Nuclear matter with the functional renormalization group

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Nuclear Matter

Why **high densities** are interesting:

- Existence of a **chiral critical endpoint** (chiral CEP)?
- If it exists, is it close to the **liquid-gas phase transition** of nuclear matter?
- Quantitative behavior of the **liquid-gas transition**? Critical temperature/density?
- Equation of state (EoS) of isospin **symmetric** nuclear matter?
- Behavior of isospin **asymmetric** nuclear matter?
- EoS of **pure neutron matter**?
- What happens in the interior of a **neutron star**?
  Two solar-mass neutron stars were observed [1, 2] $\rightarrow$ EoS has to be **sufficiently stiff**
  Does this exclude quark stars, hyperons, kaon condensates?

Study dense matter in a **chiral nucleon-meson model** around the **liquid-gas phase transition!**

Chiral nucleon-meson model

Matter at high density/low temperature described by nuclear physics
$\rightarrow$ chiral **nucleon-meson model** based on the linear sigma model.

Lagrangian [3–7]:

$$\mathcal{L} = \bar\psi i\gamma_\mu\partial^\mu\psi + \frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \frac{1}{4}F^{(\omega)}_{\mu\nu}F^{(\omega)\mu\nu} - \frac{1}{4}\boldsymbol{F}^{(\rho)}_{\mu\nu}\cdot\boldsymbol{F}^{(\rho)\mu\nu}$$
$$- \bar\psi\Big[g(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}) + \gamma_\mu(g_\omega\,\omega^\mu + g_\rho\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu)\Big]\psi$$
$$+ \frac{1}{2}m_\omega^2\,\omega_\mu\,\omega^\mu + \frac{1}{2}m_\rho^2\,\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu - \mathcal{U}(\sigma,\boldsymbol{\pi}),$$

Contains protons, neutrons, vector-mesons, a scalar field and pions.

Mean field

Exchange bosons acquire expectation values:

$$\sigma \quad\rightarrow\quad m_{\text{eff}} = g_s\sigma\,,$$
$$\omega_0, \rho_0^3 \quad\rightarrow\quad \mu^{\text{eff}}_{n,p} = \mu_{n,p} - g_\omega\omega_0 \pm g_\rho\rho_0^3\,.$$

Mean field potential (without bosonic fluctuations):

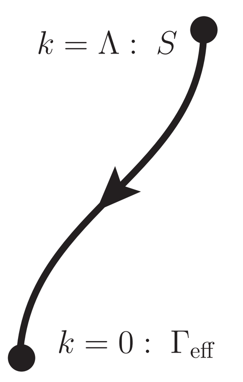
$$U^{\text{MF}} = -2\sum_{\substack{i=n,p\\ r=\pm1}}\int\frac{d^3p}{(2\pi)^3}\,\frac{p^2}{3E}\,\frac{1}{\mathrm{e}^{(E-\mu_i^{\text{eff}})/T}+1} + \sum_{n=1}^{4}\frac{a_n}{n!}(\sigma^2 - f_\pi^2)^n$$
$$- m_\pi^2 f_\pi(\sigma - f_\pi) - \frac{1}{2}m_\omega^2{\omega_0}^2 - \frac{1}{2}m_\rho^2(\rho_0^3)^2\,.$$

Free **parameters** are fitted to nuclear matter properties: compressibility, binding energy, saturation density $n_0 = 0.16$ fm^{-3}, nuclear surface tension, symmetry energy.
Potential is expanded around **liquid-gas phase transition** at $T = 0$ and $\mu = 923$ MeV.

