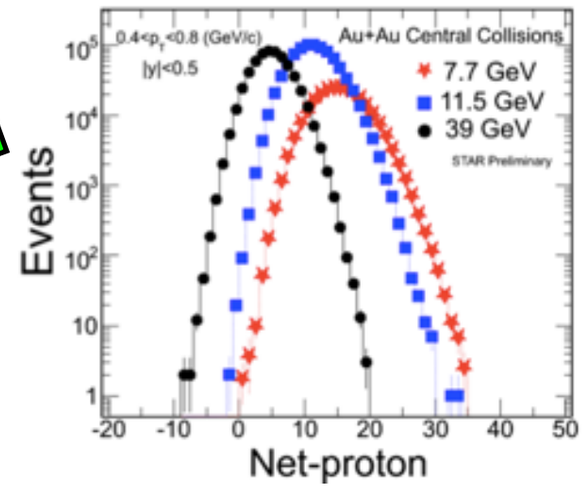
Atsushi Nakamura
in Collaboration with K.Nagata

QM2014

May 19 - 24, 2014, Darmstadt

We can get Canonical Partition Functions both from Experiments and Lattice

Experiments



Lattice QCD Simulaitions

Why $\mathbb{Z}_N$ useful ?

Yes, very useful, because

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

$(\xi \equiv e^{\mu/T}$: Fugacity$)$

$Z_n(T)$ ← $Z(\xi, T)$ at some ξ and T

$Z_n(T)$ → $Z(\xi, T)$ at ANY ξ

for both Experiments and Lattice

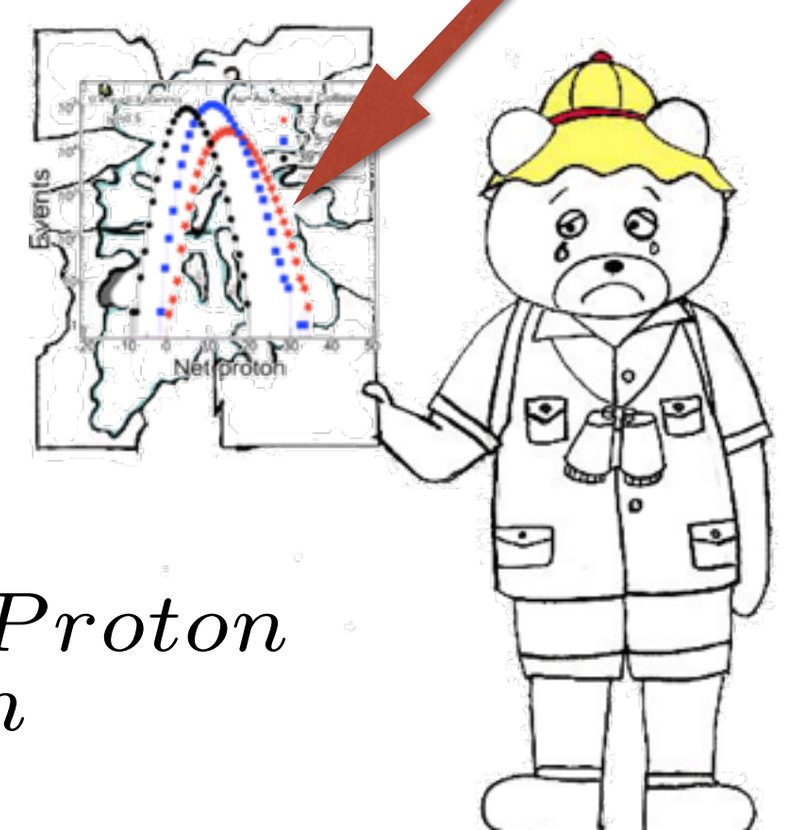
(Current) Weak Points

1) Experimental Multiplicity Data

Net-Proton and Not net-Baryon

One can prove $Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$
only for Conserved Quantities.

Possible approaches:
i) Wait for Net-Baryon data,
or Net-Charge data.
ii) Study and analyze data
assuming $Z_n^{Baryon} \sim Z_n^{Proton}$



2) N_{max} **is not very large.**

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

Lower estimation of larger density contribution.

