What are Multiplicity Distributions telling us on QCD Phase Diagram?
We can get Canonical Partition Functions both from Experiments and Lattice Simulations
Why $Z^n$ useful?
Yes, very useful, because

\[ Z(\xi, T) = \sum_n Z_n(T) \xi^n \]

(\(\xi \equiv e^{\mu/T} : \text{Fugacity}\))

\[ Z_n(T) \]

at some \(\xi\) and \(T\)

\[ Z(\xi, T) \]

at ANY \(\xi\)

for both Experiments and Lattice
(Current) Weak Points

1) Experimental Multiplicity Data
Net-Proton and **Not** net-Baryon

One can prove \[ Z(\xi, T) = \sum_n Z_n(T) \xi^n \]
only for Conserved Quantities.

Possible approaches:

i) Wait for Net-Baryon data, or Net-Charge data.

ii) Study and analyze data assuming \[ Z_n^{\text{Baryon}} \sim Z_n^{\text{Proton}} \]
2) $N_{max}$ is not very large.

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T) \xi^n$$

Lower estimation of larger density contribution.
\[ Z(\mu, T) \leftrightarrow Z_n(T) \]

**Grand Canonical**

**Canonical**

\[ Z(\mu, T) = \text{Tr} \ e^{-(H-\mu \hat{N})/T} \]

If \( [H, \hat{N}] = 0 \)

\[ = \sum_n \langle n | e^{-(H-\mu \hat{N})/T} | n \rangle \]

\[ = \sum_n \langle n | e^{-H/T} | n \rangle e^{\mu n/T} \]

\[ = \sum_n Z_n(T) \xi^n \]

\((\xi \equiv e^{\mu/T})\)

**Fugacity**
How can we extract $Z_n$ from multiplicity events?

$$P_n = Z_n \xi^n$$

$\xi$ unknown

$$Z_n = P_n / \xi^n$$

We require

$$Z_{+n} = Z_{-n}$$

(Particle-AntiParticle Symmetry)
Demand \[ Z^+ n = Z^- n \]

\[ \sqrt{s} = 62.4 \]

\[ Z_n = P_n / \xi^n \]
Fitted $\xi$ are very consistent with those by Freeze-out Analysis.

\[(\xi \equiv e^{\mu/T}) \sqrt{s} \text{ GeV}\]

This work
J.Cleymans, H.Oeschler, K.Redlich and S.Wheaton
$Z_n$ from RHIC data

$\sqrt{s} = 19.6\text{GeV}$

$\sqrt{s} = 27\text{GeV}$

$\sqrt{s} = 39\text{GeV}$

$\sqrt{s} = 62.4\text{GeV}$

$\sqrt{s} = 200\text{GeV}$

Can I see Difference?

Yes, You Can! We will see it.
We can calculate $\mathbb{Z}_n$ also by Lattice QCD.
Lattice Data

Can I see Difference?

Zn

Yes, You Can!
Wait a moment.
\[ Z(\xi, T) = \sum_n Z_n(T) \xi^n \]

\[ \xi \equiv e^{\mu/T} \]

Is this useful?

Yes, because

1) We can calculate \( Z \) at any \( \xi \) (i.e., \( \mu \) )

2) We can calculate \( Z \) even at complex \( \xi \)
Moments $\lambda_k$

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T) \xi^n$$

$$\lambda_k \equiv \left( T \frac{\partial}{\partial \mu} \right)^k \log Z$$
Susceptibility as a function of $\mu/T$

RHIC Data

I can see beyond $\mu_{Exp}$

$\sqrt{s} = 19.6$

$\sqrt{s} = 27$

$\sqrt{s} = 39$

$\sqrt{s} = 62.4$

$\sqrt{s} = 200$

Observed here
RHIC Data

\[ R42 \quad S_{NN}^{1/2} = 11.5 \]

\[ \sqrt{s} = 11.5 \]

Kurtosis \( \frac{\lambda_4}{\lambda_2} \) as a function of \( \frac{\mu}{T} \)

\[ R42, s_{NN}^{1/2} = 19.6 \]

\[ \sqrt{s} = 19.6 \]

\[ R42, s_{NN}^{1/2} = 27 \]

\[ \sqrt{s} = 27 \]

\[ R42, s_{NN}^{1/2} = 39 \]

\[ \sqrt{s} = 39 \]

\[ R42, s_{NN}^{1/2} = 62.4 \]

\[ \sqrt{s} = 62.4 \]

\[ R42, s_{NN}^{1/2} = 200 \]

\[ \sqrt{s} = 200 \]
Lee-Yang Zeros \quad (1952)

Zeros of $Z(\xi)$ in **Complex Fugacity Plane**.

\[ Z(\xi, T) = \sum_{n=-N_{\text{max}}}^{+N_{\text{max}}} Z_n(T) \xi^n \]

Great Idea to investigate a Statistical System
Lee-Yang Zeros
Experimental Data (RHIC)
Lee-Yang Zeros: RHIC Experiments

\[ \sqrt{s} = 11.5 \]

\[ \sqrt{s} = 19.6 \]

\[ \sqrt{s} = 27 \]

\[ \sqrt{s} = 39 \]

\[ \sqrt{s} = 62.4 \]

\[ \sqrt{s} = 200 \]
\[ \beta = 1.85 \quad T/T_c \sim 0.99 \]

Roberge-Weise Transition!

\[ T/T_c \sim 1.20 \]
Summary
What are Multiplicity Distributions telling us on QCD Phase Diagram?

Experimental Data

Extract $Z_n(T)$

Construct

$$Z(\xi, T) = \sum_{n} Z_n(T) \xi^n$$
Construct

\[ Z(\xi, T) = \sum_{n} Z_n(T) \xi^n \]

Calculate Moments at \( \mu \geq \mu_{\text{Experiment}} \)

Construct Lee-Yang Zeros

The current Net-Proton data is a Test-Bed. But even they suggest the phase boundary.