Functional renormalization group

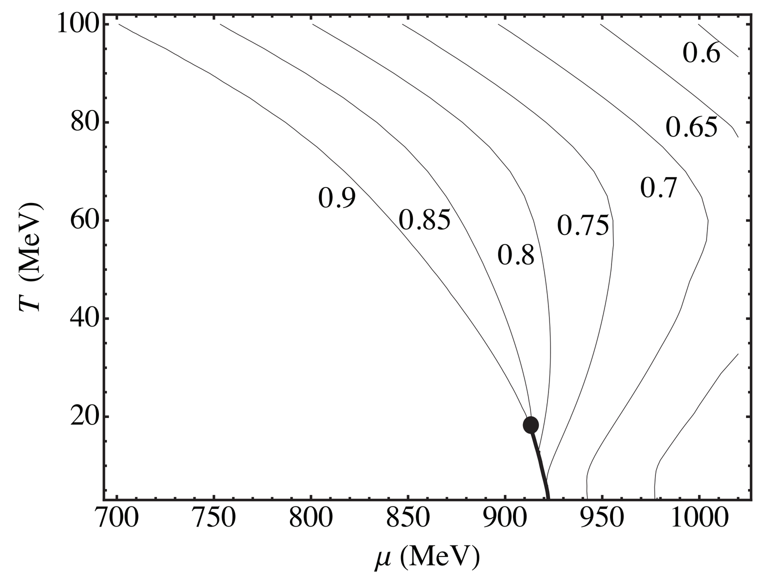
Include **bosonic fluctuations** with the **functional renormalization group (FRG)**.
Compute **effective action** Γ_k depending on RG scale k. Flow Equation [8]:

$$\partial_k\Gamma_k[\Phi_k] = \frac{1}{2}\mathrm{Tr}\;\partial_k R_k\Big(\Gamma_k^{(2)}[\Phi_k] + R_k\Big)^{-1}$$
$$= \frac{1}{2}\;\otimes\!\!\bigcirc\!\!\bullet\,.$$

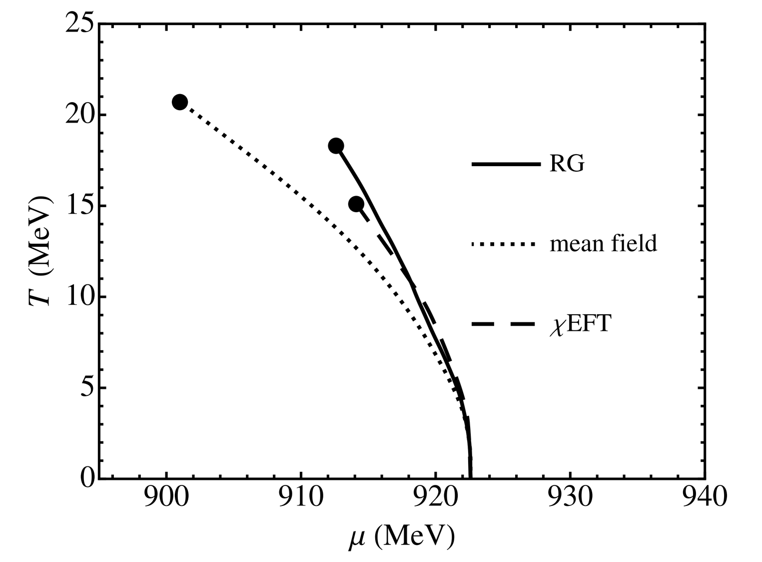


Interpolation between **UV-action** S at $k = \Lambda_{\text{UV}}$ and **full effective action** Γ_{eff} at $k = 0$.
Include **thermal fluctuations** (of bosons and nucleons) around the potential at the liquid gas phase transition [9].