$$Z(\mu, T) \Leftrightarrow Z_n(T)$$

Grand Canonical Canonical

$$Z(\mu, T) = \mathrm{Tr}\, e^{-(H-\mu\hat{N})/T}$$

$$= \sum_n \langle n | e^{-(H-\mu\hat{N})/T} | n \rangle$$

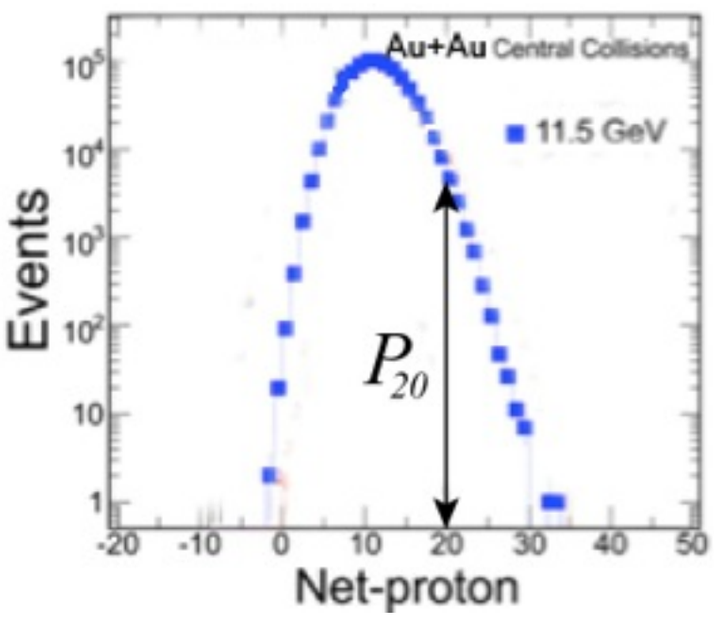
If $[H, \hat{N}] = 0$

$$= \sum_n \boxed{\langle n | e^{-H/T} | n \rangle}\, e^{\mu n/T}$$

$$= \sum_n Z_n(T) \xi^n \qquad \left(\xi \equiv e^{\mu/T} \right)$$

Fugacity

How can we extract Z_n from multiplicity events



$$P_n = Z_n \xi^n \qquad \left(\xi \equiv e^{\mu/T}\right)$$

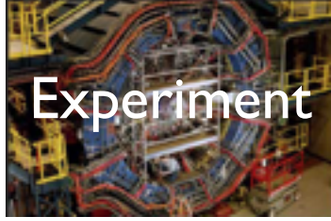
ξ unknown

$$Z_n = P_n / \xi^n$$

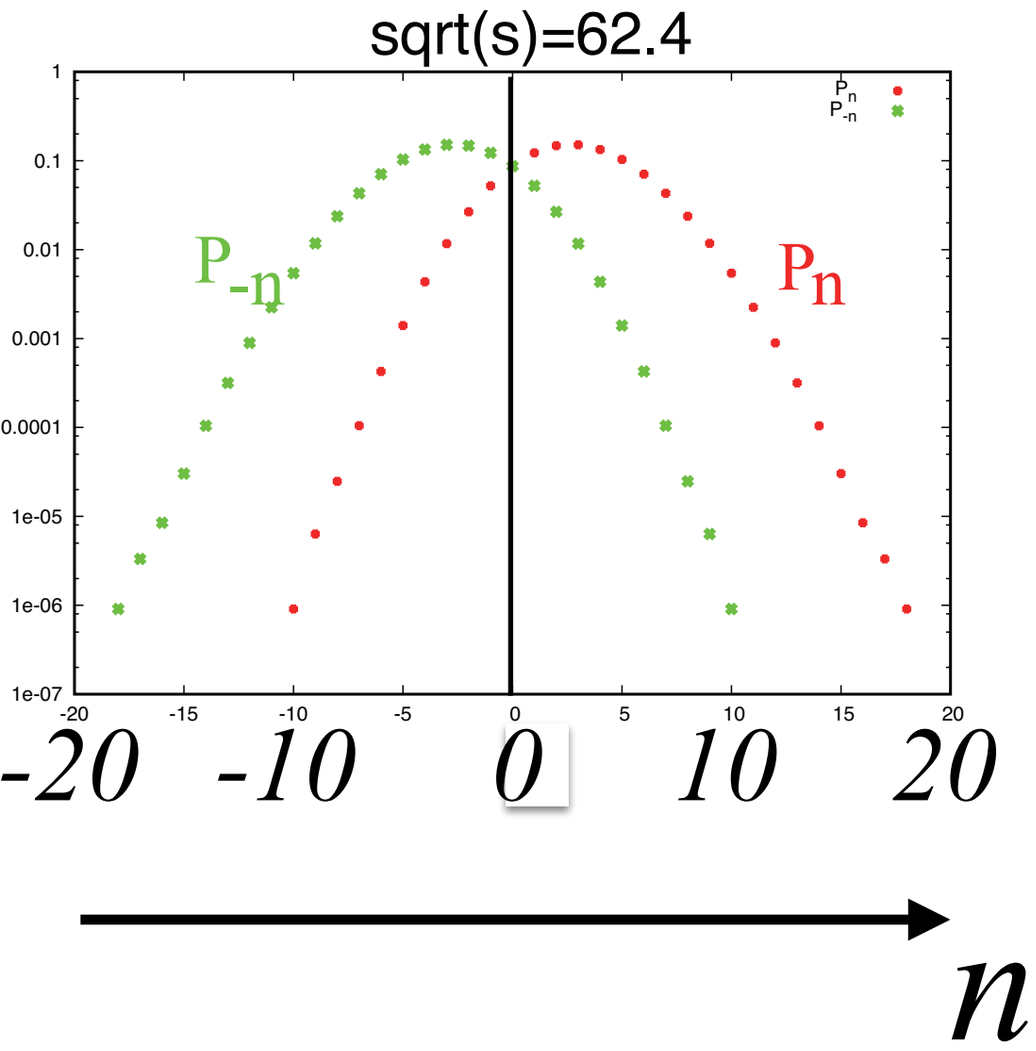
We require

$$Z_{+n} = Z_{-n}$$

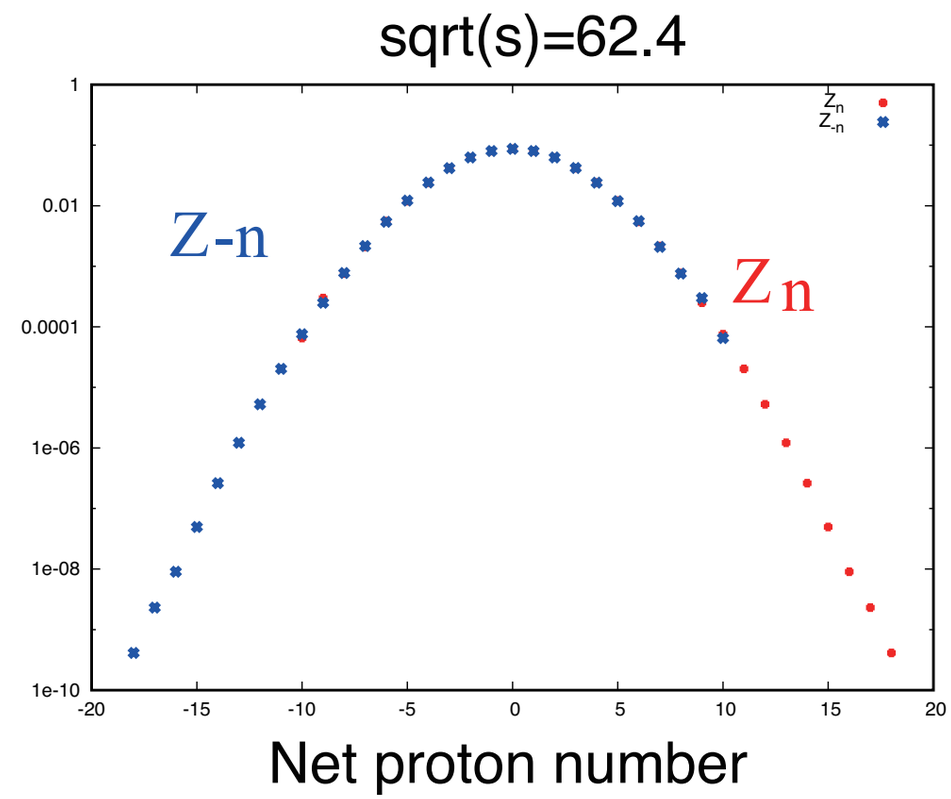
(Particle-AntiParticle Symmetry)



Demand $Z_{+n} = Z_{-n}$

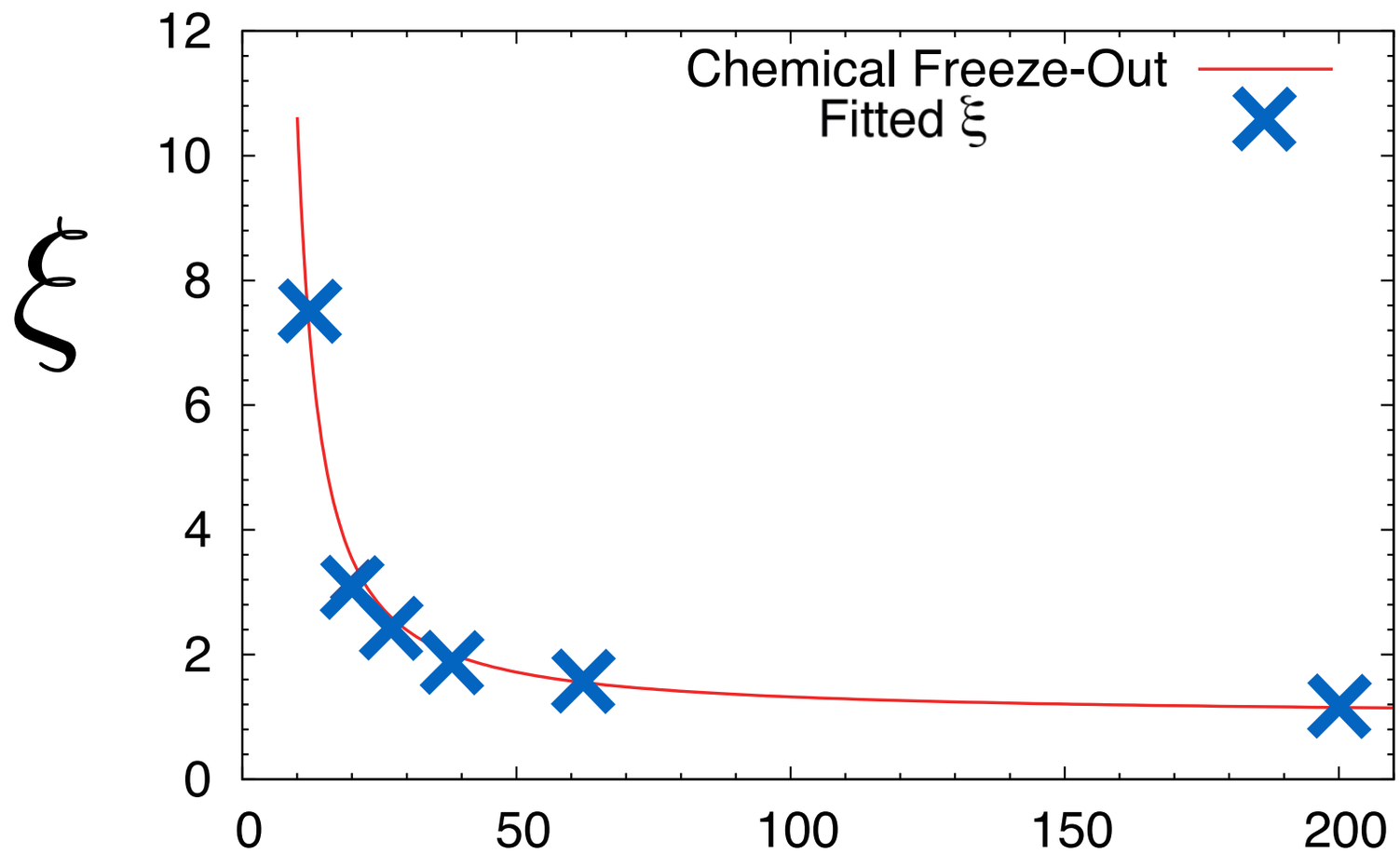


Tune ξ



$$Z_n = P_n/\xi^n$$

Fitted ξ are very consistent with those by Freeze-out Analysis.



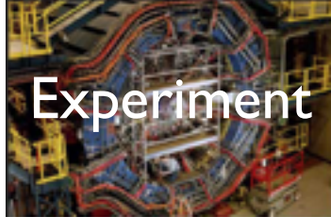
$\left(\xi \equiv e^{\mu/T}\right)$ $\sqrt{s}$ GeV

× This work

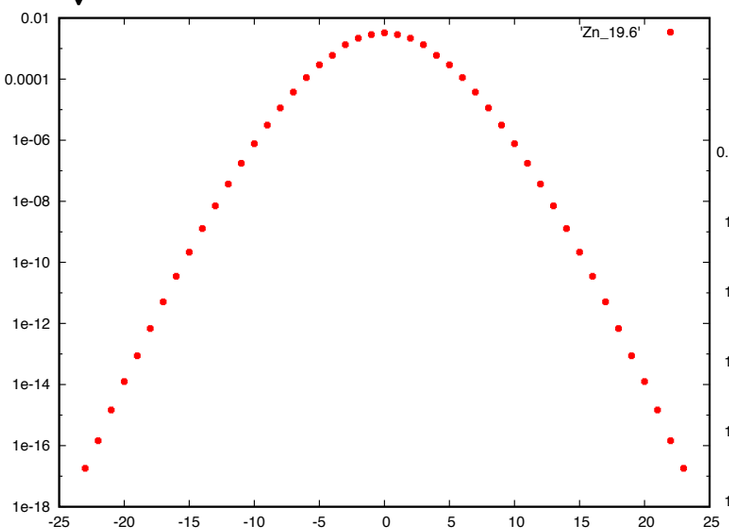
— Freeze-out

J.Cleymans, H.Oeschler, K.Redlich and S.Wheaton Phys. Rev. C73, 034905 (2006)