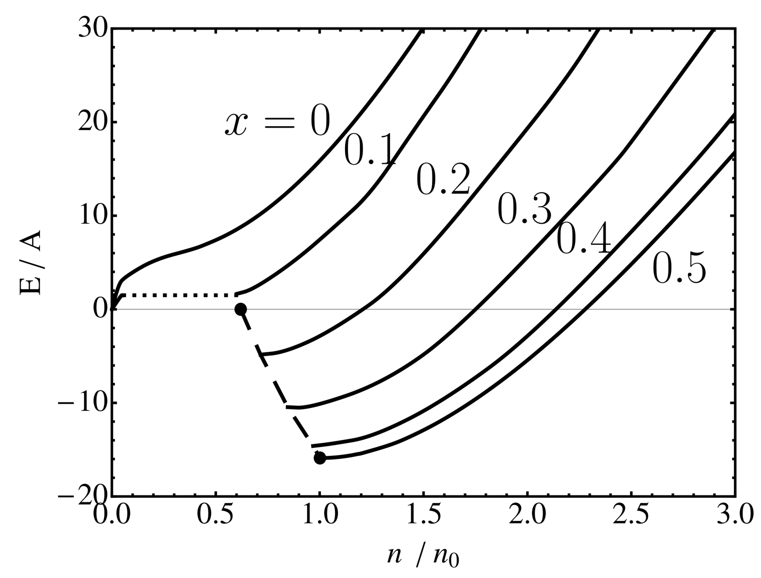
Results



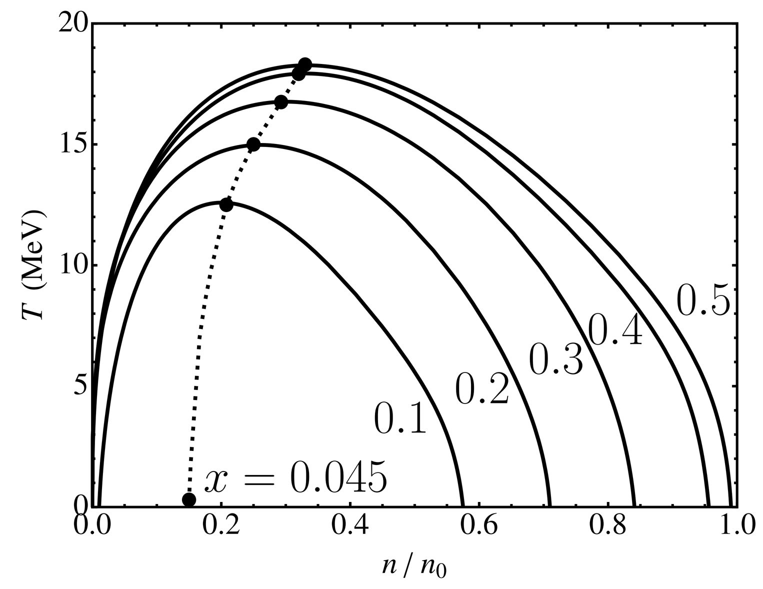
(a) Plateau lines of the chiral condensate σ/f_π. Chiral symmetry is not restored in the region showed above, so the chiral CEP is farther away from the liquid gas transition.



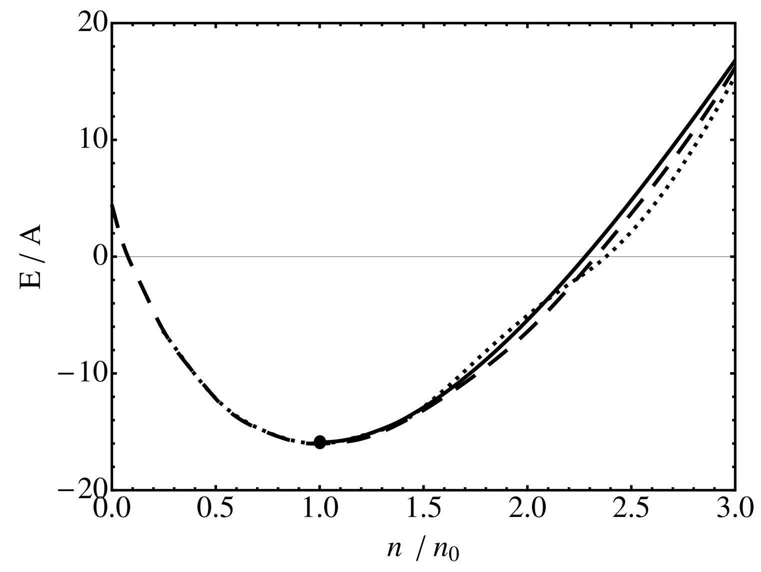
(b) Liquid gas transition: FRG (full line), mean field (dotted line), ChEFT (dashed, [10]). Critical temperature $T = 18.3$ MeV in good agreement with empirical data ($T = 17.9 \pm 0.4$ MeV, [11]).