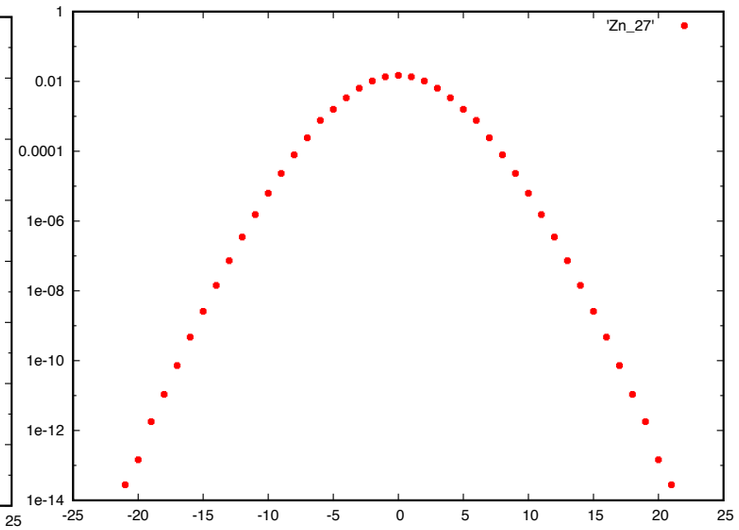
Z_n from RHIC data



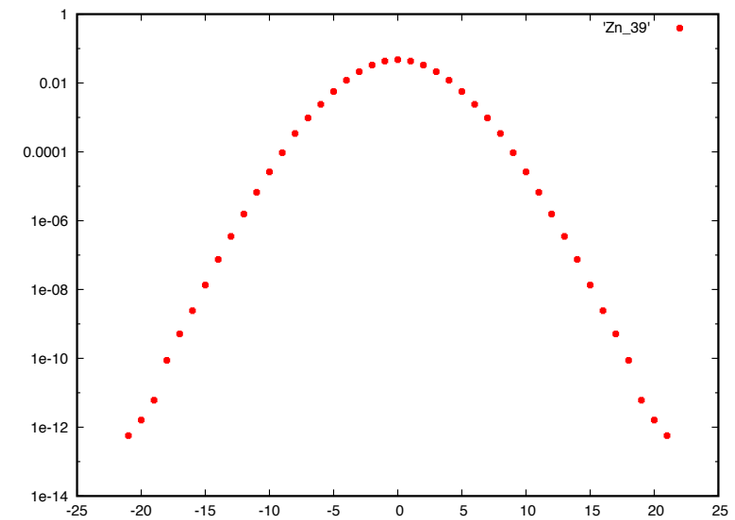
$\sqrt{s} = 19.6\mathrm{GeV}$

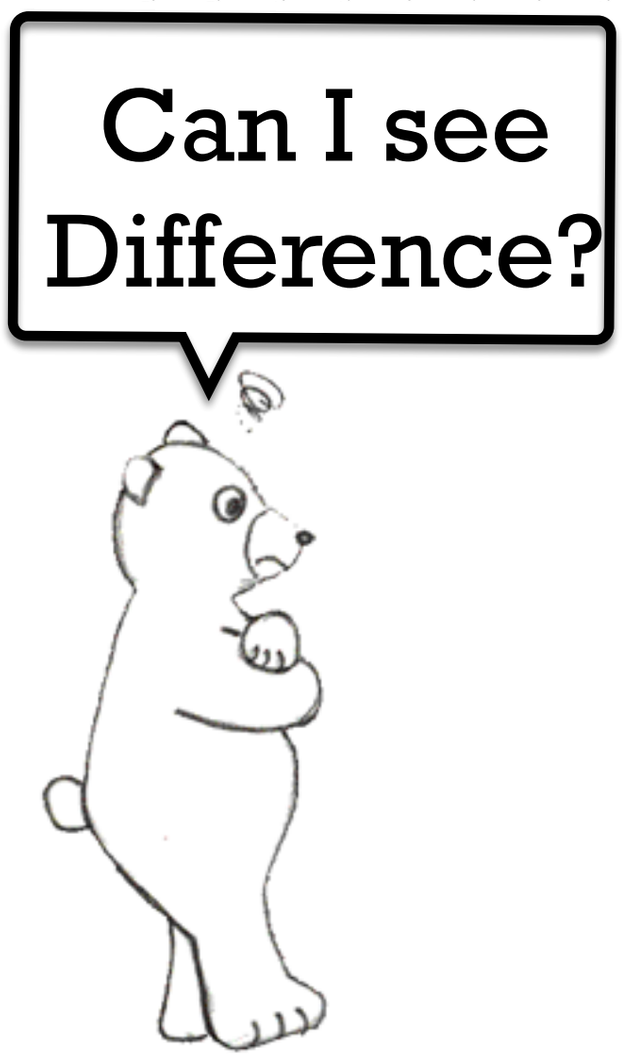


$\sqrt{s} = 27\mathrm{GeV}$

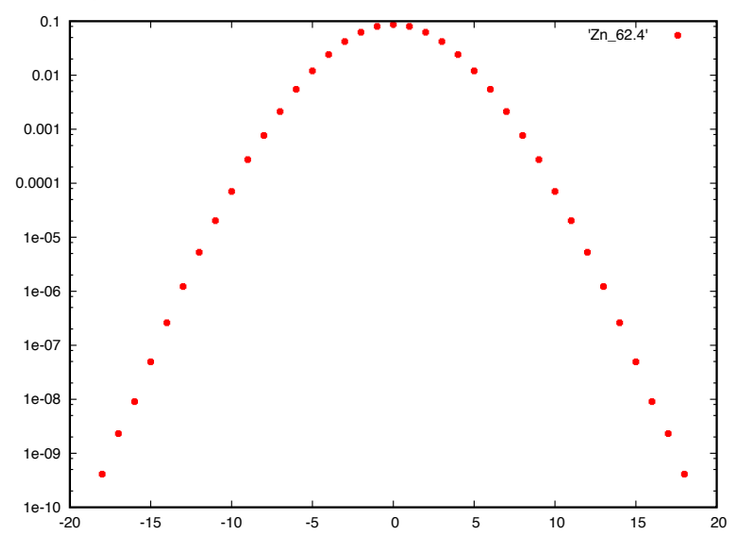


$\sqrt{s} = 39\mathrm{GeV}$

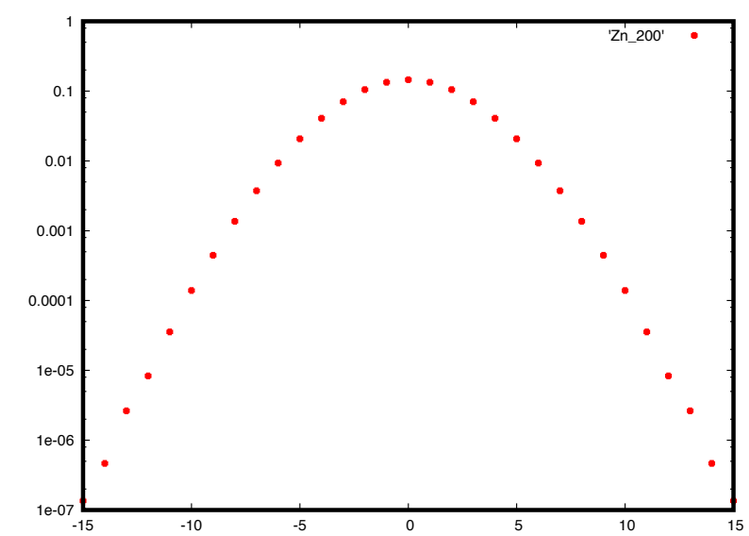




$\sqrt{s} = 62.4\mathrm{GeV}$



$\sqrt{s} = 200\mathrm{GeV}$



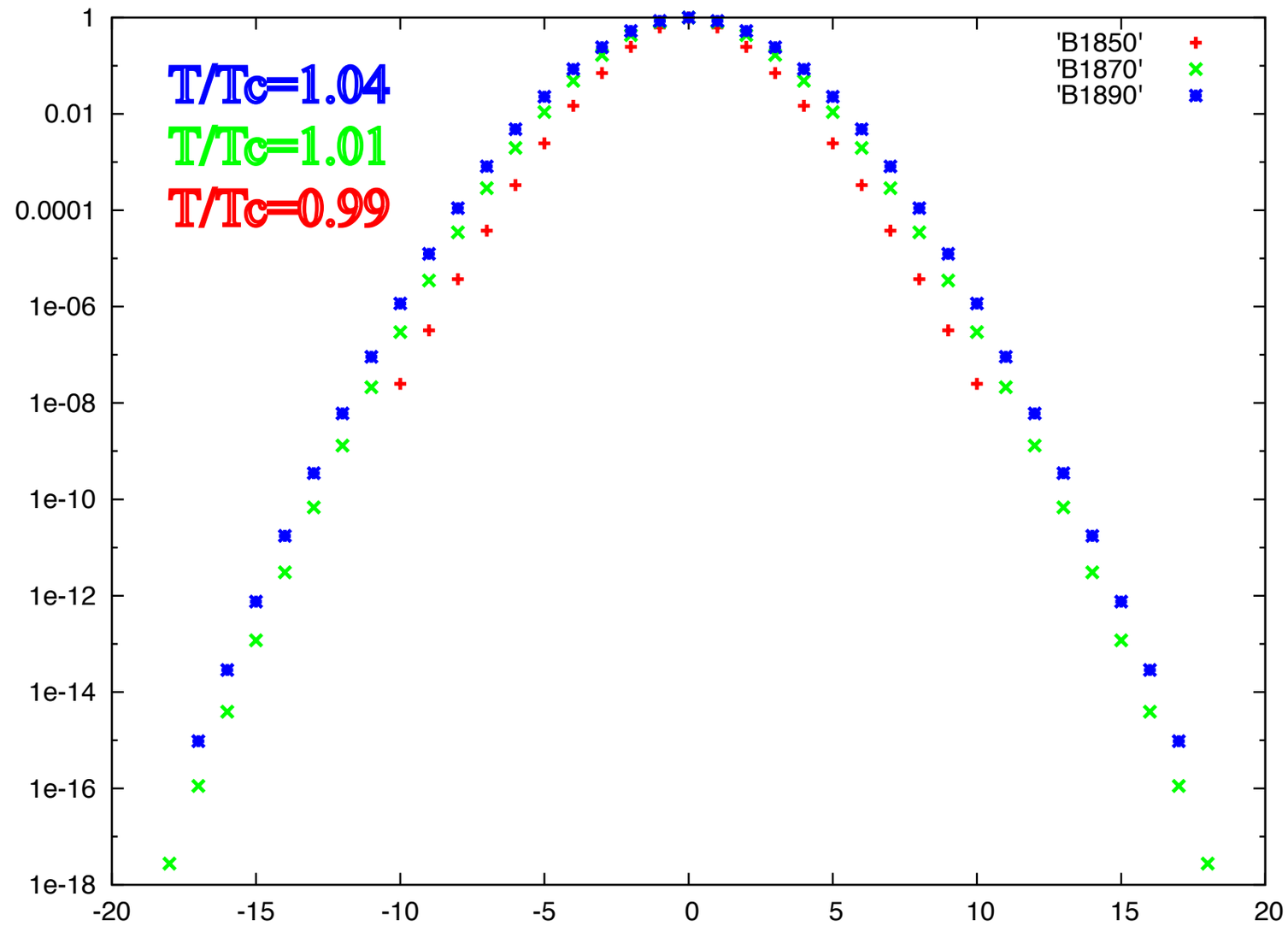
Yes, You Can !
We will see it.

We can calculate Z_n
also by Lattice QCD

Lattice Data

Can I see Difference?

Zn



Yes, You Can !
Wait a moment.

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

$$\xi \equiv e^{\mu/T}$$

Is this useful ?

Yes, because

1) We can calculate Z at any ξ (i.e., μ)

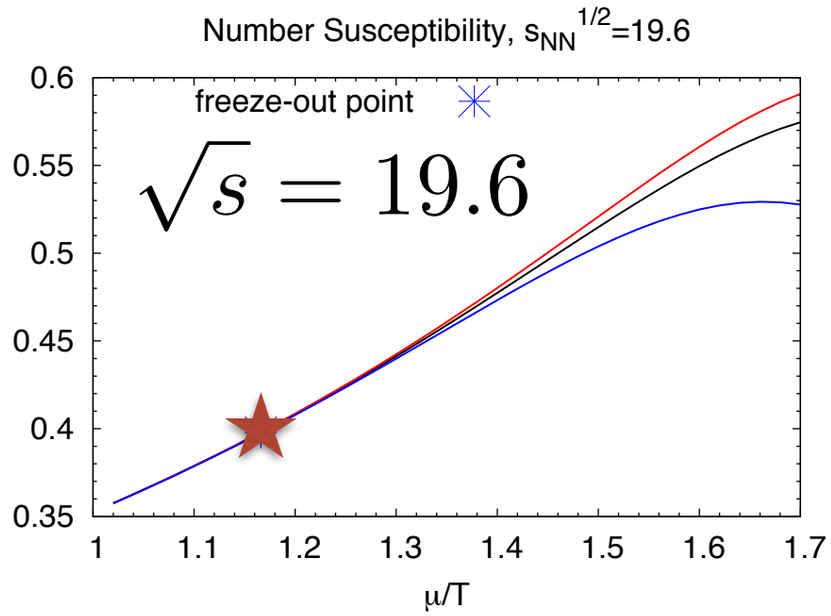
2) We can calculate Z even at complex ξ

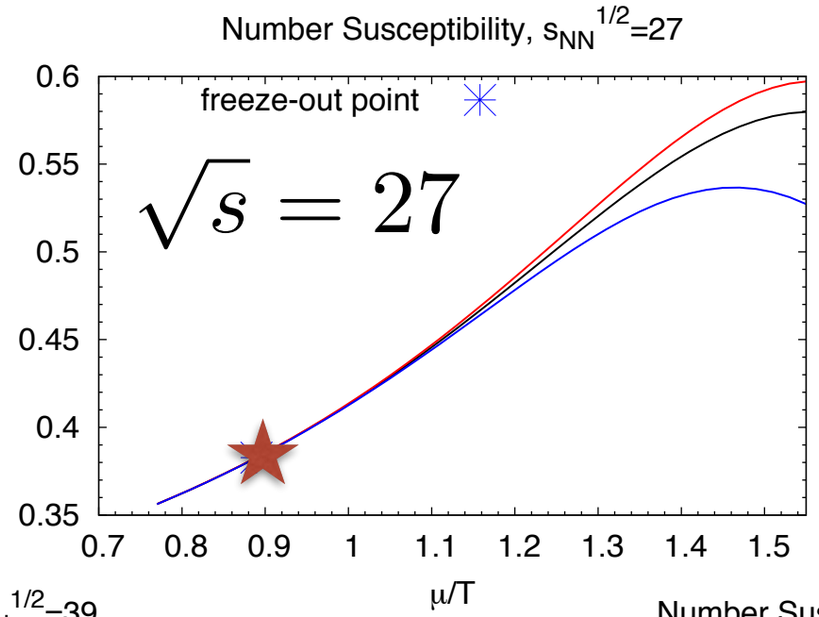
Moments λ_k

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

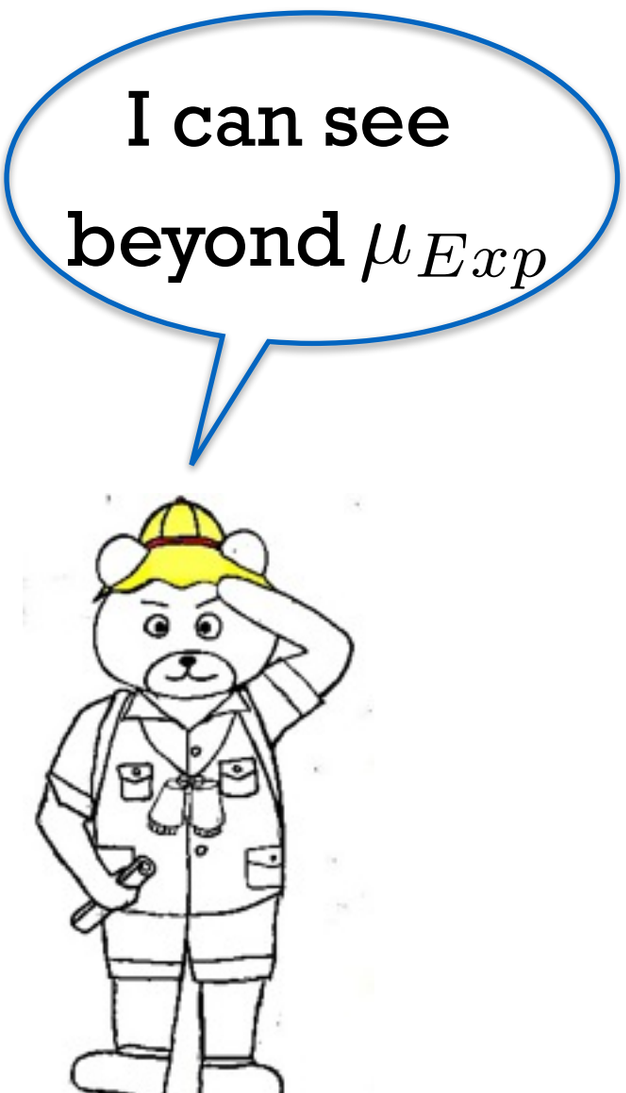
$$\lambda_k \equiv \left(T \frac{\partial}{\partial \mu} \right)^k \log Z$$