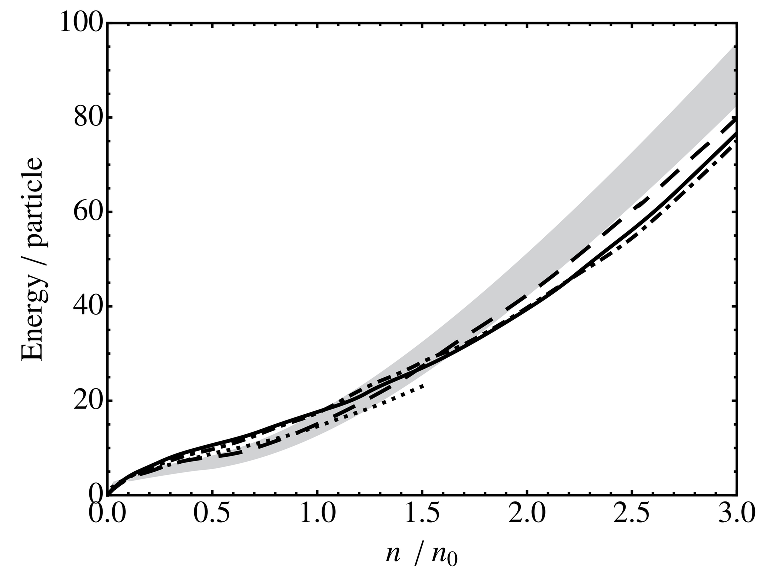
(c) EoS for different proton fractions $x = n_p/(n_p + n_n)$ at $T = 0$. Dashed curve: absolute minimum. Dotted line, $x = 0.1$: Maxwell construction.



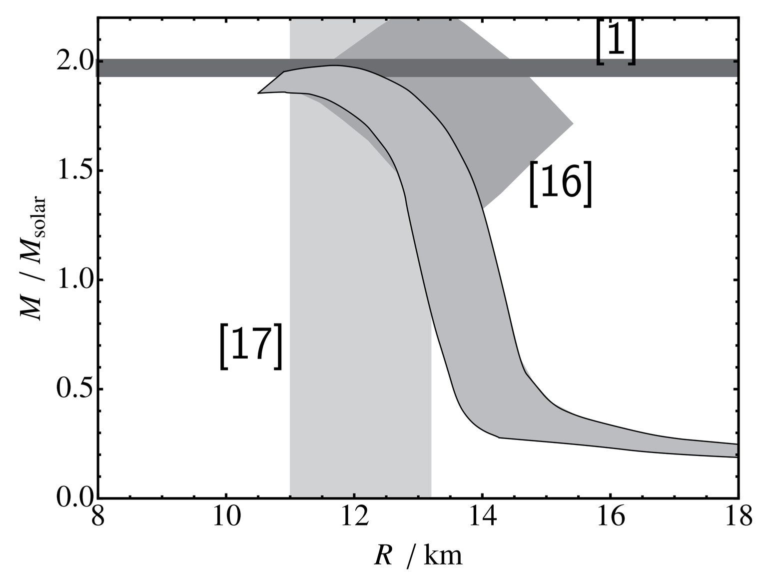
(d) Liquid-gas coexistence regions for different proton fractions x. The transition disappears for $x = 0.045$.