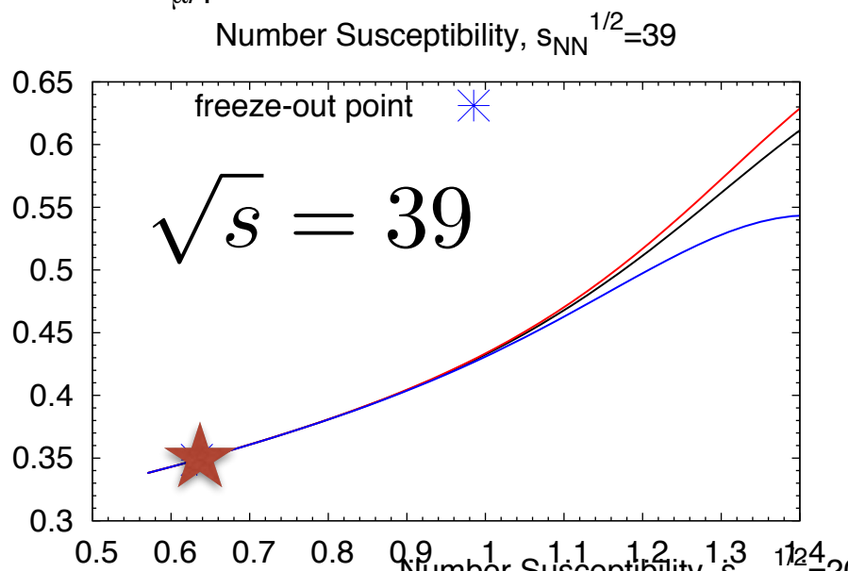
Susceptivility as a function of μ/T

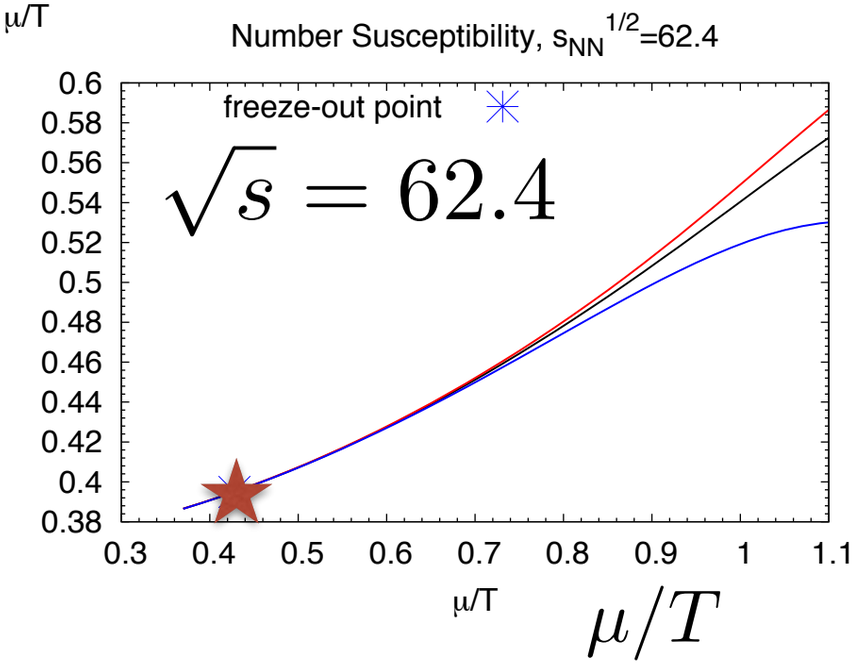


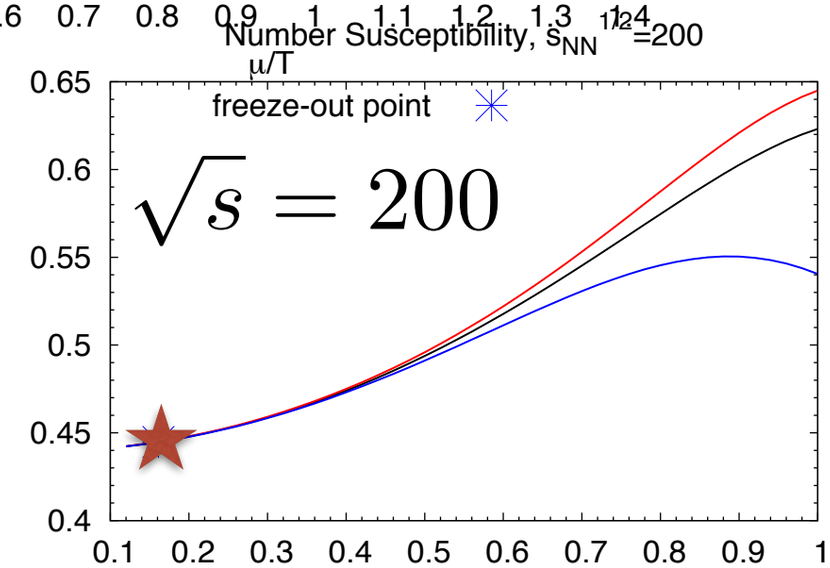


RHIC Data





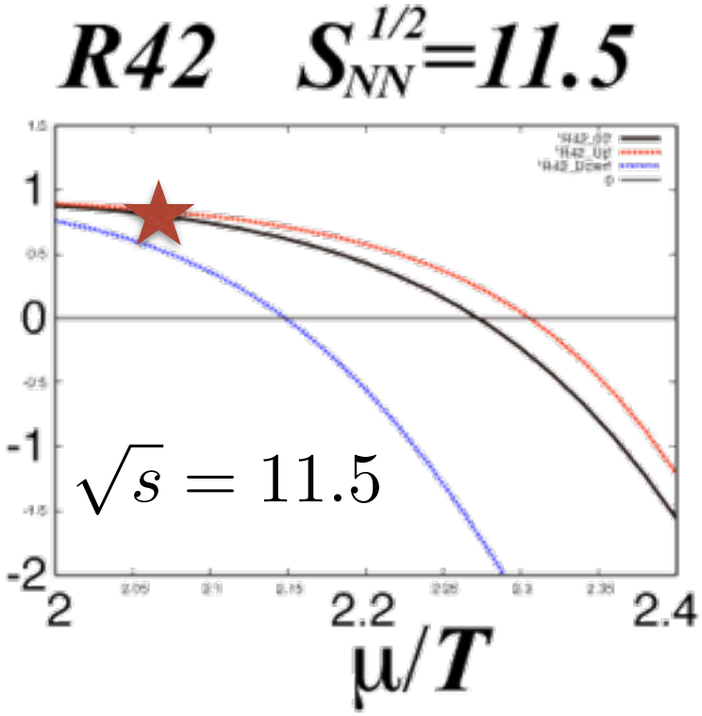


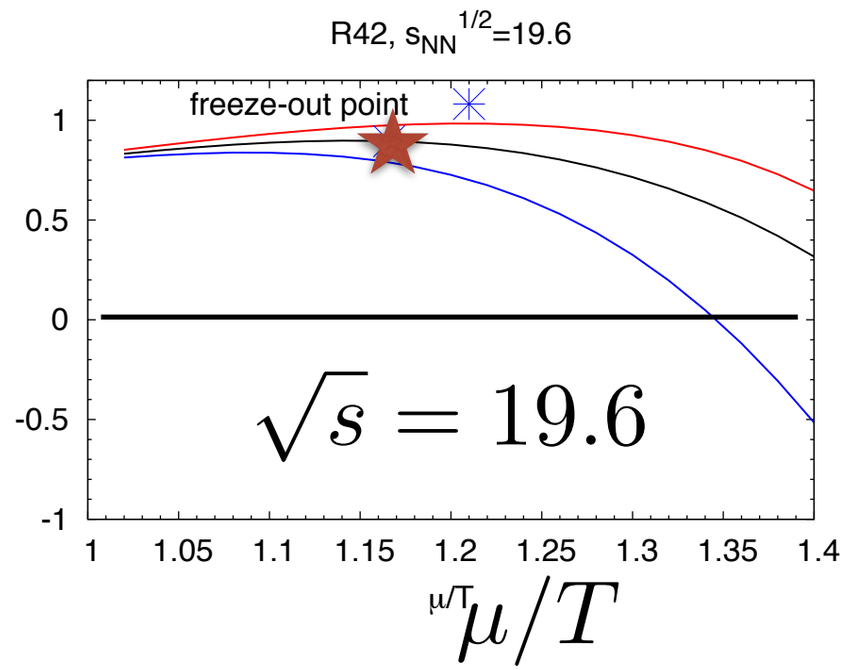


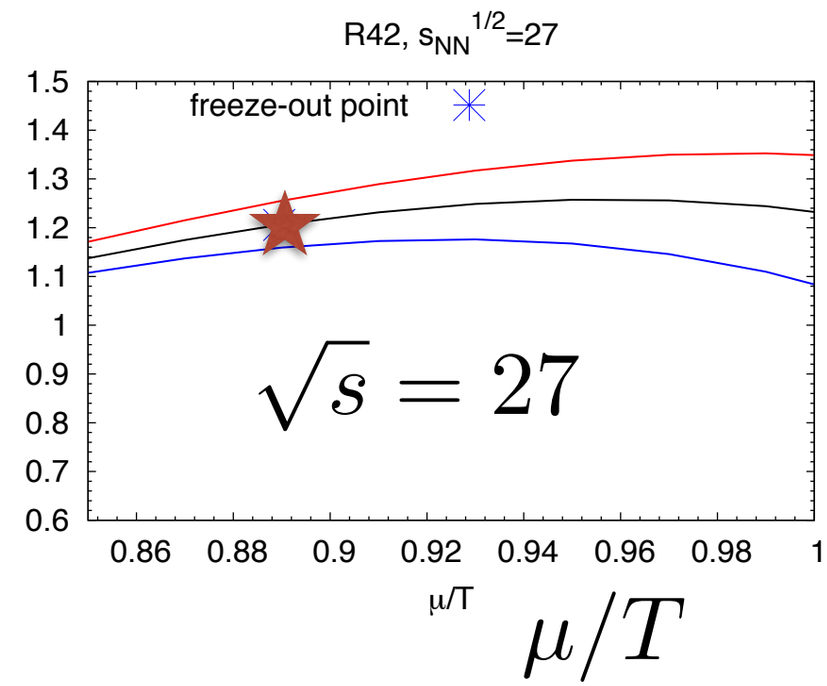
★ Observed here

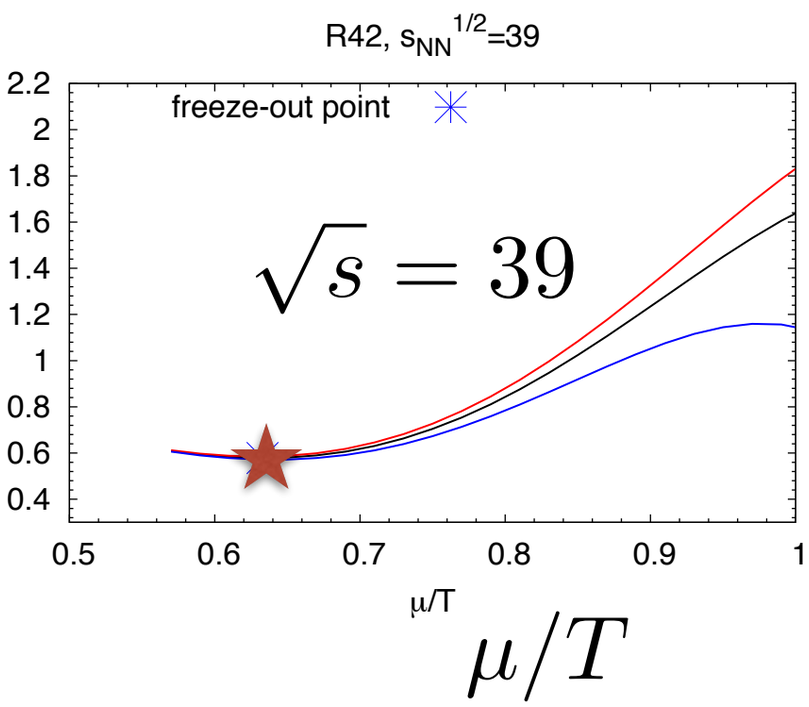
RHIC Data

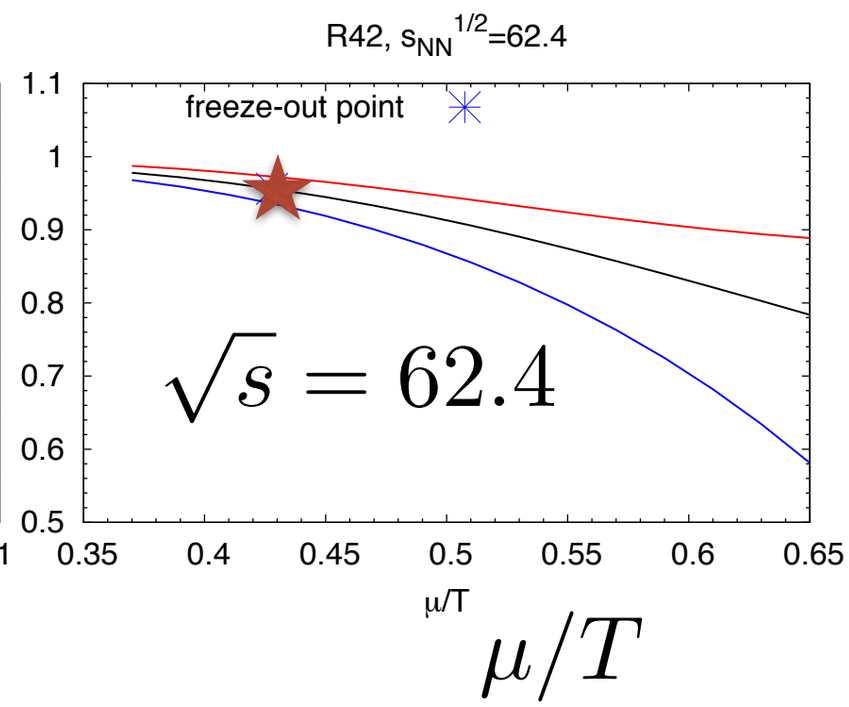
Kurtosis $\frac{\lambda_4}{\lambda_2}$ as a function of $\frac{\mu}{T}$

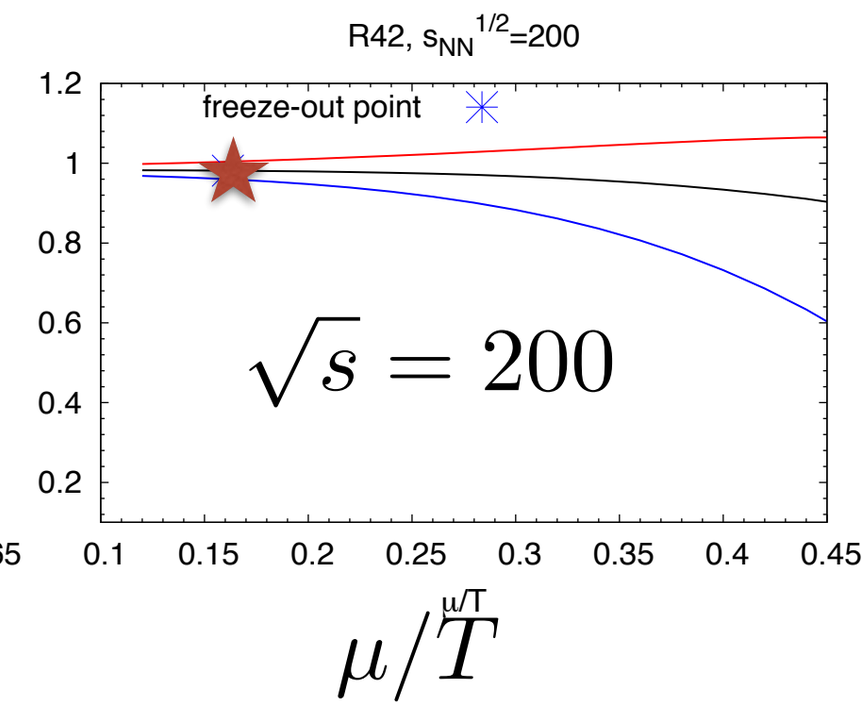


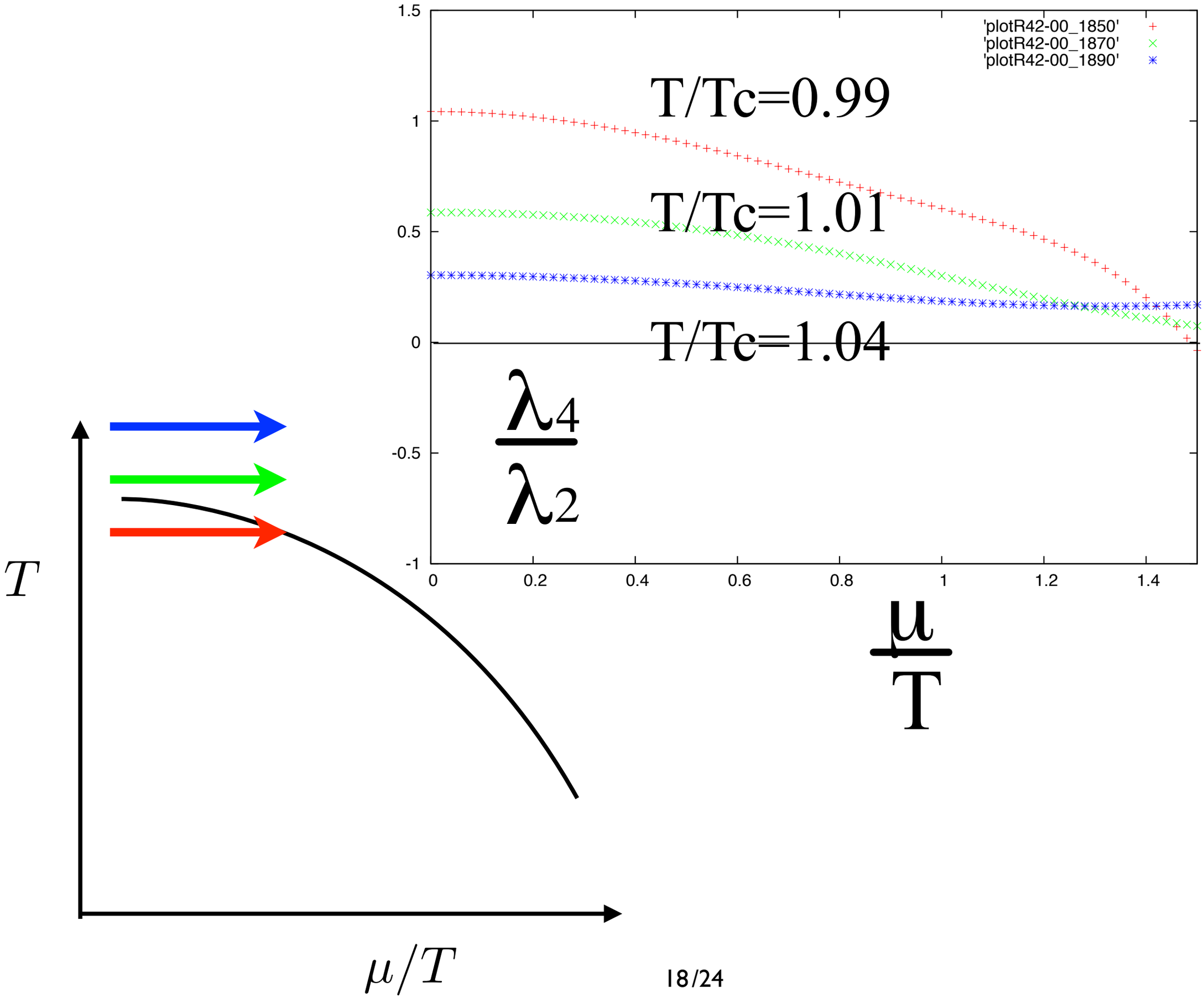











'plotR42-00_1850'
'plotR42-00_1870'
'plotR42-00_1890'
T/Tc=0.99
T/Tc=1.01
T/Tc=1.04
λ4/λ2
μ/T
1.5
1
0.5
0
-0.5
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
T
μ/T