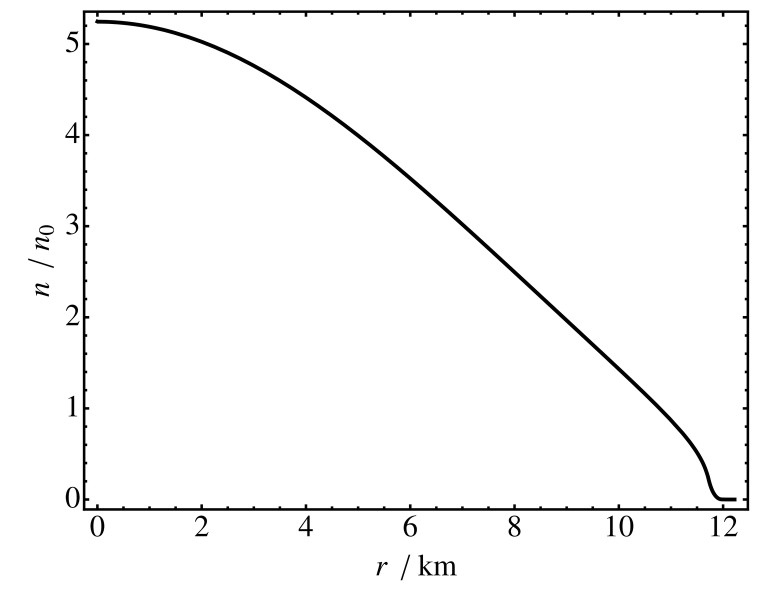
(e) EoS of symmetric nuclear matter: FRG (solid line), APR (dotted, [12]), QMC (dashed, [13]



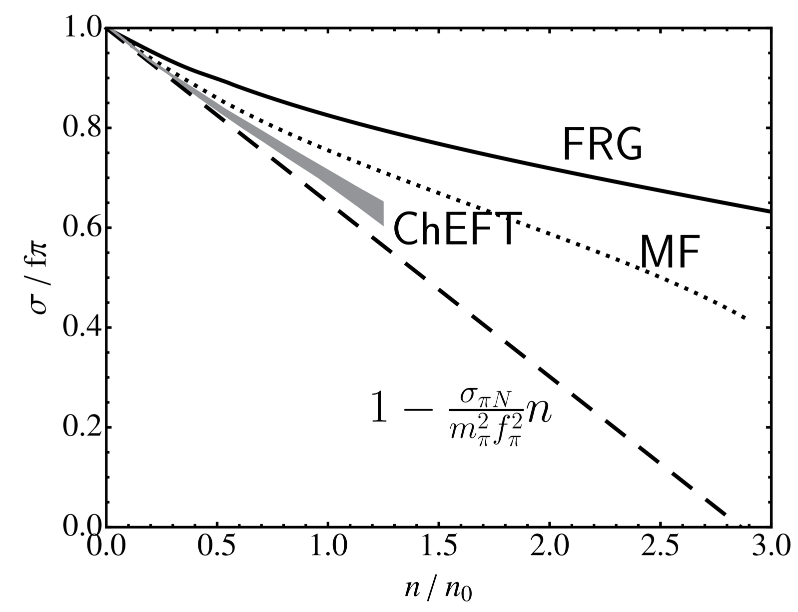
(f) EoS of pure neutron matter. FRG (gray band, with 29 MeV $\leq E_{\text{sym}} \leq 33$ MeV), ChEFT (full line, [14]), QMC based on realistic potentials (dashed, [13]), QMC based on chiral potentials (dotted, [15]), APR (dashed-dotted, [12])



(g) Mass radius relation for neutron star matter. Oberservational constraints: 2 $M_\odot$ neutron stars [1, 2], mass-radius constraints [16, 17]



(h) Density profile for a neutron star with mass $M = 1.97\ M_\odot$ and $R = 12.2$ km for $G_\rho = 1.46$ fm^{-2}



(i) Chiral condensate for pure neutron matter. Chiral symmetry still unbroken at large densities.

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