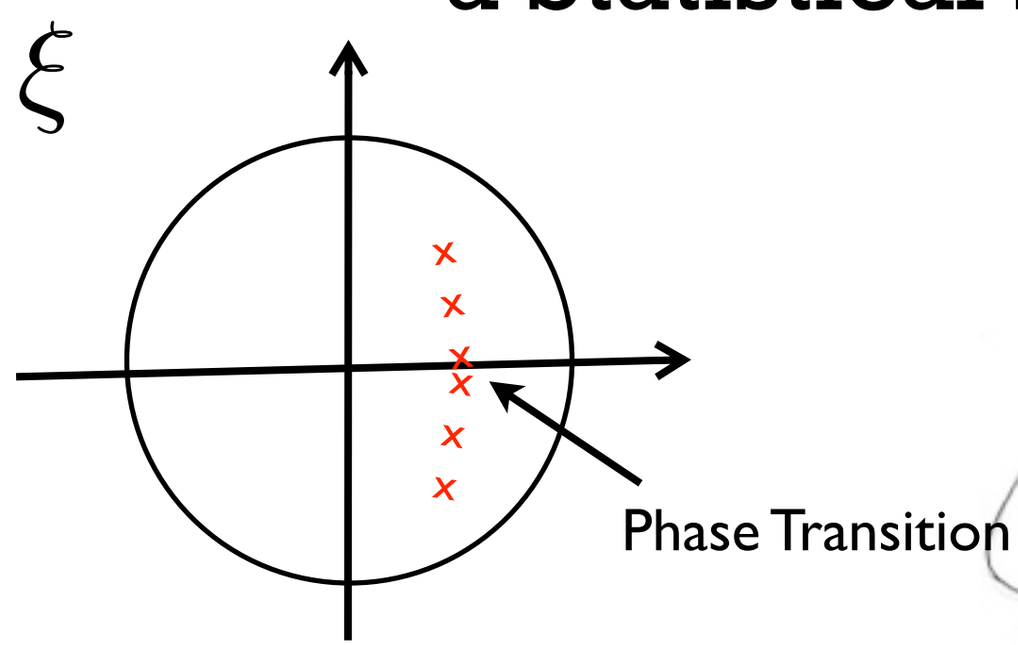
$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\,\xi^n$$

Lee-Yang Zeros (1952)

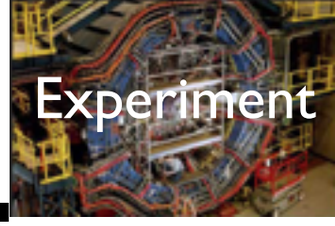
Zeros of $Z(\xi)$ in Complex Fugacity Plane.

$$Z(\alpha_k) = 0$$

Great Idea to investigate a Statistical System

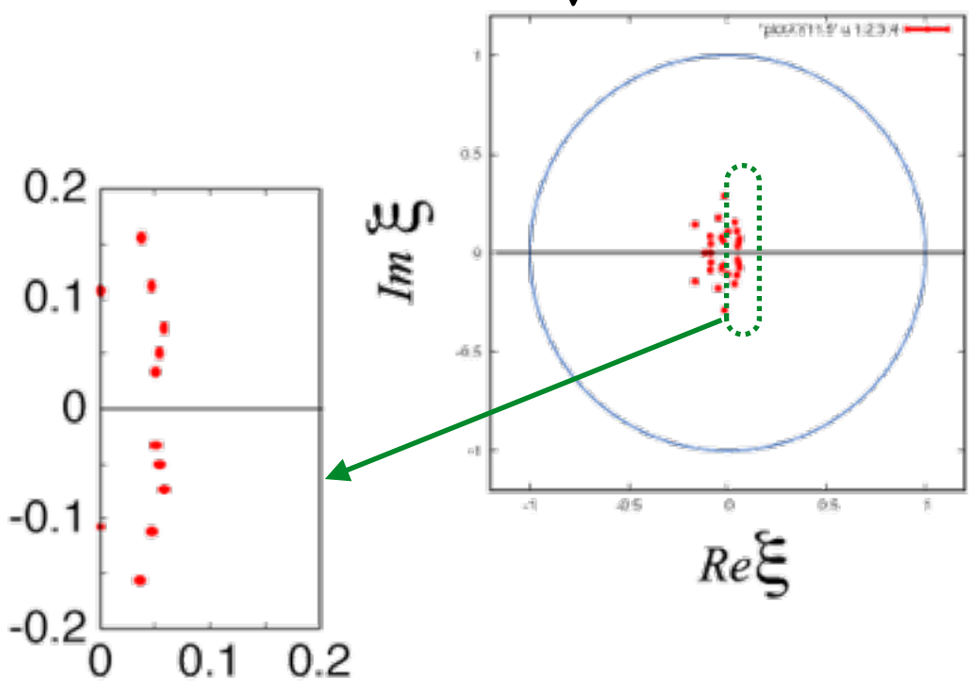


Lee-Yang Zeros Experimental Data (RHIC)

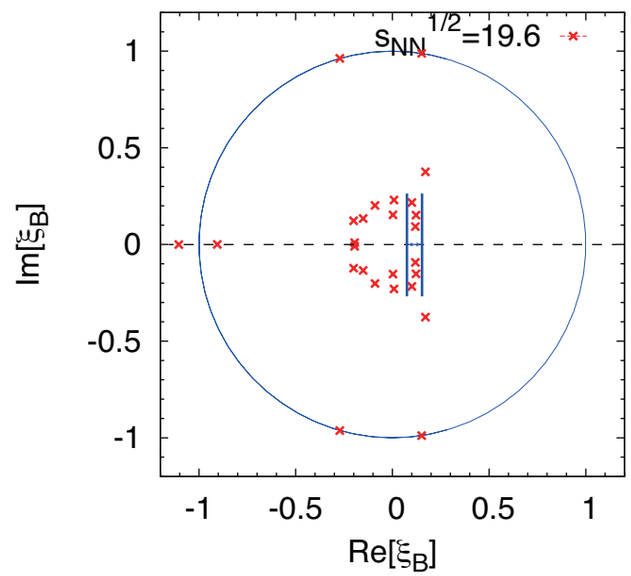


Lee-Yang Zeros: RHIC Experiments

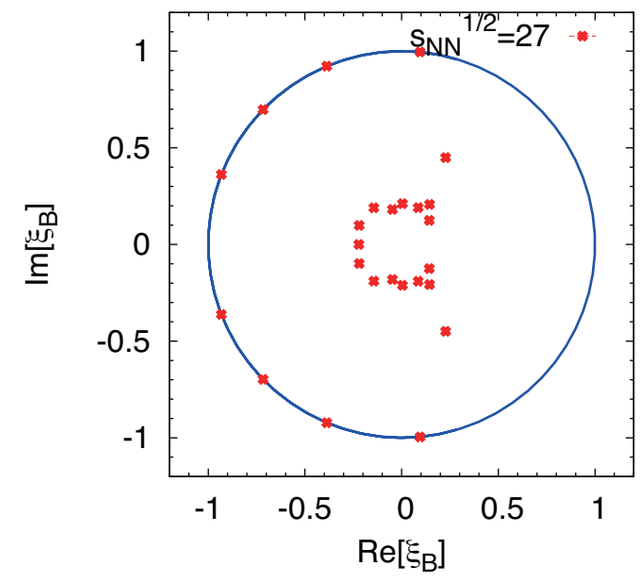
$\sqrt{s} = 11.5$



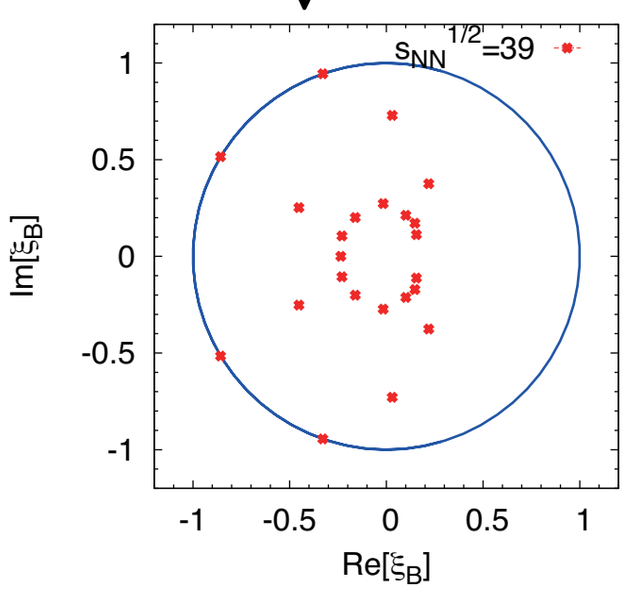
$\sqrt{s} = 19.6$



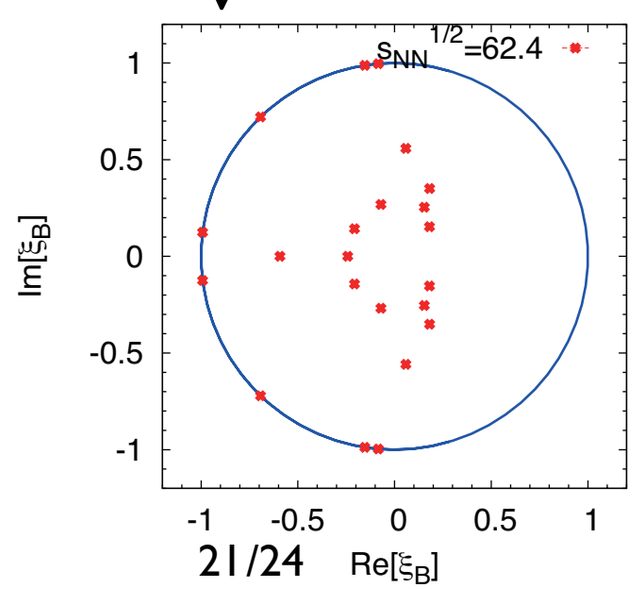
$\sqrt{s} = 27$



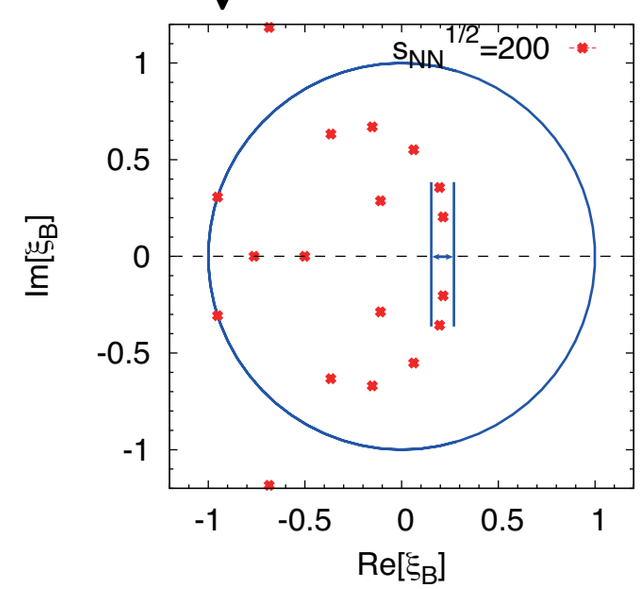
$\sqrt{s} = 39$



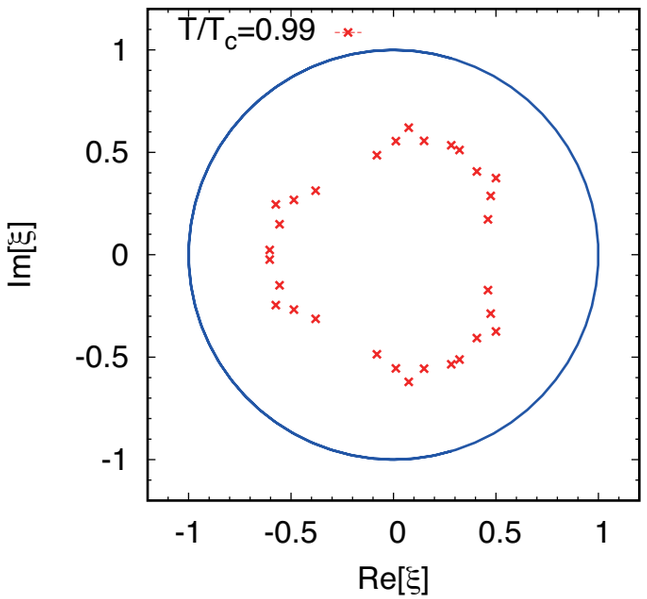
$\sqrt{s} = 62.4$



$\sqrt{s} = 200$

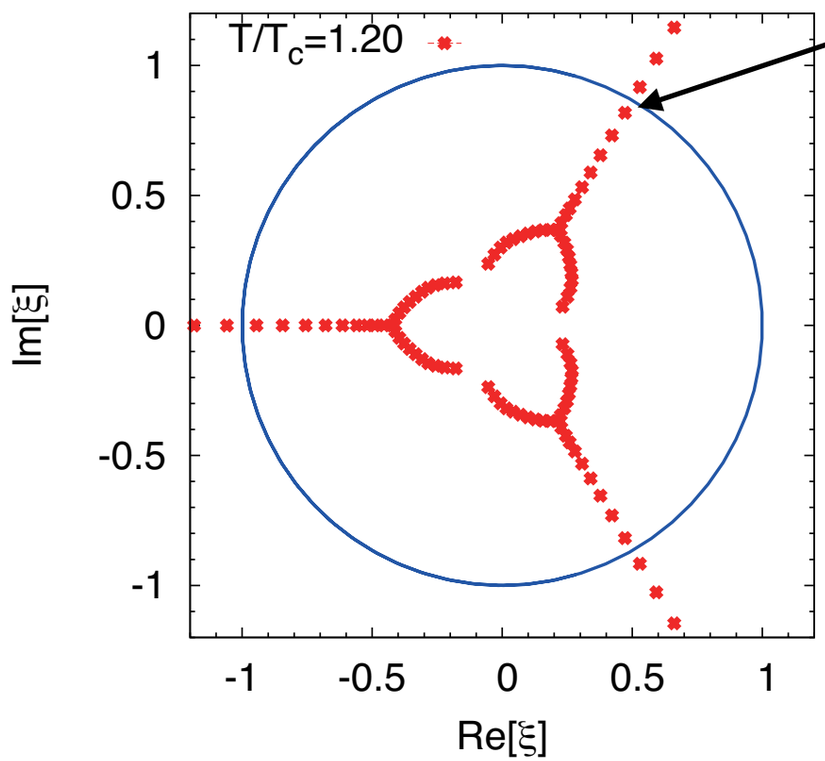


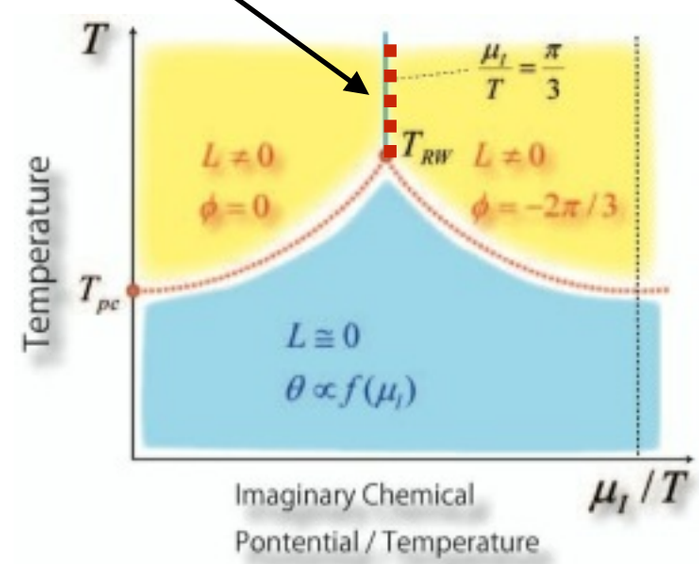
Lattice Data



$\beta = 1.85$ $\qquad$ $T/T_c \sim 0.99$

Roberge-Weise Transition !

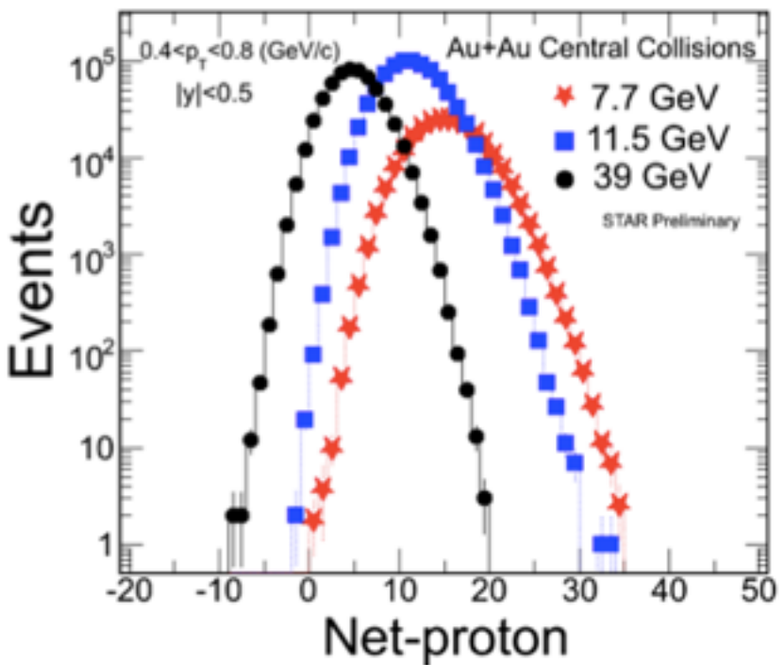




$T/T_c \sim 1.20$

Summary

What are Multiplicity Distributions and How telling us on QCD Phase Diagram ?



Experimental Data

↓

Extract $Z_n(T)$

↓

Construct

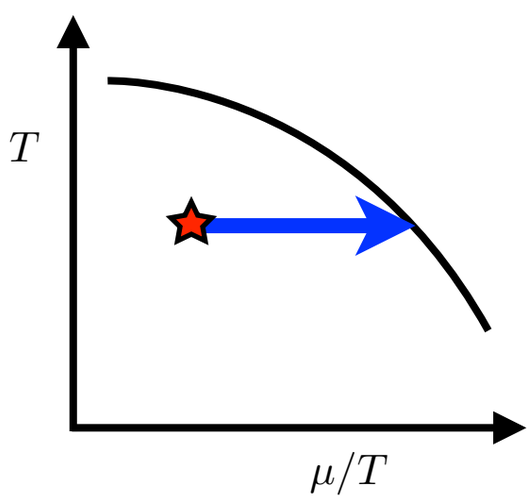
$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

Construct

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

Calculate Moments at $\mu \geq \mu_{\text{Experiment}}$

Construct Lee-Yang Zeros



The current Net-Proton data is a Test-Bed. But even they suggest the phase boundary.